The nilpotent Lie rings of order p^k for $k \leq 7$

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1 Introduction

We give presentations for the nilpotent Lie rings of order p^k for $k \leq 7$, valid for all $p > 3$. The presentations give a complete classification of these Lie rings. The class 2 presentations also give a classification for the class 2 Lie rings of order 3^k for $k \leq 7$. (On a historical note, the classification was originally obtained with the aim of classifying the groups of order p^k for $k \leq 7$ via the Baker-Campbell-Hausdorff formula. See [2] and [4]. The Baker-Campbell-Hausdorff formula only applies for p-groups of class at most $p-1$, and so computing the nilpotent Lie rings of order 2^k was of no use in this project, and computing the nilpotent Lie rings of order 3^k was only useful in this project for classifying 3-groups of nilpotency class 2.)

Our method of classifying nilpotent Lie rings of order p^n closely follows the p-group generation algorithm (see Newman [1] and O'Brien [3]). A Lie ring L is an abelian group under $+$ (addition) together with a bilinear product which satisfies

$$\begin{aligned} aa &= 0 \text{ for all } a \in L, \\ (ab)c + (bc)a + (ca)b &= 0 \text{ for all } a, b, c \in L. \end{aligned}$$

(We use ab to denote the Lie product of a and b, rather than the more familiar $[a, b]$.) Note that the axiom $aa = 0$ together with bilinearity implies that $ba = -ab$. The identity $(ab)c + (bc)a + (ca)b = 0$ is the Jacobi identity. Since the Lie product is not associative the bracketing of a product is significant. We adopt the left-normed convention whereby

$$a_1 a_2 \ldots a_n = (\ldots((a_1 a_2)a_3) \ldots a_{n-1})a_n.$$

For a Lie ring L, we define the *lower p-central series*

$$L = L_1 \geq L_2 \geq L_3 \geq \ldots \geq L_c \geq \ldots$$

in an analogous way to groups. We set $L_1 = L$, $L_2 = L^2 + pL$, and for $c > 1$ we set $L_{c+1} = L_c L + pL_c$. (Here $L^2 = \langle ab \,|\, a, b \in L \rangle$, and $L_c L = \langle ab \,|\, a \in L_c,\, b \in L \rangle$.) The ideal L_c consists of all linear combinations of terms of the form

$$a_1 a_2 \ldots a_c,\, p a_1 a_2 \ldots a_{c-1},\, p^2 a_1 a_2 \ldots a_{c-2}, \ldots,\, p^{c-1} a_1,$$

with $a_i \in L$. We say that L has p-class c if $L_{c+1} = \{0\}$, $L_c \neq \{0\}$. If L is a nilpotent Lie ring with finite order p^n for some prime p, then L_{c+1} will equal $\{0\}$ for some c. In fact if L is nilpotent of class k, and if the exponent of L as a finite abelian group is p^m then L has p-class c for some c with $k \leq c < k + m$.

If L and M are two finite nilpotent Lie rings with prime-power order, then L is a *descendant* of M if $L/L_c \cong M$ for some $c \geq 2$. If $L/L_c \cong M$ and L has p-class c (so that $L_c \neq \{0\}$, $L_{c+1} = \{0\}$) then L is an *immediate descendant* of M. Note that if L is a descendant of M then $L/L_2 \cong M/M_2$, so that L and M have the same generator number.

The key idea for calculating the nilpotent Lie rings L with order p^n is as follows. If L has p-class 1 then L is the direct sum of n copies of $\mathbb{Z}_p$; call L *elementary abelian*. If L has p-class $c > 1$, then L is an immediate descendant of L/L_c, which is a nilpotent Lie ring with order p^m for some $m < n$. To compute the nilpotent Lie rings of order p^n, first compute the nilpotent Lie rings of order p^k for $k < n$, and for each of these Lie rings compute their immediate descendants of order p^n. A method for doing this is described in [2]. Our presentations are grouped according to their immediate ancestors.

In their simplest form, the presentations below are fairly standard. For example, here is a presentation for one of the nilpotent Lie rings of order p^4:

$$\langle a, b \,|\, bab,\ pa,\ pb - baa,\ \text{class } 3\rangle. \tag{4.11}$$

Note that we specify the lower p-central class of the Lie ring as well as giving generators and relations. So if L is the Lie ring with presentation $\langle a, b \,|\, bab,\ pa,\ pb - baa\rangle$, algebra 4.11 is L/L_4. One key property of these presentations is that the prime p is symbolic. If we substitute the prime 5 for p in this presentation then we get a presentation for a nilpotent Lie ring of order 5^4. And if we substitute 7 for p in this presentation, then we get a presentation for a nilpotent Lie ring of order 7^4. However many of the presentations are more complicated than this, and involve parameters. For example algebra 4.12 has presentation

$$\langle a, b \,|\, bab,\ pa,\ pb - \omega baa,\ \text{class } 3\rangle. \tag{4.12}$$

The parameter ω occurs in hundreds of the presentations below, and is assumed throughout to be a primitive element modulo p. If you want to use the presentations to generate a complete and irredundant list of the nilpotent Lie rings of order p^k for some $k \leq 7$ and some specific prime p, then it is important that you choose a primitive element modulo p at the outset, and use this one value for ω throughout. Actually, this is not strictly necessary, but if you use different values for ω in different presentations, then no guarantees are given!

It is perhaps worthwhile to take a moment to explore algebras 4.11 and 4.12 in more detail. Consider a presentation

$$\langle a, b \,|\, bab,\ pa,\ pb - xbaa,\ \text{class } 3\rangle$$

where x is an integer. Then (for any given p) this presentation defines a nilpotent Lie ring L_x of order p^4. Since L_x has class 3, and since $baa \in L_3$, the element baa has additive order p, and so L_x is isomorphic to L_y whenever $x = y \bmod p$. It turns out (in this example) that if x and y are non-zero modulo p, then L_x is isomorphic to L_y if and only if $\frac{x}{y}$ is a square modulo p. So (for $p > 2$) there are two isomorphism classes of algebras L_x with $x \neq 0 \bmod p$: one where x is a square modulo p, and one where x is not a square modulo p. Taking $x = 1, \omega$ we obtain representatives for these two isomorphism classes. (In this case, ω does not need to be a primitive element modulo p, and any integer which is not a square modulo p would do.)

Many presentations involve parameters other than ω. Here are two simple examples for Lie rings of order p^6 and p^7:

$$\langle a, b, c \,|\, pa - ba,\ pb - cb,\ pc - xba - ca,\ \text{class } 2\rangle \ (\text{all } x) \tag{6.114}$$

$$\langle a, b \,|\, baaaa, baab - baaab, babb - baaab, pa - xbaaab, pb - ybaaab,\ \text{class } 5\rangle \ (\text{all } x, y) \tag{7.351}$$

Presentation 6.114 has a single parameter x, and since x appears as the coefficient of ba, and since the algebra has class 2, it is clear that two parameters x and x' give isomorphic algebras if $x = x' \bmod p$. The somewhat cryptic comment "all x" signifies that this is the only restriction on the parameter x, and that there are p distinct isomorphism classes of algebras with presentation 6.114, with x and x' giving isomorphic algebras if and only if $x = x' \bmod p$. This means that you can obtain p non-isomorphic algebras by taking $x = 0, 1, \ldots, p-1$. However, sometimes you might want to make a different choice of parameters. For example, it might be that the algebra is terminal unless $x = -1 \bmod p$. In that case you might want to take $x = -1, 0, 1, \ldots, p-2$ as a representative set of parameters. Similarly, the comment "all x, y" in presentation 7.351 indicates that this family of presentations gives p^2 non-isomorphic algebras. Parameters x, y and x', y' give isomorphic algebras if and only if $x = x' \bmod p$ and $y = y' \bmod p$. Would that things were always this simple! Often the parameters are given a limited range, such as "$x \neq 0$" or "all x with $1 + 4x$ not a square mod p". Here $x \neq 0$ is to be understood to mean $x \neq 0 \bmod p$. More generally, if we have a one parameter family of Lie rings L_x then we define an equivalence relation $\sim$ on the parameters x by setting $x \sim y$ if L_x is isomorphic to L_y. It is to be understood that $x \sim y$ whenever $x = y \bmod p$, but the comments associated with the presentations below often define slightly more complex equivalence relations. To save space the comments are rather cryptic. Here are some examples:

$$x \sim x' \text{ if } x^3 = x'^3 \bmod p,$$

$$x \sim x' \text{ if } x^4 = x'^4 \bmod p,$$

$$x \sim -x,$$

$$x \neq 0,\ x \sim x^{-1}.$$

The first two examples are clear enough. In the third example the cryptic comment $x \sim -x$ is intended to mean that $x \sim y$ if and only if $x = \pm y \bmod p$. In the fourth example, we restrict to parameters x with $x \neq 0 \bmod p$, and $x \sim y$ if and only if $x = y \bmod p$ or $xy = 1 \bmod p$. The simplest way of thinking about these equivalence relations is to think of them as being defined on subsets of the field $\mathbb{Z}_p$, and if you if you want to compute a set of representatives for the equivalence classes for a particular prime p, it is often easiest to compute in $\mathbb{Z}_p$. But the nevertheless, the parameters take values in the integers. This can be confusing, but the same applies in the p-group generation algorithm — the isomorphism problem for the immediate descendants of a finite p-group is solved over $\mathbb{Z}_p$, but the presentations for the descendants have integer exponents.

The presentations for the nilpotent Lie rings of order p^k for $k \leq 4$ do not involve parameters, apart from the parameter ω which is taken to be a primitive element modulo p whenever it appears. The presentations for the nilpotent Lie rings of order p^5 involve at most one parameter other than ω. But many of the presentations for the nilpotent Lie rings of order p^6 and p^7 involve two or more parameters other than ω. Just as with a single parameter, we associate a comment with these presentations to describe the restrictions on the values that the parameters can take, and to describe the equivalence relation $\sim$ associated with the isomorphism problem. Here are a few examples:

$$\text{all } x,\, y,\, (x,\, y) \sim (y,\, x),$$

$$x \neq 0,\, y \neq 0,\, 1, -1,\; y \sim y^{-1},$$

$$x \neq 0,\; \text{all } y, z,\; x \sim x' \text{ if } x^4 = x'^4 \bmod p.$$

In the last of these examples we have a presentation with three parameters x, y, z, and for any choice of y, z the algebra defined by parameters x, y, z is isomorphic to the algebra defined by parameters x', y, z if $x^4 = x'^4 \bmod p$.

In a few cases I have been unable to give simply defined equivalence relations on the parameter sets, and I need to give more complicated definitions. See for example algebra 6.62 below.

There is one further type of comment attached to many of the presentations — some of the presentations only apply for certain residue classes modulo p. A comment "$p = 1 \bmod 3$", for example, implies that the presentation should only be included for the prime p if $p = 1 \bmod 3$.

The classification given here has been criticized on the grounds that to produce a complete and irredundant set of Lie rings for any given p you have to do some computations "on the fly". Now, even with a one parameter presentation with restriction $x \neq 0$, $x \sim x^{-1}$, if you want a complete and irredundant set of representatives for the equivalence classes of the parameter x for a given prime p, you have to do a computation on the fly. And one parameter presentations like that are about as good as it gets! So I think the criticism is that there are some presentations with several parameters, and quite complicated equivalence relations. There are

$$3p^2 + 39p + 344 + 24\gcd(p-1,3) + 11\gcd(p-1,4) + 2\gcd(p-1,5)$$

nilpotent Lie rings of order p^6 for every prime $p \geq 5$, and so there have to be some presentations for the Lie rings of order p^6 involving at least two parameters (as well as possibly involving the parameter ω). The presentations for the Lie rings of order p^6 involve at most 2 parameters (other than ω) in all except one case. The one exception is algebra 6.178 which has a presentation involving four parameters. Actually one of the four parameters can be restricted to values $0, 1$, and so this four parameter presentation quickly reduces to two three parameter presentations. For $p \geq 5$ the number of nilpotent Lie rings of order p^7 is

$$\begin{aligned}
&3p^5 + 12p^4 + 44p^3 + 170p^2 + 707p + 2455 \\
&+(4p^2 + 44p + 291)\gcd(p-1,3) + (p^2 + 19p + 135)\gcd(p-1,4) \\
&+(3p + 31)\gcd(p-1,5) + 4\gcd(p-1,7) + 5\gcd(p-1,8) + \gcd(p-1,9),
\end{aligned}$$

and so there have to be presentations for Lie rings of order p^7 involving at least 5 parameters. The ideal, I suppose, would be a number of presentations with at most five parameters, and with no restrictions on the parameters. There is no reason to suppose that this ideal is achievable, and even if it were it would still take p^5 work to list all the different algebras given by a five parameter presentation. If you had a five parameter presentation with anything but the simplest restrictions on the parameters, and anything but the simplest equivalence relation solving the isomorphism problem, it could easily take more than p^5 work to list them all.

So the real issue is the amount of work required to produce a complete list of algebras of order p^7 for a given p. Ideally we would not need more than p^5 work, and perhaps that is achievable. But I have been unable to reach that goal in every case. The amount of work required to produce a complete list of algebras for any given prime p for any of the parametrized presentations in the classification is of order at most p^5, except in one case. The one exception is presentation 7.3285. This is a 12 parameter presentation for (some) immediate descendants of algebra 4.1. The equivalence relation defined on these parameters to solve the isomorphism problem is the most complicated of all the conditions appearing in this classification. It is fairly easy to see that there are 11 orbits for the values of the first 6 parameters, so that the problem of finding a set of representatives for the 12 parameters quickly reduces to 11 problems with 6 parameters. I have a program which computes a set of representatives for the orbits with work of order somewhere between p^6 and p^7. For primes up to 23 the time taken to run the program is roughly proportional to $p^{6.2}$. It would be nice to do better than this, as this particular presentation creates a serious bottleneck in producing a complete list of Lie rings of order p^7 for any p greater than 5 or 7.

2 Order p

$$\langle a \,|\, \text{class } 1\rangle \tag{1.0}$$

3 Order p^2

$$\langle a \,|\, \text{class } 2\rangle \tag{2.0}$$

$$\langle a, b \,|\, \text{class } 1\rangle \tag{2.1}$$

4 Order p^3

$$\langle a \,|\, \text{class } 3\rangle \tag{3.0}$$

$$\langle a, b, c \,|\, \text{class } 1\rangle \tag{3.1}$$

$$\langle a, b \,|\, pa,\, pb,\, \text{class } 2\rangle \tag{3.2}$$

$$\langle a, b \,|\, pa - ba,\, pb,\, \text{class } 2\rangle \tag{3.3}$$

$$\langle a, b \,|\, ba,\, pb,\, \text{class } 2\rangle \tag{3.4}$$

5 Order p^4

$$\langle a \,|\, \text{class } 4\rangle \tag{4.0}$$

$$\langle a, b, c, d \,|\, \text{class } 1\rangle \tag{4.1}$$

$$\langle a, b, c \,|\, ba\,, ca,\, cb,\, pb,\, pc,\, \text{class } 2\rangle \tag{4.2}$$

$$\langle a, b, c \,|\, ca,\, cb,\, pa,\, pb,\, pc,\, \text{class } 2\rangle \tag{4.3}$$

$$\langle a, b, c \,|\, ca,\, cb,\, pa - ba,\, pb,\, pc,\, \text{class } 2\rangle \tag{4.4}$$

$$\langle a, b, c \,|\, ca,\ cb,\ pa,\ pb,\ pc - ba,\ \text{class } 2\rangle \tag{4.5}$$

$$\langle a, b \,|\, ba,\ \text{class } 2\rangle \tag{4.6}$$

$$\langle a, b \,|\, pb,\ \text{class } 2\rangle \tag{4.7}$$

$$\langle a, b \,|\, pb - ba,\ \text{class } 2\rangle \tag{4.8}$$

$$\langle a, b \,|\, bab,\ pa,\ pb,\ \text{class } 3\rangle \tag{4.9}$$

$$\langle a, b \,|\, bab,\ pa - baa,\ pb,\ \text{class } 3\rangle \tag{4.10}$$

$$\langle a, b \,|\, bab,\ pa,\ pb - baa,\ \text{class } 3\rangle \tag{4.11}$$

$$\langle a, b \,|\, bab,\ pa,\ pb - \omega baa,\ \text{class } 3\rangle \tag{4.12}$$

$$\langle a, b \,|\, ba,\ \ pb,\ \text{class } 3\rangle \tag{4.13}$$

$$\langle a, b \,|\, ba - p^2 a,\ \ pb,\ \text{class } 3\rangle \tag{4.14}$$

6 Order p^5

There are $2p + 61 + 2\gcd(p-1, 3) + \gcd(p-1, 4)$ nilpotent Lie rings of order p^5 for $p \geq 5$. We first list the one generator and five generator Lie rings of order p^5, and then we group the rest according to their immediate ancestors.

$$\langle a \,|\, \text{class } 5\rangle \tag{5.0}$$

$$\langle a, b, c, d, e \,|\, \text{class } 1\rangle. \tag{5.1}$$

6.1 Descendants of 4.1

$$\langle a, b, c, d \,|\, ba, ca, da, cb, db, dc, pb, pc, pd,\ \text{class } 2\rangle. \tag{5.2}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, dc, pa, pb, pc, pd,\ class2\rangle \tag{5.3}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, dc, pa - ba, pb, pc, pd,\ \text{class } 2\rangle. \tag{5.4}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, dc, pa, pb, pc - ba, pd,\ \text{class } 2\rangle. \tag{5.5}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, dc - ba, pa, pb, pc, pd,\ \text{class } 2\rangle, \tag{5.6}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, dc - ba, pa - ba, pb, pc, pd,\ \text{class } 2\rangle, \tag{5.7}$$

6.2 Descendants of 3.1

$$\langle a,b,c \,|\, ba,\ ca,\ cb,\ pc,\ \text{class } 2\rangle. \tag{5.8}$$
$$\langle a,b,c \,|\, ca,\ cb,\ pb,\ pc,\ \text{class } 2\rangle, \tag{5.9}$$
$$\langle a,b,c \,|\, ca,\ cb,\ pb-ba,\ pc,\ \text{class } 2\rangle. \tag{5.10}$$
$$\langle a,b,c \,|\, ca,\ cb,\ pb,\ pc-ba,\ \text{class } 2\rangle. \tag{5.11}$$
$$\langle a,b,c \,|\, ca,\ cb,\ pa,\ pb,\ \text{class } 2\rangle, \tag{5.12}$$
$$\langle a,b,c \,|\, ca,\ cb,\ pa-ba,\ pb,\ \text{class } 2\rangle. \tag{5.13}$$
$$\langle a,b,c \,|\, cb,\ pa,\ pb,\ pc,\ \text{class } 2\rangle, \tag{5.14}$$
$$\langle a,b,c \,|\, cb,\ pa-ba,\ pb,\ pc,\ \text{class } 2\rangle. \tag{5.15}$$
$$\langle a,b,c \,|\, cb,\ pa,\ pb-ba,\ pc,\ \text{class } 2\rangle, \tag{5.16}$$
$$\langle a,b,c \,|\, cb,\ pa-ca,\ pb-ba,\ pc,\ \text{class } 2\rangle, \tag{5.17}$$
$$\langle a,b,c \,|\, cb,\ pa,\ pb-ca,\ pc,\ \text{class } 2\rangle, \tag{5.18}$$
$$\langle a,b,c \,|\, cb,\ pa-ba,\ pb-ca,\ pc,\ \text{class } 2\rangle. \tag{5.19}$$
$$\langle a,b,c \,|\, cb,\ pa,\ pb-ba,\ pc-xca,\ \text{class } 2\rangle\ (x \neq 0,\ x \sim x^{-1}), \tag{5.20}$$
$$\langle a,b,c \,|\, cb,\ pa,\ pb-ba-ca,\ pc-ca,\ \text{class } 2\rangle; \tag{5.21}$$
$$\langle a,b,c \,|\, cb,\ pa,\ pb-\omega ca,\ pc-ba,\ \text{class } 2\rangle; \tag{5.22}$$
$$\langle a,b,c \,|\, cb,\ pa,\ pb-xca,\ pc-ba-ca,\ \text{class } 2\rangle\ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p), \tag{5.23}$$

6.3 Descendants of 4.2

$$\langle a,b,c \,|\, ba,\ ca,\ cb,\ pb,\ pc,\ \text{class } 3\rangle. \tag{5.24}$$

$$\langle a,b,c \,|\, ba,\ ca,\ cb-p^2a,\ pb,\ pc,\ \text{class } 3\rangle. \tag{5.25}$$

$$\langle a,b,c \,|\, ba-p^2a,\ ca,\ cb,\ pb,\ pc,\ \text{class } 3\rangle. \tag{5.26}$$

6.4 Descendants of 4.3

$$\langle a,b,c \,|\, ca,\ cb,\ bab,\ pa,\ pb,\ pc,\ \text{class } 3\rangle, \tag{5.27}$$

$$\langle a,b,c \,|\, ca,\ cb,\ bab,\ pa-baa,\ pb,\ pc,\ \text{class } 3\rangle, \tag{5.28}$$

$$\langle a,b,c \,|\, ca,\ cb,\ bab,\ pa,\ pb-baa,\ pc,\ \text{class } 3\rangle, \tag{5.29}$$

$$\langle a,b,c \,|\, ca,\ cb,\ bab,\ pa,\ pb-\omega baa,\ pc,\ \text{class } 3\rangle. \tag{5.30}$$

$$\langle a,b,c \,|\, ca,\ cb,\ bab,\ pa,\ pb,\ pc-baa,\ \text{class } 3\rangle. \tag{5.31}$$

$$\langle a,b,c \,|\, ca,\ cb-baa,\ bab,\ pa,\ pb,\ pc,\ \text{class } 3\rangle, \tag{5.32}$$

$$\langle a,b,c \,|\, ca,\ cb-baa,\ bab,\ pa-baa,\ pb,\ pc,\ \text{class } 3\rangle, \tag{5.33}$$

$$\langle a,b,c \,|\, ca,\ cb-baa,\ bab,\ pa,\ pb-baa,\ pc,\ \text{class } 3\rangle, \tag{5.34}$$

$$\langle a,b,c \,|\, ca,\ cb-baa,\ bab,\ pa,\ pb-\omega baa,\ pc,\ \text{class } 3\rangle. \tag{5.35}$$

$$\langle a,b,c \,|\, ca,\ cb-baa,\ bab,\ pa,\ pb,\ pc-baa,\ \text{class } 3\rangle. \tag{5.36}$$

6.5 Descendants of 4.4

Algebra 4.4 is terminal.

6.6 Descendants of 4.5

Algebra 4.5 is terminal.

6.7 Descendants of 2.1

$$\langle a, b \,|\, \text{class } 2\rangle. \tag{5.37}$$

6.8 Descendants of 3.2

$$\langle a, b \,|\, pa,\, pb,\, \text{class } 3\rangle. \tag{5.38}$$

$$\langle a, b \,|\, pa - bab,\, pb,\, \text{class } 3\rangle, \tag{5.39}$$

$$\langle a, b \,|\, pa - \omega bab,\, pb,\, \text{class } 3\rangle, \tag{5.40}$$

$$\langle a, b \,|\, pa - baa,\, pb,\, \text{class } 3\rangle. \tag{5.41}$$

$$\langle a, b \,|\, pa - baa,\, pb - xbab,\, \text{class } 3\rangle \ (x \neq 0,\ x \sim x^{-1}), \tag{5.42}$$

$$\langle a, b \,|\, pa - baa - bab,\, pb - bab,\, \text{class } 3\rangle, \tag{5.43}$$

$$\langle a, b \,|\, pa - baa - \omega bab,\, pb - bab,\, \text{class } 3\rangle, \tag{5.44}$$

$$\langle a, b \,|\, pa - \omega bab,\, pb - baa,\, \text{class } 3\rangle; \tag{5.45}$$

$$\langle a, b \,|\, pa - xbab,\, pb - baa - bab,\, \text{class } 3\rangle \ (\text{all } x \text{ with } 1 + 4x \text{ not a square } \bmod p), \tag{5.46}$$

6.9 Descendants of 3.3

Algebra 3.3 is terminal.

6.10 Descendants of 3.4

Algebra 3.4 is terminal.

6.11 Descendants of 4.6

$$\langle a, b \,|\, ba,\, p^2 b,\, \text{class } 3\rangle, \tag{5.47}$$

$$\langle a, b \,|\, ba - p^2 a,\, p^2 b,\, \text{class } 3\rangle. \tag{5.48}$$

6.12 Descendants of 4.7

$$\langle a, b \,|\, baa,\ bab,\ pb,\ \text{class } 3\rangle. \tag{5.49}$$

$$\langle a, b \,|\, bab,\ p^2a,\ pb,\ \text{class } 3\rangle, \tag{5.50}$$

$$\langle a, b \,|\, bab,\ p^2a,\ pb - baa,\ \text{class } 3\rangle, \tag{5.51}$$

$$\langle a, b \,|\, bab,\ p^2a,\ pb - \omega baa,\ \text{class } 3\rangle, \tag{5.52}$$

$$\langle a, b \,|\, bab,\ p^2a - baa,\ pb,\ \text{class } 3\rangle. \tag{5.53}$$

$$\langle a, b \,|\, baa,\ p^2a,\ pb,\ \text{class } 3\rangle, \tag{5.54}$$

$$\langle a, b \,|\, baa,\ p^2a,\ pb - bab,\ \text{class } 3\rangle, \tag{5.55}$$

$$\langle a, b \,|\, baa,\ p^2a - bab,\ pb,\ \text{class } 3\rangle, \tag{5.56}$$

$$\langle a, b \,|\, baa,\ p^2a - \omega bab,\ pb,\ \text{class } 3\rangle. \tag{5.57}$$

6.13 Descendants of 4.8

$$\langle a, b \,|\, baa,\ pb - ba,\ \text{class } 3\rangle. \tag{5.58}$$

$$\langle a, b \,|\, p^2a,\ pb - ba,\ \text{class } 3\rangle. \tag{5.59}$$

6.14 Descendants of 4.9

$$\langle a, b \,|\, bab,\ pa,\ pb,\ \text{class } 4\rangle, \tag{5.60}$$

$$\langle a, b \,|\, bab,\ pa - baaa,\ pb,\ \text{class } 4\rangle. \tag{5.61}$$

$$\langle a, b \,|\, bab,\ pa,\ pb - baaa,\ \text{class } 4\rangle, \tag{5.62}$$

$$\langle a, b \,|\, bab,\ pa,\ pb - \omega baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3), \tag{5.63}$$

$$\langle a, b \,|\, bab,\ pa,\ pb - \omega^2 baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3). \tag{5.64}$$

$$\langle a, b \,|\, bab - baaa,\ pa,\ pb,\ \text{class } 4\rangle, \tag{5.65}$$

$$\langle a, b \,|\, bab - baaa,\ pa - baaa,\ pb,\ \text{class } 4\rangle, \tag{5.66}$$

$$\langle a, b \,|\, bab - baaa,\ pa - \omega baaa,\ pb,\ \text{class } 4\rangle. \tag{5.67}$$

$$\langle a, b \,|\, bab - baaa,\ pa - \omega^2 baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 4), \tag{5.68}$$

$$\langle a, b \mid bab - baaa,\ pa - \omega^3 baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 4). \tag{5.69}$$

$$\langle a, b \mid bab - baaa,\ pa,\ pb - baaa,\ \text{class } 4\rangle, \tag{5.70}$$

$$\langle a, b \mid bab - baaa,\ pa,\ pb - \omega baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3), \tag{5.71}$$

$$\langle a, b \mid bab - baaa,\ pa,\ pb - \omega^2 baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3). \tag{5.72}$$

6.15 Descendants of 4.10

Algebra 4.10 is terminal.

6.16 Descendants of 4.11 and 4.12

Algebras 4.11 and 4.12 are terminal.

6.17 Descendants of 4.13

$$\langle a, b \mid ba,\ pb,\ \text{class } 4\rangle, \tag{5.73}$$

$$\langle a, b \mid ba - p^3 a,\ pb,\ \text{class } 4\rangle. \tag{5.74}$$

6.18 Descendants of 4.14

Algebra 4.14 is terminal.

7 Order p^6

There are

$$3p^2 + 39p + 344 + 24\gcd(p-1,3) + 11\gcd(p-1,4) + 2\gcd(p-1,5)$$

nilpotent Lie rings of order p^6 for every prime $p \geq 5$.

The following table lists the nilpotent Lie rings of order at most p^5 which have immediate descendants of order p^6.

3.1	$\langle a, b, c \mid \text{ class } 1\rangle$
4.1	$\langle a, b, c, d \mid \text{ class } 1\rangle$
4.3	$\langle a, b, c \mid ca, cb, pa, pb, pc, \text{ class } 2\rangle$
4.6	$\langle a, b \mid ba, \text{ class } 2\rangle$
4.7	$\langle a, b \mid pb, \text{ class } 2\rangle$
4.8	$\langle a, b \mid pb - ba, \text{ class } 2\rangle$
5.0	$\langle a \mid \text{ class } 5\rangle$
5.1	$\langle a, b, c, d, e \mid \text{ class } 1\rangle$
5.2	$\langle a, b, c, d \mid ba, ca, da, cb, db, dc, pb, pc, pd, \text{ class } 2\rangle$
5.3	$\langle a, b, c, d \mid ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 2\rangle$
5.8	$\langle a, b, c \mid ba, ca, cb, pc, \text{ class } 2\rangle$
5.9	$\langle a, b, c \mid ca, cb, pb, pc, \text{ class } 2\rangle$
5.10	$\langle a, b, c \mid ca, cb, pb - ba, pc, \text{ class } 2\rangle$
5.11	$\langle a, b, c \mid ca, cb, pb, pc - ba, \text{ class } 2\rangle$
5.12	$\langle a, b, c \mid ca, cb, pa, pb, \text{ class } 2\rangle$
5.13	$\langle a, b, c \mid ca, cb, pa - ba, pb, \text{ class } 2\rangle$
5.14	$\langle a, b, c \mid cb, pa, pb, pc, \text{ class } 2\rangle$
5.15	$\langle a, b, c \mid cb, pa - ba, pb, pc, \text{ class } 2\rangle$
5.16	$\langle a, b, c \mid cb, pa, pb - ba, pc, \text{ class } 2\rangle$
5.18	$\langle a, b, c \mid cb, pa, pb - ca, pc, \text{ class } 2\rangle$
5.19	$\langle a, b, c \mid cb, pa - ba, pb - ca, pc, \text{ class } 2\rangle$
5.24	$\langle a, b, c \mid ba, ca, cb, pb, pc, \text{ class } 3\rangle$
5.27	$\langle a, b, c \mid bab, ca, cb, pa, pb, pc, \text{ class } 3\rangle$
5.32	$\langle a, b, c \mid bab, ca, cb - baa, pa, pb, pc, \text{ class } 3\rangle$
5.37	$\langle a, b \mid \text{ class } 2\rangle$
5.38	$\langle a, b \mid pa, pb, \text{ class } 3\rangle$
5.39	$\langle a, b \mid pa - bab, pb, \text{ class } 3\rangle$
5.40	$\langle a, b \mid pa - \omega bab, pb, \text{ class } 3\rangle$
5.41	$\langle a, b \mid pa - baa, pb, \text{ class } 3\rangle$
5.42	$\langle a, b \mid pa - baa, pb + bab, \text{ class } 3\rangle$
5.45	$\langle a, b \mid pa + bab, pb + \omega baa, \text{ class } 3\rangle$
5.47	$\langle a, b \mid ba, p^2b, \text{ class } 3\rangle$
5.48	$\langle a, b \mid ba - p^2a, p^2b, \text{ class } 3\rangle$
5.49	$\langle a, b \mid baa, bab, pb, \text{ class } 3\rangle$
5.50	$\langle a, b \mid bab, p^2a, pb, \text{ class } 3\rangle$
5.51	$\langle a, b \mid bab, p^2a, pb - baa, \text{ class } 3\rangle$
5.52	$\langle a, b \mid bab, p^2a, pb - \omega baa, \text{ class } 3\rangle$
5.54	$\langle a, b \mid baa, p^2a, pb, \text{ class } 3\rangle$
5.58	$\langle a, b \mid baa, pb - ba, \text{ class } 3\rangle$
5.60	$\langle a, b \mid bab, pa, pb, \text{ class } 4\rangle$
5.65	$\langle a, b \mid bab - baaa, pa, pb, \text{ class } 4\rangle$
5.73	$\langle a, b \mid ba, pb, \text{ class } 4\rangle$

We first give the cyclic and 6 generator Lie rings of order p^6, and then list the remainder, grouped according to their immediate ancestors.

$$\langle a \,|\, \text{class } 6\rangle \tag{6.0}$$

$$\langle a, b, c, d, e, f \,|\, \text{class } 1\rangle \tag{6.1}$$

7.1 Descendants of 3.1

$3p + 27$ algebras

$$\langle a, b, c \,|\, ba,\ ca,\ cb,\ \text{class } 2\rangle \tag{6.85}$$

$$\langle a, b, c \,|\, ca,\ cb,\ pc,\ \text{class } 2\rangle \tag{6.86}$$

$$\langle a, b, c \,|\, ca,\ cb,\ pc - ba,\ \text{class } 2\rangle \tag{6.87}$$

$$\langle a, b, c \,|\, ca,\ cb,\ pa,\ \text{class } 2\rangle \tag{6.88}$$

$$\langle a, b, c \,|\, ca,\ cb,\ pa - ba,\ \text{class } 2\rangle \tag{6.89}$$

$$\langle a, b, c \,|\, cb,\ pb,\ pc,\ \text{class } 2\rangle \tag{6.90}$$

$$\langle a, b, c \,|\, cb,\ pa,\ pc,\ \text{class } 2\rangle \tag{6.91}$$

$$\langle a, b, c \,|\, cb,\ pb - ba,\ pc,\ \text{class } 2\rangle \tag{6.92}$$

$$\langle a, b, c \,|\, cb,\ pb - ca,\ pc,\ \text{class } 2\rangle \tag{6.93}$$

$$\langle a, b, c \,|\, cb,\ pa,\ pc - ba,\ \text{class } 2\rangle \tag{6.94}$$

$$\langle a, b, c \,|\, cb,\ pa,\ pc - ca,\ \text{class } 2\rangle \tag{6.95}$$

$$\langle a, b, c \,|\, cb,\ pa - ba,\ pc,\ \text{class } 2\rangle \tag{6.96}$$

$$\langle a, b, c \,|\, cb,\ pa - ca,\ pc,\ \text{class } 2\rangle \tag{6.97}$$

$$\langle a, b, c \,|\, cb,\ pb - ba,\ pc - xca,\ \text{class } 2\rangle\ (x \neq 0,\ x \sim x^{-1}) \tag{6.98}$$

$$\langle a, b, c \,|\, cb,\ pb - ba - ca,\ pc - ca,\ \text{class } 2\rangle \tag{6.99}$$

$$\langle a, b, c \,|\, cb,\ pb - \omega ca,\ pc - ba,\ \text{class } 2\rangle \tag{6.100}$$

$$\langle a, b, c \,|\, cb,\ pb - xca,\ pc - ba - ca,\ \text{class } 2\rangle\ (\text{all } x \text{ with } 1 + 4x \text{ not a square } \bmod p) \tag{6.101}$$

$$\langle a, b, c \,|\, cb,\ pa - ca,\ pc - ba,\ \text{class } 2\rangle \tag{6.102}$$

$$\langle a, b, c \,|\, cb,\ pa - ba,\ pc - ca,\ \text{class } 2\rangle \tag{6.103}$$

$$\langle a, b, c \,|\, pa,\ pb,\ pc,\ \text{class } 2\rangle \tag{6.104}$$

$$\langle a, b, c \,|\, pa - cb,\ pb,\ pc,\ \text{class } 2\rangle \tag{6.105}$$

$$\langle a, b, c \,|\, pa - ba,\ pb,\ pc,\ \text{class } 2\rangle \tag{6.106}$$

$$\langle a, b, c \,|\, pa - ca,\ pb - xcb,\ pc,\ \text{class } 2\rangle\ (x \neq 0,\ x \sim x^{-1}) \tag{6.108}$$

$$\langle a, b, c \,|\, pa - ca - cb,\ pb - cb,\ pc,\ \text{class } 2\rangle \tag{6.109}$$

$$\langle a, b, c \,|\, pa - \omega cb,\ pb - ca,\ pc,\ \text{class } 2\rangle \tag{6.110}$$

$$\langle a, b, c \,|\, pa - xcb,\ pb - ca - cb,\ pc,\ \text{class } 2\rangle \ (\text{all } x \text{ with } 1 + 4x \text{ not a square } \bmod p) \tag{6.111}$$

$$\langle a, b, c \,|\, pa - ba,\ pb - ca,\ pc,\ \text{class } 2\rangle \tag{6.112}$$

$$\langle a, b, c \,|\, pa - ba,\ pb - cb,\ pc,\ \text{class } 2\rangle \tag{6.113}$$

$$\langle a, b, c \,|\, pa - ba,\ pb - cb,\ pc - xba - ca,\ \text{class } 2\rangle \ (\text{all } x) \tag{6.114}$$

$$\langle a, b, c \,|\, pa - ba,\ pb - ca,\ pc - cb,\ \text{class } 2\rangle \tag{6.115}$$

$$\langle a, b, c \,|\, pa - ba,\ pb - ca,\ pc + cb,\ \text{class } 2\rangle \tag{6.116}$$

$$\langle a, b, c \,|\, pa - ba,\ pb - ca,\ pc - \omega ba + cb,\ \text{class } 2\rangle \tag{6.117}$$

7.2 Descendants of 4.1

$4p + 48$ algebras

$$\langle a, b, c, d \,|\, ba,\ ca,\ da,\ cb,\ db,\ dc,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.9}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc,\ pa,\ pb,\ pc - ba,\ \text{class } 2\rangle \tag{6.10}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.11}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc,\ pb - ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.12}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc,\ pb,\ pc - ba,\ pd,\ \text{class } 2\rangle \tag{6.13}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc,\ pa,\ pb,\ pd,\ \text{class } 2\rangle \tag{6.14}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc,\ pa - ba,\ pb,\ pd,\ \text{class } 2\rangle \tag{6.15}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc - ba,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.16}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc - ba,\ pb - ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.17}$$

$$\langle a, b, c, d \,|\, ca,\ da,\ cb,\ db,\ dc - ba,\ pb,\ pc - ba,\ pd,\ \text{class } 2\rangle \tag{6.18}$$

$$\langle a, b, c, d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.19}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa-ba,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.20}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb-ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.21}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa-ca,\ pb-ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.22}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb-ca,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.23}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa-ba,\ pb-ca,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.24}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb-ba,\ pc-xca,\ pd,\ \text{class } 2\rangle\ (x\neq 0,\ x\sim x^{-1}) \tag{6.25}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb-ba-ca,\ pc-ca,\ pd,\ \text{class } 2\rangle \tag{6.26}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb-\omega ca,\ pc-ba,\ pd,\ \text{class } 2\rangle \tag{6.27}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb-xca,\ pc-ba-ca,\ pd,\ \text{class } 2\rangle\ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{6.28}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb,\ pc,\ pd-ca,\ \text{class } 2\rangle \tag{6.29}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa-ba,\ pb,\ pc,\ pd-ca,\ \text{class } 2\rangle \tag{6.30}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb-ba,\ pc,\ pd-ca,\ \text{class } 2\rangle \tag{6.31}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db,\ dc,\ pa,\ pb,\ pc-ba,\ pd-ca,\ \text{class } 2\rangle \tag{6.32}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.33}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.34}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa,\ pb-ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.35}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba,\ pb-ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.36}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba-ca,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.37}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba-ca,\ pb-ba-ca,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.38}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba,\ pb-ba,\ pc-ca,\ pd-ca,\ \text{class } 2\rangle \tag{6.39}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba,\ pb-ba,\ pc,\ pd-ca,\ \text{class } 2\rangle \tag{6.39B}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba+ca,\ pb-ba,\ pc,\ pd-ca,\ \text{class } 2\rangle \tag{6.40}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa+ca,\ pb-ba,\ pc,\ pd-ca,\ \text{class } 2\rangle \tag{6.41}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa,\ pb-ba,\ pc,\ pd-ca,\ \text{class } 2\rangle \tag{6.42}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba-ca,\ pb-ba-xca,\ pc,\ pd,\ \text{class } 2\rangle\ (x \neq 0,\ 1) \tag{6.43}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba-ca,\ pb-ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.44}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba-ca,\ pb-ca,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.45}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ba,\ pb-ca,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.46}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ba,\ dc,\ pa-ca,\ pb-ba,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.47}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.48}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa-ba,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.49}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa,\ pb,\ pc-ba,\ pd,\ \text{class } 2\rangle \tag{6.50}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa-ca,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.51}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa,\ pb,\ pc-ca,\ pd,\ \text{class } 2\rangle \tag{6.52}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa-ba,\ pb-ca,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.53}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa-ca,\ pb,\ pc-ba,\ pd,\ \text{class } 2\rangle \tag{6.54}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa,\ pb-ca,\ pc-ba,\ pd,\ \text{class } 2\rangle \tag{6.55}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa,\ pb-\omega ca,\ pc-ba,\ pd,\ \text{class } 2\rangle \tag{6.56}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa-ba,\ pb,\ pc-ca,\ pd,\ \text{class } 2\rangle \tag{6.57}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa,\ pb-xba,\ pc-ca,\ pd,\ \text{class } 2\rangle\ (x \neq 0) \tag{6.58}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc,\ pa,\ pb,\ pc-ba,\ pd-ca,\ \text{class } 2\rangle \tag{6.59}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc-\omega ba,\ pa,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.60}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc-\omega ba,\ pa-ba,\ pb,\ pc,\ pd,\ \text{class } 2\rangle \tag{6.60B}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc-\omega ba,\ pa,\ pb,\ pc-ba,\ pd-ca,\ \text{class } 2\rangle \tag{6.61}$$

$$\langle a,b,c,d \,|\, cb,\ da,\ db-ca,\ dc-\omega ba,\ pa,\ pb-ba,\ pc-xba-yca,\ pd,\ \text{class } 2\rangle \tag{6.62}$$

Algebra 6.62 has two parameters x, y, with $y \neq 0$. Parameter pairs (x, y) and (z, t) give isomorphic algebras if and only if

$$\begin{pmatrix} 1 & 0 \\ z & t \end{pmatrix} = \begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix} \begin{pmatrix} 1 & 0 \\ x & y \end{pmatrix} \begin{pmatrix} \mu+\nu x & \nu y \\ \omega\nu y & \mu+\nu x \end{pmatrix}^{-1}$$

for some non-singular matrix $\begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix}$. There are p distinct algebras.

7.3 Descendants of 4.3

$3p^2 + 13p + 37 + \gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, ca,\ cb,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.118}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.119}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-\omega bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.120}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-baa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.121}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-baa,\ pb-xbab,\ pc,\ \text{class } 3\rangle\ (x \neq 0,\ x \sim x^{-1}) \tag{6.122}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-baa-bab,\ pb-bab,\ pc,\ \text{class } 3\rangle \tag{6.123}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-baa-\omega bab,\ pb-bab,\ pc,\ \text{class } 3\rangle \tag{6.124}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-\omega bab,\ pb-baa,\ pc,\ \text{class } 3\rangle \tag{6.125}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-xbab,\ pb-baa-bab,\ pc,\ \text{class } 3\rangle\ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{6.126}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa,\ pb,\ pc-bab,\ \text{class } 3\rangle \tag{6.127}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa-baa,\ pb,\ pc-bab,\ \text{class } 3\rangle \tag{6.128}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa,\ pb-baa,\ pc-bab,\ \text{class } 3\rangle \tag{6.129}$$

$$\langle a,b,c \,|\, ca,\ cb,\ pa,\ pb-\omega baa,\ pc-bab,\ \text{class } 3\rangle \tag{6.130}$$

$$\langle a,b,c \,|\, ca-bab,\ cb,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.131}$$

$$\langle a,b,c \,|\, ca-bab,\ cb,\ pa-bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.132}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-\omega bab,\ pb,\ pc,\ \text{class 3}\rangle \tag{6.133}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-baa,\ pb,\ pc,\ \text{class 3}\rangle \tag{6.134}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-xbaa,\ pb-bab,\ pc,\ \text{class 3}\rangle\ (\text{all } x) \tag{6.135}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-baa-bab,\ pb-bab,\ pc,\ \text{class 3}\rangle \tag{6.136}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-baa-\omega bab,\ pb-bab,\ pc,\ \ \text{class 3}\rangle \tag{6.137}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-baa-xbab,\ pb-baa,\ pc,\ \text{class 3}\rangle\ (\text{all } x) \tag{6.138}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa,\ pb-baa,\ pc,\ \text{class 3}\rangle \tag{6.139}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-bab,\ pb-baa,\ pc,\ \text{class 3}\rangle \tag{6.140}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-\omega bab,\ pb-baa,\ pc,\ \text{class 3}\rangle \tag{6.141}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-baa-xbab,\ pb-\omega baa,\ pc,\ \ \text{class 3}\rangle\ (\text{all } x) \tag{6.142}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa,\ pb-\omega baa,\ pc,\ \text{class 3}\rangle \tag{6.143}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-bab,\ pb-\omega baa,\ pc,\ \text{class 3}\rangle \tag{6.144}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-\omega bab,\ pb-\omega baa,\ pc,\ \ \text{class 3}\rangle \tag{6.145}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa,\ pb-xbaa,\ pc-bab,\ \text{class 3}\rangle\ (\text{all } x) \tag{6.146}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-baa,\ pb,\ pc-bab,\ \text{class 3}\rangle \tag{6.147}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-xbab,\ pb,\ pc-baa,\ \text{class 3}\rangle\ (x=0,\ 1,\ \omega,\ \omega^2,\ \omega^3,\ \ p=1 \bmod 4) \tag{6.148}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-xbab,\ pb,\ pc-baa,\ \text{class 3}\rangle\ (x=0,\ 1,\ \omega,\ \ p=3 \bmod 4) \tag{6.148A}$$

$$\begin{gathered}\langle a,b,c\,|\,ca-bab,\ cb,\ pa-xbab,\ pb-ybab,\ pc-baa,\ \text{class 3}\rangle \\ (y=1,\ \omega,\ \omega^2,\ \text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ \ p=1 \bmod 3)\end{gathered} \tag{6.149}$$

$$\langle a,b,c\,|\,ca-bab,\ cb,\ pa-xbab,\ pb-bab,\ pc-baa,\ \text{class 3}\rangle\ (\text{all } x,\ \ p=2 \bmod 3) \tag{6.149A}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa,\ pb,\ pc,\ \text{class 3}\rangle \tag{6.150}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-baa,\ pb,\ pc,\ \text{class 3}\rangle \tag{6.151}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.152}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.153}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-\omega bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.154}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-\omega bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.155}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-bab,\ pb-baa,\ pc,\ \text{class } 3\rangle\ (p=1 \bmod 4) \tag{6.156}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-bab,\ pb-\omega baa,\ pc,\ \text{class } 3\rangle\ (p=1 \bmod 4) \tag{6.157}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-\omega bab,\ pb-\omega baa,\ pc,\ \text{class } 3\rangle\ (p=1 \bmod 4) \tag{6.158}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-bab,\ pb-baa,\ pc,\ \text{class } 3\rangle\ (p=3 \bmod 4) \tag{6.156A}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-bab,\ pb-\omega baa,\ pc,\ \text{class } 3\rangle\ (p=3 \bmod 4) \tag{6.157A}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-\omega bab,\ pb-baa,\ pc,\ \text{class } 3\rangle\ (p=3 \bmod 4) \tag{6.158A}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-xbab,\ pb-baa,\ pc,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.159}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-xbab,\ pb-\omega baa,\ pc,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.160}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa,\ pb-xbab,\ pc,\ \text{class } 3\rangle\ (x\neq 0,\ x\sim x^{-1}) \tag{6.160}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-bab,\ pb-xbab,\ pc,\ \text{class } 3\rangle\ (x\neq 0) \tag{6.161}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-\omega bab,\ pb-xbab,\ pc,\ \text{class } 3\rangle\ (x\neq 0) \tag{6.162}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-bab,\ pb-xbaa-ybab,\ pc,\ \text{class } 3\rangle\ (x\neq 0,\ y\neq 0,\ 1,\ -1,\ y\sim y^{-1}) \tag{6.163}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-\omega bab,\ pb-xbaa-ybab,\ pc,\ \text{class } 3\rangle\ (x\neq 0,\ y\neq 0,\ 1,\ -1,\ y\sim y^{-1}) \tag{6.164}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-bab,\ pb-x^2baa+bab,\ pc,\ \text{class } 3\rangle\ (x\neq 0,\ x\sim -x) \tag{6.165}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-bab,\ pb-x^2baa-bab,\ pc,\ \text{class } 3\rangle\ (x\neq 0,\ x\sim -x) \tag{6.165}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-bab,\ pb-x^2\omega baa+bab,\ pc,\ \text{class } 3\rangle\ (x\neq 0,\ x\sim -x) \tag{6.166}$$

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-baa-bab,\ pb-x^2\omega baa-bab,\ pc,\ \text{class } 3\rangle\ (x\neq 0,\ x\sim -x) \tag{6.166}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-baa-\omega bab,\ pb-x^2\omega baa+bab,\ pc,\ \text{class 3}\rangle\ (x\neq 0,\ x\sim -x,\ \ p=1 \bmod 4) \tag{6.167}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-baa-\omega bab,\ pb-x^2\omega baa-bab,\ pc,\ \text{class 3}\rangle\ (x\neq 0,\ x\sim -x,\ \ p=1 \bmod 4) \tag{6.167}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-baa-\omega bab,\ pb-x^2 baa+bab,\ pc,\ \text{class 3}\rangle\ (x\neq 0,\ x\sim -x,\ \ p=3 \bmod 4) \tag{6.167A}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-baa-\omega bab,\ pb-x^2 baa-bab,\ pc,\ \text{class 3}\rangle\ (x\neq 0,\ x\sim -x,\ \ p=3 \bmod 4) \tag{6.167A}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa,\ pb,\ pc-baa,\ \text{class 3}\rangle \tag{6.168}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-bab,\ pb,\ pc-baa,\ \text{class 3}\rangle \tag{6.169}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-\omega bab,\ pb,\ pc-baa,\ \text{class 3}\rangle \tag{6.170}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-xbab,\ pb-bab,\ pc-baa,\ \text{class 3}\rangle\ (\text{all } x) \tag{6.171}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-xbab,\ pb-ybab,\ pc-baa-bab,\ \ \text{class 3}\rangle\ (\text{all } x,\ y,\ (x,\ y)\sim(y,\ x)) \tag{6.172}$$

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa,\ pb,\ pc,\ \text{class 3}\rangle \tag{6.173}$$

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-xbaa-bab,\ pb,\ pc,\ \ \text{class 3}\rangle\ (\text{all } x,\ x\sim -x) \tag{6.174}$$

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-xbaa-\omega bab,\ pb,\ pc,\ \text{class 3}\rangle\ (\text{all } x,\ x\sim -x) \tag{6.175}$$

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-baa,\ pb,\ pc,\ \text{class 3}\rangle \tag{6.176}$$

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-\omega baa,\ pb,\ pc,\ \ \text{class 3}\rangle\ (p=1 \bmod 4) \tag{6.177}$$

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-tbaa-xbab,\ pb-ybaa-zbab,\ pc,\ \text{class 3}\rangle \tag{6.178}$$

Algebra 6.178 has four parameters x, y, z, t. If we write the parameters in a matrix $A = \begin{pmatrix} t & x \\ y & z \end{pmatrix}$, then two such parameter matrices A and B define isomorphic algebras if and only if

$$B = \frac{1}{\det P} P A P^{-1} \bmod p$$

for some matrix P of the form

$$\begin{pmatrix} \alpha & \beta \\ \omega\beta & \alpha \end{pmatrix} \text{ or } \begin{pmatrix} \alpha & \beta \\ -\omega\beta & -\alpha \end{pmatrix}.$$

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-xbaa,\ pb-ybaa,\ pc-bab,\ \text{class 3}\rangle\ (\text{all } x,\ y,\ x\sim -x) \tag{6.179}$$

7.4 Descendants of 4.6

2 algebras

$$\langle a, b \,|\, ba, \text{ class } 3\rangle \tag{6.366}$$

$$\langle a, b \,|\, ba - p^2 a, \text{ class } 3\rangle \tag{6.367}$$

7.5 Descendants of 4.7

$p + 15$ algebras

$$\langle a, b \,|\, p^2 a,\ pb, \text{ class } 3\rangle \tag{6.368}$$

$$\langle a, b \,|\, p^2 a,\ pb - bab, \text{ class } 3\rangle \tag{6.369}$$

$$\langle a, b \,|\, p^2 a,\ pb - baa, \text{ class } 3\rangle \tag{6.370}$$

$$\langle a, b \,|\, p^2 a,\ pb - \omega baa, \text{ class } 3\rangle \tag{6.371}$$

$$\langle a, b \,|\, p^2 a - bab,\ pb, \text{ class } 3\rangle \tag{6.372}$$

$$\langle a, b \,|\, p^2 a - bab,\ pb - baa, \text{ class } 3\rangle \tag{6.373}$$

$$\langle a, b \,|\, p^2 a - bab,\ pb - \omega baa, \text{ class } 3\rangle \tag{6.374}$$

$$\langle a, b \,|\, p^2 a - \omega bab,\ pb, \text{ class } 3\rangle \tag{6.375}$$

$$\langle a, b \,|\, p^2 a - \omega bab,\ pb - baa, \text{ class } 3\rangle \tag{6.376}$$

$$\langle a, b \,|\, p^2 a - \omega bab,\ pb - \omega baa, \text{ class } 3\rangle \tag{6.377}$$

$$\langle a, b \,|\, p^2 a - baa,\ pb - xbab, \text{ class } 3\rangle \text{ (all } x) \tag{6.378}$$

$$\langle a, b \,|\, baa,\ pb, \text{ class } 3\rangle \tag{6.379}$$

$$\langle a, b \,|\, baa,\ pb - bab, \text{ class } 3\rangle \tag{6.380}$$

$$\langle a, b \,|\, bab,\ pb, \text{ class } 3\rangle \tag{6.381}$$

$$\langle a, b \,|\, bab,\ pb - baa, \text{ class } 3\rangle \tag{6.382}$$

$$\langle a, b \,|\, bab,\ pb - \omega baa, \text{ class } 3\rangle \tag{6.383}$$

7.6 Descendants of 4.8

1 algebra

$$\langle a, b \,|\, pb - ba, \text{ class } 3\rangle \tag{6.384}$$

7.7 Descendants of 5.1

7 algebras

$$\langle a, b, c, d, e \,|\, ba, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 2\rangle \tag{6.2}$$

$$\langle a, b, c, d, e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{6.3}$$

$$\langle a, b, c, d, e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pa - ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{6.4}$$

$$\langle a, b, c, d, e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc - ba, pd, pe, \text{ class } 2\rangle \tag{6.5}$$

$$\langle a, b, c, d, e \,|\, ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{6.6}$$

$$\langle a, b, c, d, e \,|\, ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa - ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{6.7}$$

$$\langle a, b, c, d, e \,|\, ca, da, ea, cb, db, eb, dc - ba, ec, ed, pa, pb, pc, pd, pe - ba, \text{ class } 2\rangle \tag{6.8}$$

7.8 Descendants of 5.2

4 algebras

$$\langle a, b, c, d \,|\, ba, ca, da, cb, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{6.63}$$

$$\langle a, b, c, d \,|\, ba - p^2a, ca, da, cb, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{6.64}$$

$$\langle a, b, c, d \,|\, ba, ca, da, cb - p^2a, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{6.65}$$

$$\langle a, b, c, d \,|\, ba, ca, da - p^2a, cb - p^2a, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{6.66}$$

7.9 Descendants of 5.3

18 algebras

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{6.67}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa - baa, pb, pc, pd, \text{ class } 3\rangle \tag{6.68}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb - baa, pc, pd, \text{ class } 3\rangle \tag{6.69}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{6.70}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb, pc - baa, pd, \text{ class } 3\rangle \tag{6.71}$$

$$\langle a,b,c,d \,|\, ca,\ cb-baa,\ bab,\ da,\ db,\ dc,\ pa,\ pb,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.72}$$

$$\langle a,b,c,d \,|\, ca,\ cb-baa,\ bab,\ da,\ db,\ dc,\ pa-baa,\ pb,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.73}$$

$$\langle a,b,c,d \,|\, ca,\ cb-baa,\ bab,\ da,\ db,\ dc,\ pa,\ pb-baa,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.74}$$

$$\langle a,b,c,d \,|\, ca,\ cb-baa,\ bab,\ da,\ db,\ dc,\ pa,\ pb-\omega baa,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.75}$$

$$\langle a,b,c,d \,|\, ca,\ cb-baa,\ bab,\ da,\ db,\ dc,\ pa,\ pb,\ pc-baa,\ pd,\ \text{class } 3\rangle \tag{6.76}$$

$$\langle a,b,c,d \,|\, ca,\ cb-baa,\ bab,\ da,\ db,\ dc,\ pa,\ pb,\ pc,\ pd-baa,\ \text{class } 3\rangle \tag{6.77}$$

$$\langle a,b,c,d \,|\, bab,\ ca,\ cb,\ da,\ db,\ dc-baa,\ pa,\ pb,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.78}$$

$$\langle a,b,c,d \,|\, bab,\ ca,\ cb,\ da,\ db,\ dc-baa,\ pa-baa,\ pb,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.79}$$

$$\langle a,b,c,d \,|\, bab,\ ca,\ cb,\ da,\ db,\ dc-baa,\ pa,\ pb-baa,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.80}$$

$$\langle a,b,c,d \,|\, bab,\ ca,\ cb,\ da,\ db,\ dc-baa,\ pa,\ pb-\omega baa,\ pc,\ pd,\ \text{class } 3\rangle \tag{6.81}$$

$$\langle a,b,c,d \,|\, bab,\ ca,\ cb,\ da,\ db,\ dc-baa,\ pa,\ pb,\ pc-baa,\ pd,\ \text{class } 3\rangle \tag{6.82}$$

$$\langle a,b,c,d \,|\, bab,\ ca,\ cb,\ da,\ db,\ dc-baa,\ pa,\ pb-baa,\ pc-baa,\ pd,\ \text{class } 3\rangle \tag{6.83}$$

$$\langle a,b,c,d \,|\, bab,\ ca,\ cb,\ da,\ db,\ dc-baa,\ pa,\ pb-\omega baa,\ pc-baa,\ pd,\ \text{class } 3\rangle \tag{6.84}$$

7.10 Descendants of 5.8

4 algebras

$$\langle a,b,c \,|\, ba,\ ca,\ cb-p^2a,\ p^2b,\ pc,\ \text{class } 3\rangle \tag{6.180}$$

$$\langle a,b,c \,|\, ba,\ ca-p^2a,\ cb,\ p^2b,\ pc,\ \text{class } 3\rangle \tag{6.181}$$

$$\langle a,b,c \,|\, ba,\ ca,\ cb,\ p^2b,\ pc,\ \text{class } 3\rangle \tag{6.182}$$

$$\langle a,b,c \,|\, ba-p^2a,\ ca,\ cb,\ p^2b,\ pc,\ \text{class } 3\rangle \tag{6.183}$$

7.11 Descendants of 5.9

23 algebras

$$\langle a, b, c \,|\, baa,\ bab,\ ca,\ cb,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.184}$$

$$\langle a, b, c \,|\, baa,\ bab,\ ca - p^2a,\ cb,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.185}$$

$$\langle a, b, c \,|\, baa,\ bab,\ ca,\ cb - p^2a,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.186}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ p^2a,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.187}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ p^2a,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.188}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ p^2a,\ pb - baa,\ pc,\ \text{class } 3\rangle \tag{6.189}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ p^2a,\ pb - baa,\ pc,\ \text{class } 3\rangle \tag{6.190}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ p^2a,\ pb - \omega baa,\ pc,\ \text{class } 3\rangle \tag{6.191}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ p^2a,\ pb - \omega baa,\ pc,\ \text{class } 3\rangle \tag{6.192}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ p^2a,\ pb,\ pc - baa,\ \text{class } 3\rangle \tag{6.193}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ p^2a,\ pb,\ pc - baa,\ \text{class } 3\rangle \tag{6.194}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ p^2a - baa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.195}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ p^2a - baa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.196}$$

$$\langle a, b, c \,|\, baa,\ ca,\ cb,\ p^2a,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.197}$$

$$\langle a, b, c \,|\, baa,\ ca - bab,\ cb,\ p^2a,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.198}$$

$$\langle a, b, c \,|\, baa,\ ca,\ cb,\ p^2a,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.199}$$

$$\langle a, b, c \,|\, baa,\ ca - bab,\ cb,\ p^2a,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.200}$$

$$\langle a, b, c \,|\, baa,\ ca,\ cb,\ p^2a,\ pb,\ pc - bab,\ \text{class } 3\rangle \tag{6.201}$$

$$\langle a, b, c \,|\, baa,\ ca - bab,\ cb,\ p^2a,\ pb,\ pc - bab,\ \text{class } 3\rangle \tag{6.202}$$

$$\langle a, b, c \,|\, baa,\ ca,\ cb,\ p^2a - bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.203}$$

$$\langle a, b, c \,|\, baa,\ ca - bab,\ cb,\ p^2a - bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.204}$$

$$\langle a, b, c \,|\, baa,\ ca,\ cb,\ p^2a - \omega bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.205}$$

$$\langle a, b, c \,|\, baa,\ ca - bab,\ cb,\ p^2a - \omega bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.206}$$

7.12 Descendants of 5.10

5 algebras

$$\langle a, b, c \,|\, baa,\ ca,\ cb,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.207}$$

$$\langle a, b, c \,|\, baa,\ ca - p^2a,\ cb,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.208}$$

$$\langle a, b, c \,|\, baa,\ ca,\ cb - p^2a,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.209}$$

$$\langle a, b, c \,|\, ca,\ cb,\ p^2a,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.210}$$

$$\langle a, b, c \,|\, ca,\ cb,\ p^2a,\ pb - ba,\ pc - baa,\ \text{class } 3\rangle \tag{6.211}$$

7.13 Descendants of 5.11

4 algebras

$$\langle a, b, c \,|\, ca,\ cb,\ pb,\ pc - ba,\ \text{class } 3\rangle \tag{6.212}$$

$$\langle a, b, c \,|\, ca,\ cb - p^2a,\ pb,\ pc - ba,\ \text{class } 3\rangle \tag{6.213}$$

$$\langle a, b, c \,|\, ca,\ cb - \omega p^2a,\ pb,\ pc - ba,\ \text{class } 3\rangle \tag{6.214}$$

$$\langle a, b, c \,|\, ca - p^2a,\ cb,\ pb,\ pc - ba,\ \text{class } 3\rangle \tag{6.215}$$

7.14 Descendants of 5.12

12 algebras

$$\langle a, b, c \,|\, baa,\ bab,\ ca,\ cb,\ pa,\ pb,\ \text{class } 3\rangle \tag{6.216}$$

$$\langle a, b, c \,|\, baa,\ bab,\ ca - p^2c,\ cb,\ pa,\ pb,\ \text{class } 3\rangle \tag{6.217}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa,\ pb,\ p^2c,\ \text{class } 3\rangle \tag{6.218}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa - baa,\ pb,\ p^2c,\ \text{class } 3\rangle \tag{6.219}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa,\ pb - baa,\ p^2c,\ \text{class } 3\rangle \tag{6.220}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa,\ pb - \omega baa,\ p^2c,\ \text{class } 3\rangle \tag{6.221}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa,\ pb,\ p^2c,\ \text{class } 3\rangle \tag{6.222}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa - baa,\ pb,\ p^2c,\ \text{class } 3\rangle \tag{6.223}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa,\ pb - baa,\ p^2c,\ \text{class } 3\rangle \tag{6.224}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa,\ pb - \omega baa,\ p^2c,\ \text{class } 3\rangle \tag{6.225}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa,\ pb,\ p^2c - baa,\ \text{class } 3\rangle \tag{6.226}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa,\ pb,\ p^2c - baa,\ \text{class } 3\rangle \tag{6.227}$$

7.15 Descendants of 5.13

$p + 1$ algebras

$$\langle a, b, c \,|\, ca,\ cb,\ pa - ba,\ pb,\ \text{class } 3\rangle \tag{6.228}$$

$$\langle a, b, c \,|\, ca - p^2c,\ cb,\ pa - ba,\ pb,\ \text{class } 3\rangle \tag{6.229}$$

$$\langle a, b, c \,|\, ca,\ cb - xp^2c,\ pa - ba,\ pb,\ \text{class } 3\rangle\ (x \neq 0) \tag{6.230}$$

7.16 Descendants of 5.14

35 algebras

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.231}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac,\ pa,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.232}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac,\ pa - bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.233}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac,\ pa - bab,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.234}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac,\ pa - \omega bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.235}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac,\ pa - \omega bab,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.236}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac,\ pa,\ pb,\ pc - bab,\ \text{class } 3\rangle \tag{6.237}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa - bab,\ cac,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.238}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa - bab,\ cac,\ pa - bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.239}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa - bab,\ cac,\ pa - \omega bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.240}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa - bab,\ cac,\ pa,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.241}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa - bab,\ cac,\ pa,\ pb,\ pc - bab,\ \text{class } 3\rangle \tag{6.242}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa - bab,\ cac,\ pa,\ pb,\ pc - \omega bab,\ \text{class } 3\rangle \tag{6.243}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + bab,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.244}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + bab,\ pa - bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.245}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + bab,\ pa,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.246}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + bab,\ pa - bab,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.247}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + bab,\ pa - \omega bab,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.248}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + bab,\ pa,\ pb - bab,\ pc - bab,\ \text{class } 3\rangle \tag{6.249}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + bab,\ pa - bab,\ pb - bab,\ pc - bab,\ \text{class } 3\rangle \tag{6.250}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + \omega bab,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.251}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + \omega bab,\ pa - bab,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.252}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + \omega bab,\ pa,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.253}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + \omega bab,\ pa - bab,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.254}$$

$$\langle a, b, c \,|\, cb,\ baa,\ bac,\ caa,\ cac + \omega bab,\ pa - \omega bab,\ pb - bab,\ pc,\ \text{class } 3\rangle \tag{6.255}$$

$$\langle a, b, c \,|\, cb,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.256}$$

$$\langle a, b, c \,|\, cb,\ bab,\ bac,\ caa,\ cac,\ pa - baa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.257}$$

$$\langle a, b, c \,|\, cb,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb - baa,\ pc,\ \text{class } 3\rangle \tag{6.258}$$

$$\langle a, b, c \,|\, cb,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb - \omega baa,\ pc,\ \text{class } 3\rangle \tag{6.259}$$

$$\langle a, b, c \,|\, cb,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb,\ pc - baa,\ \text{class } 3\rangle \tag{6.260}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.261}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ bac,\ caa,\ cac,\ pa - baa,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.262}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb - baa,\ pc,\ \text{class } 3\rangle \tag{6.263}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb - \omega baa,\ pc,\ \text{class } 3\rangle \tag{6.264}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ bac,\ caa,\ cac,\ pa,\ pb,\ pc - baa,\ \text{class } 3\rangle \tag{6.265}$$

7.17 Descendants of 5.15

$2p + 13$ algebras

$$\langle a, b, c \,|\, cb,\ caa,\ pa - ba,\ pb - cac,\ pc,\ \text{class } 3\rangle \tag{6.266}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - ba,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.267}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - ba,\ pb,\ pc - cac,\ \text{class } 3\rangle \tag{6.268}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - ba - cac,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.269}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - ba - cac,\ pb,\ pc - cac,\ \text{class } 3\rangle \tag{6.270}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - ba - \omega cac,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.271}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - ba - \omega cac,\ pb,\ pc - cac,\ \text{class } 3\rangle \tag{6.272}$$

$$\langle a, b, c \,|\, cb,\ cac,\ pa - ba,\ pb - caa,\ pc,\ \text{class } 3\rangle \tag{6.273}$$

$$\langle a, b, c \,|\, cb,\ cac,\ pa - ba,\ pb,\ pc,\ \text{class } 3\rangle \tag{6.274}$$

$$\langle a, b, c \,|\, cb,\ cac,\ pa - ba,\ pb,\ pc - caa,\ \text{class } 3\rangle \tag{6.275}$$

$$\langle a, b, c \,|\, cb,\ cac,\ pa - ba,\ pb,\ pc - \omega caa,\ \text{class } 3\rangle \tag{6.276}$$

$$\langle a, b, c \,|\, cb - caa,\ cac,\ pa - ba,\ pb - caa,\ pc,\ \text{class } 3\rangle \tag{6.277}$$

$$\langle a, b, c \,|\, cb - caa,\ cac,\ pa - ba,\ pb,\ pc - xcaa,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.278}$$

$$\langle a, b, c \,|\, cb - \omega caa,\ cac,\ pa - ba,\ pb - caa,\ pc,\ \text{class } 3\rangle \tag{6.279}$$

$$\langle a, b, c \,|\, cb - \omega caa,\ cac,\ pa - ba,\ pb,\ pc - xcaa,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.280}$$

7.18 Descendants of 5.16

$4p + 8$ algebras

$$\langle a, b, c \,|\, cb,\ caa,\ pa,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.281}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa,\ pb - ba - cac,\ pc,\ \text{class } 3\rangle \tag{6.282}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - cac,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.283}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - cac,\ pb - ba - cac,\ pc,\ \text{class } 3\rangle \tag{6.284}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - \omega cac,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.285}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - \omega cac,\ pb - ba - cac,\ pc,\ \text{class } 3\rangle \tag{6.286}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - xcac,\ pb - ba,\ pc - cac,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.287}$$

$$\langle a, b, c \,|\, cb,\ caa,\ pa - xcac,\ pb - ba - cac,\ pc - cac,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.288}$$

$$\langle a, b, c \,|\, cb,\ cac,\ pa,\ pb - ba,\ pc - xcaa,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.289}$$

$$\langle a, b, c \,|\, cb - caa,\ cac,\ pa,\ pb - ba,\ pc - xcaa,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.290}$$

$$\langle a, b, c \,|\, cb,\ cac,\ pa - caa,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.291}$$

$$\langle a, b, c \,|\, cb - caa,\ cac,\ pa - caa,\ pb - ba,\ pc,\ \text{class } 3\rangle \tag{6.292}$$

7.19 Descendants of 5.18

$2p + 13 + 3\gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b, c \,|\, cb,\ baa,\ pa,\ pb - ca,\ pc - bab,\ \text{class } 3\rangle \tag{6.293}$$

$$\langle a, b, c \,|\, cb,\ baa,\ pa,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.294}$$

$$\langle a, b, c \,|\, cb,\ baa,\ pa - bab,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.295}$$

$$\langle a, b, c \,|\, cb,\ baa,\ pa - \omega bab,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.296}$$

$$\langle a, b, c \,|\, cb,\ baa,\ pa,\ pb - ca - bab,\ pc,\ \text{class } 3\rangle \tag{6.297}$$

$$\langle a, b, c \,|\, cb,\ baa,\ pa - bab,\ pb - ca - bab,\ pc,\ \text{class } 3\rangle \tag{6.298}$$

$$\langle a, b, c \,|\, cb,\ baa,\ pa - \omega bab,\ pb - ca - bab,\ pc,\ \text{class } 3\rangle \tag{6.299}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa,\ pb - ca,\ pc - baa,\ \text{class } 3\rangle \tag{6.300}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa,\ pb - ca,\ pc - \omega baa,\ \text{class } 3\rangle\ (p = 1 \bmod 3) \tag{6.301}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa,\ pb - ca,\ pc - \omega^2 baa,\ \text{class } 3\rangle\ (p = 1 \bmod 3) \tag{6.302}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.303}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa,\ pb - ca - baa,\ pc,\ \text{class } 3\rangle \tag{6.304}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa,\ pb - ca - \omega baa,\ pc,\ \text{class } 3\rangle \tag{6.305}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa - baa,\ pb - ca - xbaa,\ pc,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.305B}$$

$$\langle a, b, c \,|\, cb,\ bab - baa,\ pa - \omega baa,\ pb - ca - xbaa,\ pc,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.305C}$$

$$\langle a, b, c \,|\, cb,\ bab,\ pa,\ pb - ca,\ pc - baa,\ \text{class } 3\rangle \tag{6.306}$$

$$\langle a, b, c \,|\, cb,\ bab,\ pa,\ pb - ca,\ pc - \omega baa,\ \text{class } 3\rangle\ (p = 1 \bmod 3) \tag{6.307}$$

$$\langle a, b, c \,|\, cb,\ bab,\ pa,\ pb - ca,\ pc - \omega^2 baa,\ \text{class } 3\rangle\ (p = 1 \bmod 3) \tag{6.308}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ pa,\ pb - ca,\ pc - baa,\ \text{class } 3\rangle \tag{6.309}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ pa,\ pb - ca,\ pc - \omega baa,\ \text{class } 3\rangle\ (p = 1 \bmod 3) \tag{6.310}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ pa,\ pb - ca,\ pc - \omega^2 baa,\ \text{class } 3\rangle\ (p = 1 \bmod 3) \tag{6.311}$$

$$\langle a, b, c \,|\, cb,\ bab,\ pa,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.312}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ pa,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.313}$$

$$\langle a, b, c \,|\, cb,\ bab,\ pa - baa,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.314}$$

$$\langle a, b, c \,|\, cb - baa,\ bab,\ pa - baa,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.315}$$

$$\langle a, b, c \,|\, cb - \omega baa,\ bab,\ pa - baa,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.316}$$

$$\langle a, b, c \,|\, cb - \omega^2 baa,\ bab,\ pa - baa,\ pb - ca,\ pc,\ \text{class } 3\rangle\ (p = 1 \bmod 4) \tag{6.317}$$

$$\langle a, b, c \,|\, cb - \omega^3 baa,\ bab,\ pa - baa,\ pb - ca,\ pc,\ \text{class } 3\rangle\ (p = 1 \bmod 4) \tag{6.318}$$

7.20 Descendants of 5.19

3 algebras

$$\langle a, b, c \,|\, cb,\ pa - ba,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.319}$$

$$\langle a, b, c \,|\, cb - bab,\ pa - ba,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.320}$$

$$\langle a, b, c \,|\, cb - \omega bab,\ pa - ba,\ pb - ca,\ pc,\ \text{class } 3\rangle \tag{6.321}$$

7.21 Descendants of 5.24

3 algebras

$$\langle a, b, c \,|\, ba,\ ca,\ cb,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.322}$$

$$\langle a, b, c \,|\, ba - p^3 a,\ ca,\ cb,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.323}$$

$$\langle a, b, c \,|\, ba,\ ca,\ cb - p^3 a,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.324}$$

7.22 Descendants of 5.27

$11 + 4\gcd(p-1, 3) + 2\gcd(p-1, 4)$ algebras

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.325}$$

$$\langle a, b, c \,|\, bab - baaa,\ ca,\ cb,\ pa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.326}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baaa,\ pa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.327}$$

$$\langle a, b, c \,|\, bab - baaa,\ ca,\ cb - baaa,\ pa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.328}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.329}$$

$$\langle a, b, c \,|\, bab - baaa,\ ca,\ cb,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.330}$$

$$\langle a, b, c \,|\, bab - \omega baaa,\ ca,\ cb,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.331}$$

$$\langle a, b, c \,|\, bab - \omega^2 baaa,\ ca,\ cb,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.332}$$

$$\langle a, b, c \,|\, bab - \omega^3 baaa,\ ca,\ cb,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.333}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baaa,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.334}$$

$$\langle a, b, c \,|\, bab - baaa,\ ca,\ cb - baaa,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.335}$$

$$\langle a, b, c \,|\, bab - \omega baaa,\ ca,\ cb - baaa,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.336}$$

$$\langle a, b, c \,|\, bab - \omega^2 baaa,\ ca,\ cb - baaa,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.337}$$

$$\langle a, b, c \,|\, bab - \omega^3 baaa,\ ca,\ cb - baaa,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.338}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb,\ pa,\ pb - baaa,\ pc,\ \text{class } 4\rangle \tag{6.339}$$

$$\langle a, b, c \,|\, bab - baaa,\ ca,\ cb,\ pa,\ pb - baaa,\ pc,\ \text{class } 4\rangle \tag{6.340}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baaa,\ pa,\ pb-baaa,\ pc,\ \text{class } 4\rangle \tag{6.341}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb-baaa,\ pa,\ pb-baaa,\ pc,\ \text{class } 4\rangle \tag{6.342}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb,\ pa,\ pb-\omega baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.343}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb,\ pa,\ pb-\omega baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.344}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baaa,\ pa,\ pb-\omega baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.345}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb-baaa,\ pa,\ pb-\omega baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.346}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb,\ pa,\ pb-\omega^2 baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.347}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb,\ pa,\ pb-\omega^2 baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.348}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baaa,\ pa,\ pb-\omega^2 baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.349}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb-baaa,\ pa,\ pb-\omega^2 baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.350}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb,\ pa,\ pb,\ pc-baaa,\ \text{class } 4\rangle \tag{6.351}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baaa,\ pa,\ pb,\ pc-baaa,\ \text{class } 4\rangle \tag{6.352}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb,\ pa,\ pb,\ pc-baaa,\ \text{class } 4\rangle \tag{6.353}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb-baaa,\ pa,\ pb,\ pc-baaa,\ \text{class } 4\rangle \tag{6.354}$$

$$\langle a,b,c \,|\, bab-baaa,\ ca,\ cb-\omega baaa,\ pa,\ pb,\ pc-baaa,\ \text{class } 4\rangle \tag{6.355}$$

7.23 Descendants of 5.32

$4+2\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baa,\ pa,\ pb-baaa,\ pc,\ \text{class } 4\rangle \tag{6.356}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baa,\ pa,\ pb-\omega baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.357}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baa,\ pa,\ pb-\omega^2 baaa,\ pc,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.358}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baa,\ pa,\ pb-baaa,\ pc-baaa,\ \text{class } 4\rangle \tag{6.359}$$

$$\langle a,b,c \,|\, bab,\ ca,\ cb-baa,\ pa,\ pb-\omega baaa,\ pc-baaa,\ \text{class } 4\rangle\ (p=1 \bmod 3) \tag{6.360}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa,\ pb - \omega^2 baaa,\ pc - baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.361}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.362}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa - baaa,\ pb,\ pc,\ \text{class } 4\rangle \tag{6.363}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa,\ pb,\ pc - baaa,\ \text{class } 4\rangle \tag{6.364}$$

$$\langle a, b, c \,|\, bab,\ ca,\ cb - baa,\ pa - baaa,\ pb,\ pc - baaa,\ \text{class } 4\rangle \tag{6.365}$$

7.24 Descendants of 5.37

$p + 8$ algebras

$$\langle a, b \,|\, baa,\ bab,\ p^2 a,\ p^2 b,\ \text{class } 3\rangle \tag{6.385}$$

$$\langle a, b \,|\, baa,\ bab,\ pba,\ p^2 b,\ \text{class } 3\rangle \tag{6.386}$$

$$\langle a, b \,|\, baa,\ bab,\ p^2 a - pba,\ p^2 b,\ \text{class } 3\rangle \tag{6.387}$$

$$\langle a, b \,|\, bab,\ pba,\ p^2 a,\ p^2 b,\ \text{class } 3\rangle \tag{6.388}$$

$$\langle a, b \,|\, bab,\ pba,\ p^2 a - baa,\ p^2 b,\ \text{class } 3\rangle \tag{6.389}$$

$$\langle a, b \,|\, bab,\ pba,\ p^2 a,\ p^2 b - baa,\ \text{class } 3\rangle \tag{6.390}$$

$$\langle a, b \,|\, bab,\ pba,\ p^2 a,\ p^2 b - \omega baa,\ \text{class } 3\rangle \tag{6.391}$$

$$\langle a, b \,|\, bab,\ pba - baa,\ p^2 a,\ p^2 b - xbaa,\ \text{class } 3\rangle\ (\text{all } x) \tag{6.392}$$

$$\langle a, b \,|\, bab,\ pba - baa,\ p^2 a - baa,\ p^2 b,\ \text{class } 3\rangle \tag{6.393}$$

7.25 Descendants of 5.38

$5 + 3\gcd(p - 1, 3)$ algebras

$$\langle a, b \,|\, baab,\ babb,\ pa,\ pb,\ \text{class } 4\rangle \tag{6.394}$$

$$\langle a, b \,|\, baab,\ babb,\ pa - baaa,\ pb,\ \text{class } 4\rangle \tag{6.395}$$

$$\langle a, b \,|\, baab,\ babb,\ pa,\ pb - baaa,\ \text{class } 4\rangle \tag{6.396}$$

$$\langle a, b \,|\, baab,\ babb,\ pa,\ pb - \omega baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.397}$$

$$\langle a, b \,|\, baab,\ babb,\ pa,\ pb - \omega^2 baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.398}$$

$$\langle a, b \,|\, baab,\ babb + baaa,\ pa,\ pb,\ \text{class } 4\rangle \tag{6.399}$$

$$\langle a, b \,|\, baab,\ babb + baaa,\ pa - baaa,\ pb,\ \text{class } 4\rangle \tag{6.400}$$

$$\langle a, b \,|\, baab,\ babb + baaa,\ pa - \omega baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.401}$$

$$\langle a, b \,|\, baab,\ babb + baaa,\ pa - \omega^2 baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.402}$$

$$\langle a, b \,|\, baab,\ babb + baaa,\ pa - baaa,\ pb - baaa,\ \text{class } 4\rangle \tag{6.403}$$

$$\langle a, b \,|\, baab,\ babb + \omega baaa,\ pa,\ pb,\ \text{class } 4\rangle \tag{6.404}$$

$$\langle a, b \,|\, baab,\ babb + \omega baaa,\ pa - baaa,\ pb,\ \text{class } 4\rangle \tag{6.405}$$

$$\langle a, b \,|\, baab,\ babb + \omega baaa,\ pa - \omega baaa,\ pb,\ \ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.406}$$

$$\langle a, b \,|\, baab,\ babb + \omega baaa,\ pa - \omega^2 baaa,\ pb,\ \ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.407}$$

7.26 Descendants of 5.39

$1 + \gcd(p-1, 3) + (1/2)\gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, pa - bab,\ pb - baaa,\ \text{class } 4\rangle \tag{6.408}$$

$$\langle a, b \,|\, pa - bab,\ pb - \omega baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.409}$$

$$\langle a, b \,|\, pa - bab,\ pb - \omega^2 baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.410}$$

$$\langle a, b \,|\, pa - bab,\ pb,\ \text{class } 4\rangle \tag{6.411}$$

$$\langle a, b \,|\, pa - bab - baaa,\ pb,\ \text{class } 4\rangle \tag{6.412}$$

$$\langle a, b \,|\, pa - bab - \omega baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.413}$$

7.27 Descendants of 5.40

$1 + \gcd(p-1, 3) + (1/2)\gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, pa - \omega bab,\ pb - baaa,\ \text{class } 4\rangle \tag{6.408}$$

$$\langle a, b \,|\, pa - \omega bab,\ pb - \omega baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.409}$$

$$\langle a, b \,|\, pa - \omega bab,\ pb - \omega^2 baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.410}$$

$$\langle a, b \,|\, pa - \omega bab,\ pb,\ \text{class } 4\rangle \tag{6.411}$$

$$\langle a, b \,|\, pa - \omega bab - baaa,\ pb,\ \text{class } 4\rangle \tag{6.412}$$

$$\langle a, b \,|\, pa - \omega bab - \omega baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.413}$$

7.28 Descendants of 5.41

$p + 1 + \gcd(p - 1, 3)$ algebras

$$\langle a, b \,|\, pa - baa,\ pb,\ \text{class } 4\rangle \tag{6.420}$$

$$\langle a, b \,|\, pa - baa - babb,\ pb,\ \text{class } 4\rangle \tag{6.421}$$

$$\langle a, b \,|\, pa - baa - \omega babb,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.422}$$

$$\langle a, b \,|\, pa - baa - \omega^2 babb,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.423}$$

$$\langle a, b \,|\, pa - baa - xbabb,\ pb - babb,\ \text{class } 4\rangle\ (\text{all } x) \tag{6.424}$$

7.29 Descendants of 5.42

$p + 1$ algebras

$$\langle a, b \,|\, pa - baa,\ pb + bab,\ \text{class } 4\rangle \tag{6.425}$$

$$\langle a, b \,|\, pa - baa - baab,\ pb + bab - xbaab,\ \text{class } 4\rangle\ (\text{all } x) \tag{6.426}$$

7.30 Descendants of 5.45

Here we have a two parameter family of p algebras, with the isomorphism type depending on the value of $y^2 - \omega x^2 \bmod p$.

$$\langle a, b \,|\, pa + bab - xbaaa,\ pb + \omega baa - ybaaa,\ \text{class } 4\rangle \tag{6.427}$$

7.31 Descendants of 5.47

2 algebras

$$\langle a, b \,|\, ba,\ p^2 b,\ \text{class } 4\rangle \tag{6.428}$$

$$\langle a, b \,|\, ba - p^3 a,\ p^2 b,\ \text{class } 4\rangle \tag{6.429}$$

7.32 Descendants of 5.48

1 algebra

$$\langle a, b \,|\, ba - p^2 a,\ p^2 b,\ \text{class } 4\rangle \tag{6.430}$$

7.33 Descendants of 5.49

4 algebras

$$\langle a, b \,|\, baa,\ bab,\ pb,\ \text{class } 4\rangle \tag{6.431}$$

$$\langle a, b \,|\, baa - p^3 a,\ bab,\ pb,\ \text{class } 4\rangle \tag{6.432}$$

$$\langle a, b \,|\, baa,\ bab - p^3 a,\ pb,\ \text{class } 4\rangle \tag{6.433}$$

$$\langle a, b \,|\, baa,\ bab - \omega p^3 a,\ pb,\ \text{class } 4\rangle \tag{6.434}$$

7.34 Descendants of 5.50

$3 + 2\gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, bab,\ p^2 a,\ pb,\ \text{class } 4\rangle \tag{6.435}$$

$$\langle a, b \,|\, bab - baaa,\ p^2 a,\ pb,\ \text{class } 4\rangle \tag{6.436}$$

$$\langle a, b \,|\, bab,\ p^2 a - baaa,\ pb,\ \text{class } 4\rangle \tag{6.437}$$

$$\langle a, b \,|\, bab - baaa,\ p^2 a - baaa,\ pb,\ \text{class } 4\rangle \tag{6.438}$$

$$\langle a, b \,|\, bab - \omega baaa,\ p^2 a - baaa,\ pb,\ \text{class } 4\rangle \tag{6.439}$$

$$\langle a, b \,|\, bab - \omega^2 baaa,\ p^2 a - baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.440}$$

$$\langle a, b \,|\, bab - \omega^3 baaa,\ p^2 a - baaa,\ pb,\ \text{class } 4\rangle\ (p = 1 \bmod 4) \tag{6.441}$$

$$\langle a, b \,|\, bab,\ p^2 a,\ pb - baaa,\ \text{class } 4\rangle \tag{6.442}$$

$$\langle a, b \,|\, bab,\ p^2 a,\ pb - \omega baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.443}$$

$$\langle a, b \,|\, bab,\ p^2 a,\ pb - \omega^2 baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.444}$$

$$\langle a, b \,|\, bab - baaa,\ p^2 a,\ pb - baaa,\ \text{class } 4\rangle \tag{6.445}$$

$$\langle a, b \,|\, bab - baaa,\ p^2 a,\ pb - \omega baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.446}$$

$$\langle a, b \,|\, bab - baaa,\ p^2 a,\ pb - \omega^2 baaa,\ \text{class } 4\rangle\ (p = 1 \bmod 3) \tag{6.447}$$

7.35 Descendants of 5.51

$3(p+1)/2$ algebras

$$\langle a, b \,|\, bab,\ p^2a,\ pb - baa - xbaaa,\ \text{class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{6.448}$$

$$\langle a, b \,|\, bab,\ p^2a - baaa,\ pb - baa,\ \text{class } 4\rangle \tag{6.449}$$

$$\langle a, b \,|\, bab - baaa,\ p^2a - xbaaa,\ pb - baa,\ \text{class } 4\rangle \ (\text{all } x) \tag{6.450}$$

7.36 Descendants of 5.52

$3(p+1)/2$ algebras

$$\langle a, b \,|\, bab,\ p^2a,\ pb - \omega baa - xbaaa,\ \text{class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{6.451}$$

$$\langle a, b \,|\, bab,\ p^2a - baaa,\ pb - \omega baa,\ \text{class } 4\rangle \tag{6.452}$$

$$\langle a, b \,|\, bab - baaa,\ p^2a - xbaaa,\ pb - \omega baa,\ \text{class } 4\rangle \ (\text{all } x) \tag{6.453}$$

7.37 Descendants of 5.54

$3 + 2\gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, baa,\ p^2a,\ pb,\ \text{class } 4\rangle \tag{6.454}$$

$$\langle a, b \,|\, baa,\ p^2a,\ pb - babb,\ \text{class } 4\rangle \tag{6.455}$$

$$\langle a, b \,|\, baa,\ p^2a - babb,\ pb,\ \text{class } 4\rangle \tag{6.456}$$

$$\langle a, b \,|\, baa,\ p^2a - \omega babb,\ pb,\ \text{class } 4\rangle \ (p = 1 \bmod 3) \tag{6.457}$$

$$\langle a, b \,|\, baa,\ p^2a - \omega^2 babb,\ pb,\ \text{class } 4\rangle \ (p = 1 \bmod 3) \tag{6.458}$$

$$\langle a, b \,|\, baa - babb,\ p^2a,\ pb,\ \text{class } 4\rangle \tag{6.459}$$

$$\langle a, b \,|\, baa - babb,\ p^2a,\ pb - babb,\ \text{class } 4\rangle \tag{6.460}$$

$$\langle a, b \,|\, baa - babb,\ p^2a,\ pb - \omega babb,\ \text{class } 4\rangle \tag{6.461}$$

$$\langle a, b \,|\, baa - babb,\ p^2a,\ pb - \omega^2 babb,\ \text{class } 4\rangle \ (p = 1 \bmod 4) \tag{6.462}$$

$$\langle a, b \,|\, baa - babb,\ p^2a,\ pb - \omega^3 babb,\ \text{class } 4\rangle \ (p = 1 \bmod 4) \tag{6.463}$$

$$\langle a, b \,|\, baa - babb,\ p^2a - babb,\ pb,\ \text{class } 4\rangle \tag{6.464}$$

$$\langle a, b \,|\, baa - babb,\ p^2a - \omega babb,\ pb,\ \text{class } 4\rangle \ (p = 1 \bmod 3) \tag{6.465}$$

$$\langle a, b \,|\, baa - babb,\ p^2a - \omega^2 babb,\ pb,\ \text{class } 4\rangle \ (p = 1 \bmod 3) \tag{6.466}$$

7.38 Descendants of 5.58

2 algebras

$$\langle a, b \,|\, baa,\ pb - ba,\ \text{class } 4\rangle \tag{6.467}$$

$$\langle a, b \,|\, baa - p^3 a,\ pb - ba,\ \text{class } 4\rangle \tag{6.468}$$

7.39 Descendants of 5.60

$7 + 2\gcd(p-1,3) + 3\gcd(p-1,4)$ algebras

$$\langle a, b \,|\, bab,\ baaab,\ pa,\ pb,\ \text{class } 5\rangle \tag{6.469}$$

$$\langle a, b \,|\, bab,\ baaab,\ pa - baaaa,\ pb,\ \text{class } 5\rangle \tag{6.470}$$

$$\langle a, b \,|\, bab,\ baaab,\ pa,\ pb - baaaa,\ \text{class } 5\rangle \tag{6.471}$$

$$\langle a, b \,|\, bab,\ baaab,\ pa,\ pb - \omega baaaa,\ \text{class } 5\rangle \tag{6.472}$$

$$\langle a, b \,|\, bab,\ baaab,\ pa,\ pb - \omega^2 baaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.473}$$

$$\langle a, b \,|\, bab,\ baaab,\ pa,\ pb - \omega^3 baaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.474}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa,\ pb,\ \text{class } 5\rangle \tag{6.475}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa - baaaa,\ pb,\ \text{class } 5\rangle \tag{6.476}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa - \omega baaaa,\ pb,\ \text{class } 5\rangle \tag{6.477}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa - \omega^2 baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.478}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa - \omega^3 baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.479}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa - \omega^4 baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.480}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa - \omega^5 baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.481}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa,\ pb - baaaa,\ \text{class } 5\rangle \tag{6.482}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa,\ pb - \omega baaaa,\ \text{class } 5\rangle \tag{6.483}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa,\ pb - \omega^2 baaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.484}$$

$$\langle a, b \,|\, bab - baaaa,\ baaab,\ pa,\ pb - \omega^3 baaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.485}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa,\ pb,\ \text{class } 5\rangle \tag{6.486}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa - baaab,\ pb,\ \text{class } 5\rangle \tag{6.487}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa - \omega baaab,\ pb,\ \text{class } 5\rangle \tag{6.488}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa,\ pb - baaab,\ \text{class } 5\rangle \tag{6.489}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa - baaab,\ pb - baaab,\ \text{class } 5\rangle \tag{6.490}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa - \omega baaab,\ pb - baaab,\ \text{class } 5\rangle \tag{6.491}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa - \omega^2 baaab,\ pb - baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.492}$$

$$\langle a, b \,|\, bab,\ baaaa,\ pa - \omega^3 baaab,\ pb - baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.493}$$

7.40 Descendants of 5.65

$2\gcd(p-1,3) + \gcd(p-1,4) + 2\gcd(p-1,5)$ algebras

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa,\ pb,\ \text{class } 5\rangle \tag{6.494}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa,\ pb - baaab,\ \text{class } 5\rangle \tag{6.495}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa,\ pb - \omega baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.496}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa,\ pb - \omega^2 baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.497}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa,\ pb - \omega^3 baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.498}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa,\ pb - \omega^4 baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.499}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - baaab,\ pb,\ \text{class } 5\rangle \tag{6.500}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - \omega baaab,\ pb,\ \text{class } 5\rangle \tag{6.501}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - \omega^2 baaab,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.502}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - \omega^3 baaab,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.503}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - \omega^4 baaab,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.504}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - \omega^5 baaab,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 3) \tag{6.505}$$

For the next 5 algebras, we have $x \neq 0$, $x \sim x'$ if $x^5 = x'^5 \bmod p$:

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - xbaaab,\ pb - baaab,\ \text{class } 5\rangle \tag{6.506}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - xbaaab,\ pb - \omega baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.506A}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - xbaaab,\ pb - \omega^2 baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.506A}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - xbaaab,\ pb - \omega^3 baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.506A}$$

$$\langle a, b \,|\, bab - baaa,\ baaaa,\ pa - xbaaab,\ pb - \omega^4 baaab,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.506A}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa,\ pb,\ \text{class } 5\rangle \tag{6.507}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa,\ pb - baaaa,\ \text{class } 5\rangle \tag{6.508}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa,\ pb - \omega baaaa,\ \text{class } 5\rangle \tag{6.509}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa,\ pb - \omega^2 baaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.510}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa,\ pb - \omega^3 baaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 4) \tag{6.511}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - baaaa,\ pb,\ \text{class } 5\rangle \tag{6.512}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.513}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega^2 baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.514}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega^3 baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.515}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega^4 baaaa,\ pb,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.516}$$

For the next 5 algebras, we have $x \neq 0$, $x \sim x'$ if $x^5 = x'^5 \bmod p$:

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - baaaa,\ pb - xbaaaa,\ \text{class } 5\rangle \tag{6.517}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega baaaa,\ pb - xbaaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.517A}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega^2 baaaa,\ pb - xbaaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.517A}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega^3 baaaa,\ pb - xbaaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.517A}$$

$$\langle a, b \,|\, bab - baaa,\ baaab,\ pa - \omega^4 baaaa,\ pb - xbaaaa,\ \text{class } 5\rangle\ (p = 1 \bmod 5) \tag{6.517A}$$

7.41 Descendants of 5.73

2 algebras

$$\langle a, b \mid ba,\ pb,\ \text{class } 5\rangle \tag{6.518}$$

$$\langle a, b \mid ba - p^4 a,\ pb,\ \text{class } 5\rangle \tag{6.519}$$

8 Order p^7

For $p \geq 5$ the number of nilpotent Lie rings of order p^7 is

$$\begin{aligned}&3p^5 + 12p^4 + 44p^3 + 170p^2 + 707p + 2455\\&+(4p^2 + 44p + 291)\gcd(p-1,3) + (p^2 + 19p + 135)\gcd(p-1,4)\\&+(3p+31)\gcd(p-1,5) + 4\gcd(p-1,7) + 5\gcd(p-1,8) + \gcd(p-1,9).\end{aligned}$$

The following table lists the nilpotent Lie rings of order at most p^5 which have immediate descendants of order p^7, and gives the number of descendants.

3.1	$p + 14$
4.1	1361 if $p = 3$, otherwise $p^5 + 2p^4 + 7p^3 + 25p^2 + 88p + 270 + (p+4)\gcd(p-1,3) + \gcd(p-1,4)$
4.7	4
5.1	178 if $p = 3$, otherwise $p^2 + 15p + 125$
5.3	$p^4 + 5p^3 + 19p^2 + 64p + 140 + (p+6)\gcd(p-1,3) + (p+7)\gcd(p-1,4) + \gcd(p-1,5)$
5.8	$p + 8$
5.9	$4p^2 + 26p + 107 + 5\gcd(p-1,3) + (p+4)\gcd(p-1,4)$
5.10	$2p + 7$
5.12	$3p^2 + 17p + 53 + \gcd(p-1,3) + \gcd(p-1,4)$
5.14	$2p^5 + 7p^4 + 19p^3 + 49p^2 + 128p + 256 + (p^2 + 7p + 29)\gcd(p-1,3)$
	$+(p^2 + 7p + 24)\gcd(p-1,4) + (p+3)\gcd(p-1,5)$
5.15	$3p^2 + 12p + 14 + (p+2)\gcd(p-1,4)$
5.16	$p^4 + 2p^3 + 5p^2 + 14p$
5.18	$3p^3 + 6p^2 + 6p + 11 + (p+7)\gcd(p-1,3) + (p+1)\gcd(p-1,4) + \gcd(p-1,5)$
5.37	$p^2 + 8p + 25$
5.38	$p + 6 + (p^2 + 3p + 10)\gcd(p-1,3)$
5.60	$2p^2 + p + 3 + 2(p+1)\gcd(p-1,3) + (2p+4)\gcd(p-1,4) + \gcd(p-1,8)$
5.65	$p^3 + p^2 + p - 2 + 2\gcd(p-1,3) + \gcd(p-1,4) + (p+1)\gcd(p-1,5)$

In the two tables below we give the list of 42 nilpotent Lie rings of order at most p^5 which have immediate descendants of order p^6. For each of these Lie rings we give the number of immediate descendants of order p^6 which are capable, and we give the number of "grandchildren" of order p^7. Note that by "grandchildren" we mean Lie rings whose immediate ancestors have order p^6.

	capable descendants of order p^6
3.1	$3p + 27$
4.1	24
4.3	$5p + 37 + \gcd(p-1, 4)$
4.6	2
4.7	16
4.8	1
5.0	1
5.1	2
5.2	1
5.3	2
5.8	2
5.9	9
5.10	1
5.11	2
5.12	3
5.13	1
5.14	3
5.15	$2p + 9$
5.16	4
5.18	11
5.19	0
5.24	1
5.27	4
5.32	1
5.37	2
5.38	3
5.39	$1 + \gcd(p-1, 3) + \gcd(p-1, 4)/2$
5.40	$1 + \gcd(p-1, 3) + \gcd(p-1, 4)/2$
5.41	$p + 1 + \gcd(p-1, 3)$
5.42	$p + 1$
5.45	p
5.47	2
5.48	0
5.49	1
5.50	$2 + 2\gcd(p-1, 3)$
5.51	$(p+1)/2$
5.52	$(p+1)/2$
5.54	$3 + \gcd(p-1, 4)$
5.58	1
5.60	2
5.65	1
5.73	1

	grandchildren of order p^7
3.1	$2p^2+63p+362+(p+19)\gcd(p-1,3)+5\gcd(p-1,4)+\gcd(p-1,5)$
4.1	$p^3+13p^2+96p+595+(3p+21)\gcd(p-1,3)+(p+11)\gcd(p-1,4)+\gcd(p-1,5)$
4.3	$p^4+4p^3+17p^2+39p+72+(p^2+9p+47)\gcd(p-1,3)$
	$+(2p+8)\gcd(p-1,4)+2\gcd(p-1,5)+\gcd(p-1,7)$
4.6	4
4.7	$15p+41+16\gcd(p-1,3)+4\gcd(p-1,4)$
4.8	2
5.0	1
5.1	30
5.2	4
5.3	$35+(p+15)\gcd(p-1,3)+4\gcd(p-1,4)$
5.8	6
5.9	$5p+49+11\gcd(p-1,3)+4\gcd(p-1,4)$
5.10	5
5.11	7
5.12	$2p+20+7\gcd(p-1,3)+3\gcd(p-1,4)$
5.13	$p+1$
5.14	$p^2+9p+36+(p^2+5p+29)\gcd(p-1,3)+(p+7)\gcd(p-1,4)$
	$+\gcd(p-1,7)+\gcd(p-1,8)$
5.15	$10p+16+(2p+7)\gcd(p-1,3)+2\gcd(p-1,4)+2\gcd(p-1,5)$
5.16	$p^3+5p^2+13p+6+3\gcd(p-1,3)$
5.18	$2p^2+14p+10+(2p+8)\gcd(p-1,3)+7\gcd(p-1,4)+\gcd(p-1,5)$
5.19	0
5.24	3
5.27	$p^2+10p+34+(p+14)\gcd(p-1,3)+13\gcd(p-1,4)+6\gcd(p-1,5)+\gcd(p-1,7)$
5.32	$p^2+7p+3+2\gcd(p-1,3)+3\gcd(p-1,4)+\gcd(p-1,5)$
5.37	$5p+10+2\gcd(p-1,3)+\gcd(p-1,4)$
5.38	$p^3+3p^2+8p+18+5\gcd(p-1,3)+(p+5)\gcd(p-1,4)$
	$+3\gcd(p-1,5)+2\gcd(p-1,8)+\gcd(p-1,9)$
5.39	$\frac{1}{2}(p^2+2p+1+(p+5)\gcd(p-1,3)+(p+3)\gcd(p-1,4))$
5.40	$\frac{1}{2}(p^2+2p+1+(p+5)\gcd(p-1,3)+(p+3)\gcd(p-1,4))$
5.41	$3p^2+4p+(p+1)\gcd(p-1,3)+\gcd(p-1,4)$
5.42	$p+3$
5.45	$p+1$
5.47	3
5.48	0
5.49	4
5.50	$4p+5+(p+7)\gcd(p-1,3)+3\gcd(p-1,4)+2\gcd(p-1,5)$
5.51	$\frac{1}{2}(p+1)$
5.52	$\frac{1}{2}(p+1)$
5.54	$7p+9+4\gcd(p-1,3)+6\gcd(p-1,4)+2\gcd(p-1,5)$
5.58	2
5.60	$4p+3+2\gcd(p-1,3)+4\gcd(p-1,5)+\gcd(p-1,7)+\gcd(p-1,8)$
5.65	$2p^2+p+2p\gcd(p-1,3)+p\gcd(p-1,5)$
5.73	2

We now give the cyclic and seven generator nilpotent Lie rings of order p^7, and then list the rest, grouped according to their immediate ancestors.

$$\langle a \,|\, \text{class } 7\rangle \tag{7.0}$$

$$\langle a,b,c,d,e,f,g \,|\, \text{class } 1\rangle. \tag{7.1}$$

8.1 Descendants of 4.7

4 algebras

$$\langle a, b \,|\, pb, \text{ class } 3\rangle \tag{7.2}$$

$$\langle a, b \,|\, pb - baa, \text{ class } 3\rangle \tag{7.3}$$

$$\langle a, b \,|\, pb - \omega baa, \text{ class } 3\rangle \tag{7.4}$$

$$\langle a, b \,|\, pb - bab, \text{ class } 3\rangle \tag{7.5}$$

8.2 Descendants of 5.37

$p^2 + 8p + 25$ algebras

$$\langle a, b \,|\, baa, bab, pba, \text{ class } 3\rangle \tag{7.6}$$

$$\langle a, b \,|\, baa, bab, p^2b, \text{ class } 3\rangle \tag{7.7}$$

$$\langle a, b \,|\, baa, bab, p^2b - pba, \text{ class } 3\rangle \tag{7.8}$$

$$\langle a, b \,|\, bab, p^2a, p^2b - xbaa - pba, \text{ class } 3\rangle \text{ (all } x) \tag{7.9}$$

$$\langle a, b \,|\, bab, p^2a - baa, p^2b - xbaa - pba, \text{ class } 3\rangle \text{ (all } x) \tag{7.10}$$

$$\langle a, b \,|\, bab, p^2a, p^2b - baa, \text{ class } 3\rangle \tag{7.11}$$

$$\langle a, b \,|\, bab, p^2a - pba, p^2b - baa, \text{ class } 3\rangle \tag{7.12}$$

$$\langle a, b \,|\, bab, p^2a, p^2b - \omega baa, \text{ class } 3\rangle \tag{7.13}$$

$$\langle a, b \,|\, bab, p^2a - pba, p^2b - \omega baa, \text{ class } 3\rangle \tag{7.14}$$

$$\langle a, b \,|\, bab, p^2a, p^2b, \text{ class } 3\rangle \tag{7.15}$$

$$\langle a, b \,|\, bab, p^2a - baa, p^2b, \text{ class } 3\rangle \tag{7.16}$$

$$\langle a, b \,|\, bab, p^2a - pba, p^2b, \text{ class } 3\rangle \tag{7.17}$$

$$\langle a, b \,|\, bab, p^2a - baa - pba, p^2b, \text{ class } 3\rangle \tag{7.18}$$

$$\langle a, b \,|\, bab, pba, p^2a, \text{ class } 3\rangle \tag{7.19}$$

$$\langle a, b \,|\, bab, pba, p^2a - baa, \text{ class } 3\rangle \tag{7.20}$$

$$\langle a, b \,|\, bab, pba - baa, p^2a, \text{ class } 3\rangle \tag{7.21}$$

$$\langle a, b \,|\, bab, pba - baa, p^2a - baa, \text{ class } 3\rangle \tag{7.22}$$

$$\langle a, b \,|\, bab, pba, p^2b, \text{ class } 3\rangle \tag{7.23}$$

$$\langle a, b \,|\, bab, pba, p^2b - baa, \text{ class } 3\rangle \tag{7.24}$$

$$\langle a, b \,|\, bab, pba, p^2b - \omega baa, \text{ class } 3\rangle \tag{7.25}$$

$$\langle a, b \,|\, bab, pba - baa, p^2b - xbaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.26}$$

$$\langle a, b \,|\, p^2a - baa, p^2b, pba, \text{ class } 3\rangle \tag{7.27}$$

$$\langle a, b \,|\, p^2a - baa, p^2b - xbab, pba, \text{ class } 3\rangle \ (x \neq 0, x \sim x^{-1}) \tag{7.28}$$

$$\langle a, b \,|\, p^2a - baa - bab, p^2b - bab, pba, \text{ class } 3\rangle \tag{7.29}$$

$$\langle a, b \,|\, p^2a - baa - \omega bab, p^2b - bab, pba, \text{ class } 3\rangle \tag{7.30}$$

$$\langle a, b \,|\, p^2a, p^2b, pba, \text{ class } 3\rangle \tag{7.31}$$

$$\langle a, b \,|\, p^2a - bab, p^2b, pba, \text{ class } 3\rangle \tag{7.32}$$

$$\langle a, b \,|\, p^2a - \omega bab, p^2b, pba, \text{ class } 3\rangle \tag{7.33}$$

$$\langle a, b \,|\, p^2a - \omega bab, p^2b - baa, pba, \text{ class } 3\rangle \tag{7.34}$$

$$\langle a, b \,|\, p^2a - xbab, p^2b - baa - bab, pba, \text{ class } 3\rangle \text{ (all } x \text{ with } 1 + 4x \text{ not a square } \bmod p) \tag{7.35}$$

$$\langle a, b \,|\, p^2a - bab, p^2b - xbaa, pba - baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.36}$$

$$\langle a, b \,|\, p^2a - \omega bab, p^2b - xbaa, pba - baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.37}$$

$$\langle a, b \,|\, p^2a - ybab, p^2b - xbaa - bab, pba - baa, \text{ class } 3\rangle \text{ (all } x,\ y \neq 0) \tag{7.38}$$

$$\langle a, b \,|\, p^2a, p^2b - xbaa, pba - baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.39}$$

$$\langle a, b \,|\, p^2a - baa, p^2b - xbaa - bab, pba - baa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.40}$$

$$\langle a, b \,|\, p^2a, p^2b - bab, pba - baa, \text{ class } 3\rangle \tag{7.41}$$

$$\langle a, b \,|\, p^2a - baa, p^2b - xbab, pba - baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.42}$$

8.3 Descendants of 5.38

$p+6+(p^2+3p+10)\gcd(p-1,3)$ algebras

$$\langle a,b \,|\, babb, pa, pb, \text{ class } 4\rangle \tag{7.43}$$

$$\langle a,b \,|\, babb, pa, pb-baab, \text{ class } 4\rangle \tag{7.44}$$

$$\langle a,b \,|\, babb, pa-baaa, pb-xbaab, \text{ class } 4\rangle \text{ (all } x) \tag{7.45}$$

$$\langle a,b \,|\, babb, pa-baab, pb, \text{ class } 4\rangle \tag{7.46}$$

$$\langle a,b \,|\, babb, pa-baaa-baab, pb-2baab, \text{ class } 4\rangle \tag{7.47}$$

$$\langle a,b \,|\, babb, pa-baaa-\omega baab, pb-2baab, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.48}$$

$$\langle a,b \,|\, babb, pa-baaa-\omega^2 baab, pb-2baab, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.49}$$

$$\langle a,b \,|\, babb, pa, pb-baaa, \text{ class } 4\rangle \tag{7.50}$$

$$\langle a,b \,|\, babb, pa-baab, pb-baaa, \text{ class } 4\rangle \tag{7.51}$$

$$\langle a,b \,|\, babb, pa-\omega baab, pb-baaa, \text{ class } 4\rangle \tag{7.52}$$

$$\langle a,b \,|\, babb, pa-xbaab, pb-baaa-baab, \text{ class } 4\rangle \text{ (all } x) \tag{7.53}$$

$$\langle a,b \,|\, babb, pa, pb-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.54}$$

$$\langle a,b \,|\, babb, pa-baab, pb-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.55}$$

$$\langle a,b \,|\, babb, pa-\omega baab, pb-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.56}$$

$$\langle a,b \,|\, babb, pa-xbaab, pb-\omega baaa-baab, \text{ class } 4\rangle \text{ (all } x,\ p=1 \bmod 3) \tag{7.57}$$

$$\langle a,b \,|\, babb, pa, pb-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.58}$$

$$\langle a,b \,|\, babb, pa-baab, pb-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.59}$$

$$\langle a,b \,|\, babb, pa-\omega baab, pb-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.60}$$

$$\langle a,b \,|\, babb, pa-xbaab, pb-\omega^2 baaa-baab, \text{ class } 4\rangle \text{ (all } x,\ p=1 \bmod 3) \tag{7.61}$$

Algebra 5.38 has $\gcd(p-1,3)(p^2+3p+10)+p+6$ immediate descendants of order p^7. Of these $\frac{1}{2}((p^2+3p+11)\gcd(p-1,3)+1)$ come from one 4 parameter family of algebras, and $\frac{1}{2}(\gcd(p-1,3)(p^2+p+1)+5)$ come from another four parameter family. These two families have presentations:

$$\langle a, b \,|\, babb - baaa, pa - xbaaa - ybaab, pb - zbaaa - tbaab, \text{ class } 4\rangle \tag{7.62}$$

$$\langle a, b \,|\, babb - \omega baaa, pa - xbaaa - ybaab, pb - zbaaa - tbaab, \text{ class } 4\rangle \tag{7.63}$$

In both 7.62 and 7.63 we take the four parameters as entries in a 2×2 matrix

$$A = \begin{pmatrix} x & y \\ z & t \end{pmatrix},$$

and in both cases we consider the orbits of matrices A of this form over $\mathrm{GF}(p)$ under an action of the subgroup of $\mathrm{GL}(2, p)$ consisting of non-singular matrices of the form

$$\begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} \text{ or } \begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix}.$$

In 7.62 two matrices A and B give isomorphic Lie rings if and only if

$$B = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} A \begin{pmatrix} (\alpha^4 - \beta^4) & 2\alpha\beta(\alpha^2 - \beta^2) \\ 2\alpha\beta(\alpha^2 - \beta^2) & \alpha^4 - \beta^4 \end{pmatrix}^{-1}$$

or

$$B = \begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix} A \begin{pmatrix} -(\alpha^4 - \beta^4) & -2\alpha\beta(\alpha^2 - \beta^2) \\ 2\alpha\beta(\alpha^2 - \beta^2) & \alpha^4 - \beta^4 \end{pmatrix}^{-1}$$

for some α, β.

In 7.63, two matrices A and B give isomorphic Lie rings if and only if

$$B = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} A \begin{pmatrix} \alpha^4 - \omega^2\beta^4 & 2\alpha\beta(\alpha^2 - \omega\beta^2) \\ 2\omega\alpha\beta(\alpha^2 - \omega\beta^2) & \alpha^4 - \omega^2\beta^4 \end{pmatrix}^{-1}$$

or

$$B = \begin{pmatrix} \alpha & \beta \\ -\beta & -\alpha \end{pmatrix} A \begin{pmatrix} -(\alpha^4 - \omega^2\beta^4) & -2\alpha\beta(\alpha^2 - \omega\beta^2) \\ 2\omega\alpha\beta(\alpha^2 - \omega\beta^2) & \alpha^4 - \omega^2\beta^4 \end{pmatrix}^{-1}$$

for some α, β.

8.4 Descendants of 5.60

$2p^2 + p + 3 + 2(p+1)\gcd(p-1, 3) + (2p+4)\gcd(p-1, 4) + \gcd(p-1, 8)$ algebras

$$\langle a, b \,|\, bab, pa, pb, \text{ class } 5\rangle \tag{7.64}$$

$$\langle a, b \,|\, bab, pa, pb - baaab, \text{ class } 5\rangle \tag{7.65}$$

$$\langle a, b \,|\, bab, pa - baaaa, pb - xbaaab, \text{ class } 5\rangle \text{ (all } x) \tag{7.66}$$

$$\langle a, b \,|\, bab, pa - baaab, pb, \text{ class } 5\rangle \tag{7.67}$$

$$\langle a, b \,|\, bab, pa - \omega baaab, pb, \text{ class } 5\rangle \tag{7.68}$$

$$\langle a, b \,|\, bab, pa - baaaa - baaab, pb - baaab, \text{ class } 5\rangle \tag{7.69}$$

$$\langle a, b \,|\, bab, pa - baaaa - \omega baaab, pb - baaab, \text{ class } 5\rangle \tag{7.70}$$

$$\langle a, b \,|\, bab, pa - baaaa - \omega^2 baaab, pb - baaab, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.71}$$

$$\langle a, b \,|\, bab, pa - baaaa - \omega^3 baaab, pb - baaab, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.72}$$

$$\langle a, b \,|\, bab, pa, pb - baaaa, \text{ class } 5\rangle \tag{7.73}$$

$$\langle a, b \,|\, bab, pa - baaab, pb - baaaa, \text{ class } 5\rangle \tag{7.74}$$

$$\langle a, b \,|\, bab, pa - \omega baaab, pb - baaaa, \text{ class } 5\rangle \tag{7.75}$$

$$\langle a, b \,|\, bab, pa - xbaaab, pb - baaaa - baaab, \text{ class } 5\rangle \ (\text{all } x) \tag{7.76}$$

$$\langle a, b \,|\, bab, pa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.77}$$

$$\langle a, b \,|\, bab, pa - baaab, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.78}$$

$$\langle a, b \,|\, bab, pa - \omega baaab, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.79}$$

$$\langle a, b \,|\, bab, pa - xbaaab, pb - \omega baaaa - baaab, \text{ class } 5\rangle \ (\text{all } x) \tag{7.80}$$

$$\langle a, b \,|\, bab, pa, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.81}$$

$$\langle a, b \,|\, bab, pa - baaab, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.82}$$

$$\langle a, b \,|\, bab, pa - \omega baaab, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.83}$$

$$\langle a, b \,|\, bab, pa - xbaaab, pb - \omega^2 baaaa - baaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 4) \tag{7.84}$$

$$\langle a, b \,|\, bab, pa, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.85}$$

$$\langle a, b \,|\, bab, pa - baaab, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.86}$$

$$\langle a, b \,|\, bab, pa - \omega baaab, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.87}$$

$$\langle a, b \,|\, bab, pa - xbaaab, pb - \omega^3 baaaa - baaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 4) \tag{7.88}$$

$$\langle a, b \,|\, bab - baaaa, pa, pb, \text{ class } 5\rangle \tag{7.89}$$

$$\langle a, b \,|\, bab - baaaa, pa, pb - baaab, \text{ class } 5\rangle \tag{7.90}$$

$$\langle a, b \,|\, bab - baaaa, pa, pb - \omega baaab, \text{ class } 5\rangle \tag{7.91}$$

$$\langle a, b \,|\, bab - baaaa, pa, pb - \omega^2 baaab, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.92}$$

$$\langle a, b \,|\, bab - baaaa, pa, pb - \omega^3 baaab, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.93}$$

$$\langle a, b \,|\, bab - baaaa, pa, pb - \omega^4 baaab, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.94}$$

$$\langle a, b \,|\, bab - baaaa, pa, pb - \omega^5 baaab, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.95}$$

$$\langle a, b \,|\, bab - baaaa, pa - baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x) \tag{7.96}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x) \tag{7.97}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^2 baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.98}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^3 baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.99}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^4 baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.100}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^5 baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.101}$$

$$\langle a, b \,|\, bab - baaaa, pa - baaab, pb, \text{ class } 5\rangle \tag{7.102}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega baaab, pb, \text{ class } 5\rangle \tag{7.103}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^2 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.104}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^3 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.105}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^4 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 8) \tag{7.106}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^5 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 8) \tag{7.107}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^6 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 8) \tag{7.108}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^7 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 8) \tag{7.109}$$

$$\langle a, b \,|\, bab - baaaa, pa - baaaa - xbaaab, pb - baaab, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.110}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega baaaa - xbaaab, pb - \omega baaab, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.111}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^2 baaaa - xbaaab, pb - \omega^2 baaab, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.112}$$

$$\langle a, b \,|\, bab - baaaa, pa - \omega^3 baaaa - xbaaab, pb - \omega^3 baaab, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.113}$$

$$\langle a,b \,|\, bab-baaaa, pa-\omega^4 baaaa - xbaaab, pb-\omega^4 baaab, \text{ class } 5\rangle \ (x\neq 0, x\sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \quad (7.114)$$

$$\langle a,b \,|\, bab-baaaa, pa-\omega^5 baaaa - xbaaab, pb-\omega^5 baaab, \text{ class } 5\rangle \ (x\neq 0, x\sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \quad (7.115)$$

$$\langle a,b \,|\, bab-baaaa, pa-xbaaab, pb-baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y,\ p=3 \bmod 4) \quad (7.116)$$

$$\langle a,b \,|\, bab-baaaa, pa-xbaaab, pb-\omega baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y,\ p=3 \bmod 4) \quad (7.117)$$

$$\langle a,b \,|\, bab-baaaa, pa-xbaaab, pb-baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y,\ y\sim -y,\ p=1 \bmod 4) \quad (7.118)$$

$$\langle a,b \,|\, bab-baaaa, pa-xbaaab, pb-\omega baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y,\ y\sim -y,\ p=1 \bmod 4) \quad (7.119)$$

$$\langle a,b \,|\, bab-baaaa, pa-xbaaab, pb-\omega^2 baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y,\ y\sim -y,\ p=1 \bmod 4) \quad (7.120)$$

$$\langle a,b \,|\, bab-baaaa, pa-xbaaab, pb-\omega^3 baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y,\ y\sim -y,\ p=1 \bmod 4) \quad (7.121)$$

8.5 Descendants of 5.65

$p^3+p^2+p-2+2\gcd(p-1,3)+\gcd(p-1,4)+(p+1)\gcd(p-1,5)$ algebras

$$\langle a,b \,|\, bab-baaa, pa-xbaaaa - ybaaab, pb-baaaa - zbaaab, \text{ class } 5\rangle \ (x\neq 0, \text{all } y,z,\ x\sim x' \text{ if } x^4 = x'^4 \bmod p) \quad (7.122)$$

$$\langle a,b \,|\, bab-baaa, pa-xbaaaa - ybaaab, pb-\omega baaaa - zbaaab, \text{ class } 5\rangle \ (x\neq 0, \text{all } y,z,\ x\sim x' \text{ if } x^4 = x'^4 \bmod p) \quad (7.123)$$

$$\langle a,b \,|\, bab-baaa, pa-xbaaaa - ybaaab, pb-\omega^2 baaaa - zbaaab, \text{ class } 5\rangle \ (x\neq 0, \text{all } y,z,\ x\sim x' \text{ if } x^4 = x'^4 \bmod p,\ p=1 \bmod 4) \quad (7.124)$$

$$\langle a,b \,|\, bab-baaa, pa-xbaaaa - ybaaab, pb-\omega^3 baaaa - zbaaab, \text{ class } 5\rangle \ (x\neq 0, \text{all } y,z,\ x\sim x' \text{ if } x^4 = x'^4 \bmod p,\ p=1 \bmod 4) \quad (7.125)$$

$$\langle a,b \,|\, bab-baaa, pa-xbaaab, pb-baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p) \quad (7.126)$$

$$\langle a,b \,|\, bab-baaa, pa-xbaaab, pb-\omega baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p) \quad (7.127)$$

$$\langle a,b \,|\, bab-baaa, pa-xbaaab, pb-\omega^2 baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p,\ p=1 \bmod 4) \quad (7.128)$$

$$\langle a,b \,|\, bab-baaa, pa-xbaaab, pb-\omega^3 baaaa - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p,\ p=1 \bmod 4) \quad (7.129)$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - baaaa, \text{ class } 5\rangle \ (x \neq 0,\ p = 3 \bmod 4) \tag{7.130}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega baaaa, \text{ class } 5\rangle \ (x \neq 0,\ p = 3 \bmod 4) \tag{7.131}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - baaaa, \text{ class } 5\rangle \ (x \neq 0,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.132}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega baaaa, \text{ class } 5\rangle \ (x \neq 0,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.133}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (x \neq 0,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.134}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (x \neq 0,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.135}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - baaaa, \text{ class } 5\rangle \tag{7.136}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.137}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.138}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.139}$$

$$\langle a, b \,|\, bab - baaa, pa - baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x) \tag{7.140}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 5) \tag{7.141}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^2 baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 5) \tag{7.142}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^3 baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 5) \tag{7.143}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^4 baaaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 5) \tag{7.144}$$

$$\langle a, b \,|\, bab - baaa, pa - baaaa - ybaaab, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ \ y \neq 0,\ y \sim y' \text{ if } y^5 = y'^5 \bmod p) \tag{7.145}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega baaaa - ybaaab, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ \ y \neq 0,\ y \sim y' \text{ if } y^5 = y'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.146}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^2 baaaa - ybaaab, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ y \neq 0,\ y \sim y' \text{ if } y^5 = y'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.147}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^3 baaaa - ybaaab, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ y \neq 0,\ y \sim y' \text{ if } y^5 = y'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.148}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^4 baaaa - ybaaab, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ y \neq 0,\ y \sim y' \text{ if } y^5 = y'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.149}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - baaab, \text{ class } 5\rangle \tag{7.150}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - \omega baaab, \text{ class } 5\rangle \ (p = 1 \bmod 5) \tag{7.151}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - \omega^2 baaab, \text{ class } 5\rangle \ (p = 1 \bmod 5) \tag{7.152}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - \omega^3 baaab, \text{ class } 5\rangle \ (p = 1 \bmod 5) \tag{7.153}$$

$$\langle a, b \,|\, bab - baaa, pa, pb - \omega^4 baaab, \text{ class } 5\rangle \ (p = 1 \bmod 5) \tag{7.154}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - baaab, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^5 = x'^5 \bmod p) \tag{7.155}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega baaab, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.156}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega^2 baaab, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.157}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega^3 baaab, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.158}$$

$$\langle a, b \,|\, bab - baaa, pa - xbaaab, pb - \omega^4 baaab, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.159}$$

$$\langle a, b \,|\, bab - baaa, pa, pb, \text{ class } 5\rangle \tag{7.160}$$

$$\langle a, b \,|\, bab - baaa, pa - baaab, pb, \text{ class } 5\rangle \tag{7.161}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega baaab, pb, \text{ class } 5\rangle \tag{7.162}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^2 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.163}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^3 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.164}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^4 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.165}$$

$$\langle a, b \,|\, bab - baaa, pa - \omega^5 baaab, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.166}$$

8.6 Descendants of 6.366

2 algebras

$$\langle a, b \,|\, ba, p^3 b, \text{ class } 4\rangle \tag{7.167}$$

$$\langle a, b \,|\, ba - p^3 a, p^3 b, \text{ class } 4\rangle \tag{7.168}$$

8.7 Descendants of 6.367

2 algebras

$$\langle a, b \,|\, ba - p^2a, p^3a, \text{ class } 4\rangle \tag{7.169}$$

$$\langle a, b \,|\, ba - p^2a, p^3b, \text{ class } 4\rangle \tag{7.170}$$

8.8 Descendants of 6.368

$9 + 6\gcd(p-1, 3)$ algebras

$$\langle a, b \,|\, baaa, baab, p^2a, pb, \text{ class } 4\rangle \tag{7.171}$$

$$\langle a, b \,|\, baaa, baab, p^2a, pb - babb, \text{ class } 4\rangle \tag{7.172}$$

$$\langle a, b \,|\, baaa, baab, p^2a - babb, pb, \text{ class } 4\rangle \tag{7.173}$$

$$\langle a, b \,|\, baaa, baab, p^2a - \omega babb, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.174}$$

$$\langle a, b \,|\, baaa, baab, p^2a - \omega^2 babb, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.175}$$

$$\langle a, b \,|\, baab, babb - baaa, p^2a, pb, \text{ class } 4\rangle \tag{7.176}$$

$$\langle a, b \,|\, baab, babb - baaa, p^2a, pb - baaa, \text{ class } 4\rangle \tag{7.177}$$

$$\langle a, b \,|\, baab, babb - baaa, p^2a, pb - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.178}$$

$$\langle a, b \,|\, baab, babb - baaa, p^2a, pb - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.179}$$

$$\langle a, b \,|\, baab, babb - baaa, p^2a - baaa, pb, \text{ class } 4\rangle \tag{7.180}$$

$$\langle a, b \,|\, baab, babb - baaa, p^2a - \omega baaa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.181}$$

$$\langle a, b \,|\, baab, babb - baaa, p^2a - \omega^2 baaa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.182}$$

$$\langle a, b \,|\, baab, babb - \omega baaa, p^2a, pb, \text{ class } 4\rangle \tag{7.183}$$

$$\langle a, b \,|\, baab, babb - \omega baaa, p^2a, pb - baaa, \text{ class } 4\rangle \tag{7.184}$$

$$\langle a, b \,|\, baab, babb - \omega baaa, p^2a, pb - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.185}$$

$$\langle a, b \,|\, baab, babb - \omega baaa, p^2a, pb - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.186}$$

$$\langle a, b \,|\, baab, babb - \omega baaa, p^2a - baaa, pb, \text{ class } 4\rangle \tag{7.187}$$

$$\langle a, b \,|\, baab, babb - \omega baaa, p^2a - \omega baaa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.188}$$

$$\langle a, b \,|\, baab, babb - \omega baaa, p^2a - \omega^2 baaa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.189}$$

$$\langle a, b \,|\, baaa, babb, p^2a, pb, \text{ class } 4\rangle \tag{7.190}$$

$$\langle a, b \,|\, baaa, babb, p^2a, pb - baab, \text{ class } 4\rangle \tag{7.191}$$

$$\langle a, b \,|\, baaa, babb, p^2a - baab, pb, \text{ class } 4\rangle \tag{7.192}$$

$$\langle a, b \,|\, baab, babb, p^2a, pb, \text{ class } 4\rangle \tag{7.193}$$

$$\langle a, b \,|\, baab, babb, p^2a, pb - baaa, \text{ class } 4\rangle \tag{7.194}$$

$$\langle a, b \,|\, baab, babb, p^2a, pb - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.195}$$

$$\langle a, b \,|\, baab, babb, p^2a, pb - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.196}$$

$$\langle a, b \,|\, baab, babb, p^2a - baaa, pb, \text{ class } 4\rangle \tag{7.197}$$

8.9 Descendants of 6.369

$5 + \gcd(p-1, 3)$ algebras

$$\langle a, b \,|\, baaa, p^2a, pb - bab, \text{ class } 4\rangle \tag{7.198}$$

$$\langle a, b \,|\, baaa, p^2a, pb - bab - baab, \text{ class } 4\rangle \tag{7.199}$$

$$\langle a, b \,|\, baaa, p^2a - baab, pb - bab, \text{ class } 4\rangle \tag{7.200}$$

$$\langle a, b \,|\, baab, p^2a, pb - bab, \text{ class } 4\rangle \tag{7.201}$$

$$\langle a, b \,|\, baab, p^2a, pb - bab - baaa, \text{ class } 4\rangle \tag{7.202}$$

$$\langle a, b \,|\, baab, p^2a, pb - bab - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.203}$$

$$\langle a, b \,|\, baab, p^2a, pb - bab - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.204}$$

$$\langle a, b \,|\, baab, p^2a - baaa, pb - bab, \text{ class } 4\rangle \tag{7.205}$$

8.10 Descendants of 6.370

$2p + 2 + (p+1)/2 + \gcd(p-1,3) + \gcd(p-1,4)/2$ algebras

$$\langle a, b \,|\, baaa, p^2a, pb - baa, \text{ class } 4\rangle \tag{7.206}$$

$$\langle a, b \,|\, baaa, p^2a, pb - baa - babb, \text{ class } 4\rangle \tag{7.207}$$

$$\langle a, b \,|\, baaa, p^2a, pb - baa - \omega babb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.208}$$

$$\langle a, b \,|\, baaa, p^2a - babb, pb - baa, \text{ class } 4\rangle \tag{7.209}$$

$$\langle a, b \,|\, baaa, p^2a - \omega babb, pb - baa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.210}$$

$$\langle a, b \,|\, baaa, p^2a - \omega^2 babb, pb - baa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.211}$$

$$\langle a, b \,|\, babb, p^2a, pb - baa - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.212}$$

$$\langle a, b \,|\, babb, p^2a - baaa, pb - baa, \text{ class } 4\rangle \tag{7.213}$$

$$\langle a, b \,|\, babb - baaa, p^2a, pb - baa - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.214}$$

$$\langle a, b \,|\, babb - baaa, p^2a - xbaaa, pb - baa, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.215}$$

$$\langle a, b \,|\, babb - \omega baaa, p^2a, pb - baa - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.216}$$

$$\langle a, b \,|\, babb - \omega baaa, p^2a - xbaaa, pb - baa, \text{ class } 4\rangle \ (x \neq 0,\ x \sim -x) \tag{7.217}$$

8.11 Descendants of 6.371

$2p + 2 + (p+1)/2 + \gcd(p-1,3) + \gcd(p-1,4)/2$ algebras

$$\langle a, b \,|\, baaa, p^2a, pb - \omega baa, \text{ class } 4\rangle \tag{7.218}$$

$$\langle a, b \,|\, baaa, p^2a, pb - \omega baa - babb, \text{ class } 4\rangle \tag{7.219}$$

$$\langle a, b \,|\, baaa, p^2a, pb - \omega baa - \omega babb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.220}$$

$$\langle a, b \,|\, baaa, p^2a - babb, pb - \omega baa, \text{ class } 4\rangle \tag{7.221}$$

$$\langle a, b \,|\, baaa, p^2a - \omega babb, pb - \omega baa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.222}$$

$$\langle a, b \,|\, baaa, p^2a - \omega^2 babb, pb - \omega baa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.223}$$

$$\langle a, b \,|\, babb, p^2a, pb - \omega baa - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.224}$$

$$\langle a, b \,|\, babb, p^2a - baaa, pb - \omega baa, \text{ class } 4\rangle \tag{7.225}$$

$$\langle a, b \,|\, babb - baaa, p^2a, pb - \omega baa - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x) \tag{7.226}$$

$$\langle a, b \,|\, babb - baaa, p^2a - xbaaa, pb - \omega baa, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.227}$$

$$\langle a, b \,|\, babb - \omega baaa, p^2a, pb - \omega baa - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x) \tag{7.228}$$

$$\langle a, b \,|\, babb - \omega baaa, p^2a - xbaaa, pb - \omega baa, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.229}$$

8.12 Descendants of 6.372

$(p+1)/2 + \gcd(p-1,3) + \gcd(p-1,4)/2$ algebras

$$\langle a, b \,|\, p^2a - bab, pb, \text{ class } 4\rangle \tag{7.230}$$

$$\langle a, b \,|\, p^2a - bab, pb - baaa, \text{ class } 4\rangle \tag{7.231}$$

$$\langle a, b \,|\, p^2a - bab, pb - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.232}$$

$$\langle a, b \,|\, p^2a - bab, pb - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.233}$$

$$\langle a, b \,|\, p^2a - bab - baaa, pb - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.234}$$

$$\langle a, b \,|\, p^2a - bab - \omega baaa, pb - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.235}$$

$$\langle a, b \,|\, p^2a - bab - baaa, pb - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x,\ p = 3 \bmod 4) \tag{7.236}$$

8.13 Descendants of 6.373

p algebras

$$\langle a, b \,|\, p^2a - bab, pb - baa - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x) \tag{7.237}$$

$$\langle a, b \,|\, p^2a - bab - xbaaa, pb - baa, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.238}$$

8.14 Descendants of 6.374

p algebras

$$\langle a, b \,|\, p^2a - bab, pb - \omega baa - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x) \tag{7.239}$$

$$\langle a, b \,|\, p^2a - bab - xbaaa, pb - \omega baa, \text{ class } 4\rangle \ (x \neq 0,\ x \sim -x) \tag{7.240}$$

8.15 Descendants of 6.375

$(p+1)/2 + \gcd(p-1,3) + \gcd(p-1,4)/2$ algebras

$$\langle a, b \,|\, p^2 a - \omega bab, pb, \text{ class } 4\rangle \tag{7.241}$$

$$\langle a, b \,|\, p^2 a - \omega bab, pb - baaa, \text{ class } 4\rangle \tag{7.242}$$

$$\langle a, b \,|\, p^2 a - \omega bab, pb - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.243}$$

$$\langle a, b \,|\, p^2 a - \omega bab, pb - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.244}$$

$$\langle a, b \,|\, p^2 a - \omega bab - baaa, pb - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.245}$$

$$\langle a, b \,|\, p^2 a - \omega bab - \omega baaa, pb - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim x'\ if\ x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.246}$$

$$\langle a, b \,|\, p^2 a - \omega bab - baaa, pb - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x,\ p = 3 \bmod 4) \tag{7.247}$$

8.16 Descendants of 6.376

p algebras

$$\langle a, b \,|\, p^2 a - \omega bab, pb - baa - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.248}$$

$$\langle a, b \,|\, p^2 a - \omega bab - xbaaa, pb - baa, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.249}$$

8.17 Descendants of 6.377

p algebras

$$\langle a, b \,|\, p^2 a - \omega bab, pb - \omega baa - xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.250}$$

$$\langle a, b \,|\, p^2 a - \omega bab - xbaaa, pb - \omega baa, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.251}$$

8.18 Descendants of 6.378

$p + 1 + \gcd(p-1,3)$ algebras

$$\langle a, b \,|\, p^2 a - baa, pb, \text{ class } 4\rangle \tag{7.252}$$

$$\langle a, b \,|\, p^2 a - baa - babb, pb, \text{ class } 4\rangle \tag{7.253}$$

$$\langle a, b \,|\, p^2 a - baa - \omega babb, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.254}$$

$$\langle a, b \,|\, p^2 a - baa - \omega^2 babb, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.255}$$

$$\langle a, b \,|\, p^2 a - baa - xbabb, pb - babb, \text{ class } 4\rangle \ (\text{all } x) \tag{7.256}$$

8.19 Descendants of 6.379

$5 + 2\gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a, b \,|\, p^3a, baa, pb, \text{ class } 4\rangle \tag{7.257}$$

$$\langle a, b \,|\, p^3a, baa, pb - babb, \text{ class } 4\rangle \tag{7.258}$$

$$\langle a, b \,|\, p^3a, baa - babb, pb, \text{ class } 4\rangle \tag{7.259}$$

$$\langle a, b \,|\, p^3a, baa - babb, pb - babb, \text{ class } 4\rangle \tag{7.260}$$

$$\langle a, b \,|\, p^3a, baa - babb, pb - \omega babb, \text{ class } 4\rangle \tag{7.261}$$

$$\langle a, b \,|\, p^3a, baa - babb, pb - \omega^2 babb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.262}$$

$$\langle a, b \,|\, p^3a, baa - babb, pb - \omega^3 babb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.263}$$

$$\langle a, b \,|\, babb, baa, pb, \text{ class } 4\rangle \tag{7.264}$$

$$\langle a, b \,|\, babb, baa - p^3a, pb, \text{ class } 4\rangle \tag{7.265}$$

$$\langle a, b \,|\, p^3a - babb, baa, pb, \text{ class } 4\rangle \tag{7.266}$$

$$\langle a, b \,|\, p^3a - babb, baa - babb, pb, \text{ class } 4\rangle \tag{7.267}$$

$$\langle a, b \,|\, p^3a - \omega babb, baa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.268}$$

$$\langle a, b \,|\, p^3a - \omega babb, baa - babb, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.269}$$

$$\langle a, b \,|\, p^3a - \omega^2 babb, baa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.270}$$

$$\langle a, b \,|\, p^3a - \omega^2 babb, baa - babb, pb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.271}$$

8.20 Descendants of 6.380

p algebras

$$\langle a, b \,|\, baa - xp^3a, pb - bab, \text{ class } 4\rangle \ (\text{all } x) \tag{7.272}$$

8.21 Descendants of 6.381

$6 + 2\gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a, b \,|\, baaa, bab, pb, \text{ class } 4\rangle \tag{7.273}$$

$$\langle a, b \,|\, baaa, bab - p^3a, pb, \text{ class } 4\rangle \tag{7.274}$$

$$\langle a, b \,|\, baaa, bab - \omega p^3a, pb, \text{ class } 4\rangle \tag{7.275}$$

$$\langle a, b \,|\, p^3a, bab, pb, \text{ class } 4\rangle \tag{7.276}$$

$$\langle a, b \,|\, p^3a, bab - baaa, pb, \text{ class } 4\rangle \tag{7.277}$$

$$\langle a, b \,|\, p^3a, bab, pb - baaa, \text{ class } 4\rangle \tag{7.278}$$

$$\langle a, b \,|\, p^3a, bab - baaa, pb - baaa, \text{ class } 4\rangle \tag{7.279}$$

$$\langle a, b \,|\, p^3a, bab, pb - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.280}$$

$$\langle a, b \,|\, p^3a, bab - baaa, pb - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.281}$$

$$\langle a, b \,|\, p^3a, bab, pb - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.282}$$

$$\langle a, b \,|\, p^3a, bab - baaa, pb - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.283}$$

$$\langle a, b \,|\, p^3a - baaa, bab, pb, \text{ class } 4\rangle \tag{7.284}$$

$$\langle a, b \,|\, p^3a - baaa, bab - baaa, pb, \text{ class } 4\rangle \tag{7.285}$$

$$\langle a, b \,|\, p^3a - baaa, bab - \omega baaa, pb, \text{ class } 4\rangle \tag{7.286}$$

$$\langle a, b \,|\, p^3a - baaa, bab - \omega^2 baaa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.287}$$

$$\langle a, b \,|\, p^3a - baaa, bab - \omega^3 baaa, pb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.288}$$

8.22 Descendants of 6.382

$p + 5 + (p - 1)/2$ algebras

$$\langle a, b \,|\, baaa, bab, pb - baa, \text{ class } 4\rangle \tag{7.289}$$

$$\langle a, b \,|\, baaa, bab - p^3 a, pb - baa, \text{ class } 4\rangle \tag{7.290}$$

$$\langle a, b \,|\, baaa, bab - \omega p^3 a, pb - baa, \text{ class } 4\rangle \tag{7.291}$$

$$\langle a, b \,|\, p^3 a, bab, pb - baa - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x) \tag{7.292}$$

$$\langle a, b \,|\, p^3 a, bab - baaa, pb - baa, \text{ class } 4\rangle \tag{7.293}$$

$$\langle a, b \,|\, p^3 a - baaa, bab - xbaaa, pb - baa, \text{ class } 4\rangle \text{ (all } x) \tag{7.294}$$

8.23 Descendants of 6.383

$p + 5 + (p - 1)/2$ algebras

$$\langle a, b \,|\, baaa, bab, pb - \omega baa, \text{ class } 4\rangle \tag{7.295}$$

$$\langle a, b \,|\, baaa, bab - p^3 a, pb - \omega baa, \text{ class } 4\rangle \tag{7.296}$$

$$\langle a, b \,|\, baaa, bab - \omega p^3 a, pb - \omega baa, \text{ class } 4\rangle \tag{7.297}$$

$$\langle a, b \,|\, p^3 a, bab, pb - \omega baa - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x) \tag{7.298}$$

$$\langle a, b \,|\, p^3 a, bab - baaa, pb - \omega baa, \text{ class } 4\rangle \tag{7.299}$$

$$\langle a, b \,|\, p^3 a - baaa, bab - xbaaa, pb - \omega baa, \text{ class } 4\rangle \text{ (all } x) \tag{7.300}$$

8.24 Descendants of 6.384

2 algebras

$$\langle a, b \,|\, baaa, pb - ba, \text{ class } 4\rangle \tag{7.301}$$

$$\langle a, b \,|\, p^3 a, pb - ba, \text{ class } 4\rangle \tag{7.302}$$

8.25 Descendants of 6.386

$p + 5$ algebras

$$\langle a, b \,|\, baa, bab, pba, p^2b, \text{ class } 4\rangle \tag{7.303}$$

$$\langle a, b \,|\, baa - p^3a, bab, pba, p^2b, \text{ class } 4\rangle \tag{7.304}$$

$$\langle a, b \,|\, baa, bab - p^3a, pba, p^2b, \text{ class } 4\rangle \tag{7.305}$$

$$\langle a, b \,|\, baa, bab - \omega p^3a, pba, p^2b, \text{ class } 4\rangle \tag{7.306}$$

$$\langle a, b \,|\, baa, bab - xp^3a, pba - p^3a, p^2b, \text{ class } 4\rangle \text{ (all } x) \tag{7.307}$$

$$\langle a, b \,|\, baa - p^3a, bab, pba - p^3a, p^2b, \text{ class } 4\rangle \tag{7.308}$$

8.26 Descendants of 6.388

$4p + 5 + 2\gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b \,|\, bab, pba, p^2a, p^2b, \text{ class } 4\rangle \tag{7.309}$$

$$\langle a, b \,|\, bab - baaa, pba, p^2a, p^2b, \text{ class } 4\rangle \tag{7.310}$$

$$\langle a, b \,|\, bab, pba - baaa, p^2a, p^2b, \text{ class } 4\rangle \tag{7.311}$$

$$\langle a, b \,|\, bab - baaa, pba - baaa, p^2a, p^2b, \text{ class } 4\rangle \tag{7.312}$$

$$\langle a, b \,|\, bab, pba - \omega baaa, p^2a, p^2b, \text{ class } 4\rangle \tag{7.313}$$

$$\langle a, b \,|\, bab - baaa, pba - \omega baaa, p^2a, p^2b, \text{ class } 4\rangle \tag{7.314}$$

$$\langle a, b \,|\, bab, pba, p^2a - baaa, p^2b, \text{ class } 4\rangle \tag{7.315}$$

$$\langle a, b \,|\, bab - baaa, pba, p^2a - baaa, p^2b, \text{ class } 4\rangle \tag{7.316}$$

$$\langle a, b \,|\, bab - \omega baaa, pba, p^2a - baaa, p^2b, \text{ class } 4\rangle \tag{7.317}$$

$$\langle a, b \,|\, bab - \omega^2 baaa, pba, p^2a - baaa, p^2b, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.318}$$

$$\langle a, b \,|\, bab - \omega^3 baaa, pba, p^2a - baaa, p^2b, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.319}$$

$$\langle a, b \,|\, bab - xbaaa, pba - baaa, p^2a - baaa, p^2b, \text{ class } 4\rangle \text{ (all } x) \tag{7.320}$$

$$\langle a, b \,|\, bab - xbaaa, pba - \omega baaa, p^2a - baaa, p^2b, \text{ class } 4\rangle \text{ (all } x) \tag{7.321}$$

$$\langle a, b \,|\, bab, pba, p^2a, p^2b - baaa, \text{ class } 4\rangle \tag{7.322}$$

$$\langle a, b \,|\, bab - baaa, pba, p^2a, p^2b - baaa, \text{ class } 4\rangle \tag{7.323}$$

$$\langle a, b \,|\, bab, pba - xbaaa, p^2a, p^2b - baaa, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.324}$$

$$\langle a, b \,|\, bab - baaa, pba - xbaaa, p^2a, p^2b - baaa, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.325}$$

$$\langle a, b \,|\, bab, pba, p^2a, p^2b - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.326}$$

$$\langle a, b \,|\, bab - baaa, pba, p^2a, p^2b - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.327}$$

$$\langle a, b \,|\, bab, pba - xbaaa, p^2a, p^2b - \omega baaa, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.328}$$

$$\langle a, b \,|\, bab - baaa, pba - xbaaa, p^2a, p^2b - \omega baaa, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.329}$$

$$\langle a, b \,|\, bab, pba, p^2a, p^2b - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.330}$$

$$\langle a, b \,|\, bab - baaa, pba, p^2a, p^2b - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.331}$$

$$\langle a, b \,|\, bab, pba - xbaaa, p^2a, p^2b - \omega^2 baaa, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.332}$$

$$\langle a, b \,|\, bab - baaa, pba - xbaaa, p^2a, p^2b - \omega^2 baaa, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.333}$$

8.27 Descendants of 6.394

$p^2 + 3p + 10 + 2\gcd(p-1,3) + 3\gcd(p-1,4) + 2\gcd(p-1,5) + \gcd(p-1,8) + \gcd(p-1,9)$ algebras

$$\langle a, b \,|\, baaaa, baab, babb, pa, pb, \text{ class } 5\rangle \tag{7.334}$$

$$\langle a, b \,|\, baaaa, baab, babb, pa - baaab, pb, \text{ class } 5\rangle \tag{7.335}$$

$$\langle a, b \,|\, baaaa, baab, babb, pa - \omega baaab, pb, \text{ class } 5\rangle \tag{7.336}$$

$$\langle a, b \,|\, baaaa, baab, babb, pa, pb - baaab, \text{ class } 5\rangle \tag{7.337}$$

$$\langle a, b \,|\, baaaa, baab, babb, pa - baaab, pb - baaab, \text{ class } 5\rangle \tag{7.338}$$

$$\langle a, b \,|\, baaaa, baab, babb, pa - \omega baaab, pb - baaab, \text{ class } 5\rangle \tag{7.339}$$

$$\langle a, b \,|\, baaaa, baab, babb, pa - \omega^2 baaab, pb - baaab, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.340}$$

$$\langle a, b \,|\, baaaa, baab, babb, pa - \omega^3 baaab, pb - baaab, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.341}$$

$$\langle a, b \,|\, baaaa, baab, babb - baaab, pa, pb, \text{ class } 5\rangle \tag{7.342}$$

$$\langle a, b \,|\, baaaa, baab, babb - baaab, pa, pb - baaab, \text{ class } 5\rangle \tag{7.343}$$

$$\langle a, b \,|\, baaaa, baab, babb - baaab, pa, pb - xbaaab, \text{ class } 5\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.344}$$

In the following presentation we have $y = 1, \omega, \omega^2, \omega^3, \omega^4, \omega^5$, all x, $x \sim x'$ if $x^6 = x'^6 \bmod p$, $p = 1 \bmod 3$:

$$\langle a, b \,|\, baaaa, baab, babb - baaab, pa - ybaaab, pb - xbaaab, \text{ class } 5\rangle \tag{7.345}$$

$$\langle a, b \,|\, baaaa, baab, babb - baaab, pa - ybaaab, pb - xbaaab, \text{ class } 5\rangle \ (y = 1, \omega, \text{all } x,\ x \sim -x,\ p = 2 \bmod 3) \tag{7.346}$$

$$\langle a, b \,|\, baaaa, baab - baaab, babb, pa, pb, \text{ class } 5\rangle \tag{7.347}$$

$$\langle a, b \,|\, baaaa, baab - baaab, babb, pa - baaab, pb, \text{ class } 5\rangle \tag{7.348}$$

$$\langle a, b \,|\, baaaa, baab - baaab, babb, pa - \omega baaab, pb, \text{ class } 5\rangle \tag{7.349}$$

$$\langle a, b \,|\, baaaa, baab - baaab, babb, pa - xbaaab, pb - baaab, \text{ class } 5\rangle \ (\text{all } x) \tag{7.350}$$

$$\langle a, b \,|\, baaaa, baab - baaab, babb - baaab, pa - xbaaab, pb - ybaaab, \text{ class } 5\rangle \ (\text{all } x,\ y) \tag{7.351}$$

$$\langle a, b \,|\, baaab, baab, babb, pa, pb, \text{ class } 5\rangle \tag{7.352}$$

$$\langle a, b \,|\, baaab, baab, babb, pa, pb - baaaa, \text{ class } 5\rangle \tag{7.353}$$

$$\langle a, b \,|\, baaab, baab, babb, pa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.354}$$

$$\langle a, b \,|\, baaab, baab, babb, pa, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.355}$$

$$\langle a, b \,|\, baaab, baab, babb, pa, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.356}$$

$$\langle a, b \,|\, baaab, baab, babb, pa - baaaa, pb, \text{ class } 5\rangle \tag{7.357}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb, pa, pb, \text{ class } 5\rangle \tag{7.358}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb, pa - baaaa, pb, \text{ class } 5\rangle \tag{7.359}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb, pa - xbaaaa, pb, \text{ class } 5\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.360}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb, pa, pb - baaaa, \text{ class } 5\rangle \tag{7.361}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb, pa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.362}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb, pa, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.363}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb, pa, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.364}$$

$$\langle a, b \,|\, baaab, baab, babb - baaaa, pa, pb, \text{ class } 5\rangle \tag{7.365}$$

$$\langle a, b \,|\, baaab, baab, babb - baaaa, pa - baaaa, pb, \text{ class } 5\rangle \tag{7.366}$$

$$\langle a, b \,|\, baaab, baab, babb - baaaa, pa - ybaaaa, pb, \text{ class } 5\rangle \ (y = \omega, \omega^2, \ p = 1 \bmod 3) \tag{7.367}$$

$$\langle a, b \,|\, baaab, baab, babb - baaaa, pa - ybaaaa, pb, \text{ class } 5\rangle \ (y = \omega^3, \omega^4, \omega^5, \omega^6, \omega^7, \omega^8, \ p = 1 \bmod 9) \tag{7.368}$$

$$\langle a, b \,|\, baaab, baab, babb - baaaa, pa - xbaaaa, pb - ybaaaa, \text{ class } 5\rangle \ (y = 1, \omega, \text{all } x, \ x \sim x' \text{ if } x^8 = x'^8 \bmod p) \tag{7.369}$$

$$\langle a, b \,|\, baaab, baab, babb - baaaa, pa - xbaaaa, pb - ybaaaa, \text{ class } 5\rangle \ (y = \omega^2, \omega^3, \text{all } x, \ x \sim x' \text{ if } x^8 = x'^8 \bmod p, \ p = 1 \bmod 4) \tag{7.370}$$

$$\langle a, b \,|\, baaab, baab, babb - baaaa, pa - xbaaaa, pb - ybaaaa, \text{ class } 5\rangle \ (y = \omega^4, \omega^5, \omega^6, \omega^7, \text{all } x, \ x \sim x' \text{ if } x^8 = x'^8 \bmod p, \ p = 1 \bmod 8) \tag{7.371}$$

8.28 Descendants of 6.399

$(1/2)p^3 + (3/2)p^2 + 4p + 7 + 3\gcd(p-1,3) + (p+3)\gcd(p-1,4)/2 + \gcd(p-1,5) + \gcd(p-1,8)/2$ algebras

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa, pb, \text{ class } 5\rangle \tag{7.372}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - baaaa, pb, \text{ class } 5\rangle \tag{7.373}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - \omega baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.374}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - \omega^2 baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 8) \tag{7.375}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - \omega^3 baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 8) \tag{7.376}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - xbaaaa, pb - baaaa, \text{ class } 5\rangle \ (\text{all } x, \ x \sim -x) \tag{7.377}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - xbaaaa, pb - \omega baaaa, \text{ class } 5\rangle \ (\text{all } x, \ x \sim -x) \tag{7.378}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - xbaaaa, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (\text{all } x, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.379}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa, pa - xbaaaa, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (\text{all } x, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.380}$$

$$\langle a, b \,|\, baaab, baab - baaaa, babb + baaa, pa - xbaaaa, pb - ybaaaa, \text{ class } 5\rangle \ (\text{all } x, \ y, x \sim -x) \tag{7.381}$$

$$\langle a, b \,|\, baaab, baab, babb + baaa - baaaa, pa - xbaaaa, pb - ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x \sim -x) \tag{7.382}$$

$$\langle a, b \,|\, baaab, baab - xbaaaa, babb + baaa - baaaa, pa - ybaaaa, pb - zbaaaa, \text{ class } 5\rangle \ (x \neq 0,\ \text{ all } y, z, x \sim -x) \tag{7.383}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa, pb, \text{ class } 5\rangle \tag{7.384}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa, pb - baaaa, \text{ class } 5\rangle \tag{7.385}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.386}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.387}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.388}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa - baaaa, pb - baaaa, \text{ class } 5\rangle \tag{7.389}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa - \omega baaaa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.390}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa, pa - baaaa, pb + baaaa, \text{ class } 5\rangle \tag{7.391}$$

$$\langle a, b \,|\, baaab - baaaa, baab, babb + baaa - baaaa, pa - xbaaaa, pb - ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, (x,\ y) \sim (y, x)) \tag{7.392}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa, pb - xbaaaa, \text{ class } 5\rangle \text{ (all } x) \tag{7.393}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - baaaa, pb - baaaa, \text{ class } 5\rangle \tag{7.394}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - \omega baaaa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.395}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - baaaa, pb + baaaa, \text{ class } 5\rangle \tag{7.396}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - \omega baaaa, pb + \omega baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.397}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa - 2baaaa, pa - \omega^2 baaaa, pb + \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.398}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa, pb - xbaaaa, \text{ class } 5\rangle \text{ (all } x) \tag{7.399}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa - baaaa, pb - baaaa, \text{ class } 5\rangle \tag{7.400}$$

$$\langle a, b \,|\, baaab - baaaa, baab - baaaa, babb + baaa + 2baaaa, pa - \omega baaaa, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.401}$$

$$\langle a,b\,|\,baaab-baaaa, baab-baaaa, babb+baaa+2baaaa, pa-xbaaaa, pb-xbaaaa, \text{ class } 5\rangle \ (x=\omega^2,\omega^3,\omega^4,\omega^5,\ p=1 \bmod 3) \tag{7.402}$$

$$\langle a,b\,|\,baaab-baaaa, baab-baaaa, babb+baaa+2baaaa, pa-baaaa, pb+baaaa, \text{ class } 5\rangle \tag{7.403}$$

$$\langle a,b\,|\,baaab-baaaa, baab-baaaa, babb+baaa+2baaaa, pa-xbaaaa, pb+xbaaaa, \text{ class } 5\rangle \ (x=\omega,\omega^2,\omega^3,\omega^4,\ p=1 \bmod 5) \tag{7.404}$$

In the following presentation, z is chosen so as to be any *one* integer such that z^2-4 is not a square $\bmod p$:

$$\langle a,b\,|\,baaab-baaaa, baab-baaaa, babb+baaa-zbaaaa, pa-xbaaaa, pb-ybaaaa, \text{ class } 5\rangle \ (\text{all } x,\ y, (x,\ y)\sim(y,x)) \tag{7.405}$$

8.29 Descendants of 6.404

$(p^3+p^2+2p+2+(p+1)\gcd(p-1,4)+\gcd(p-1,8))/2$ algebras

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa, pb, \text{ class } 5\rangle \tag{7.406}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-baaaa, pb, \text{ class } 5\rangle \tag{7.407}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-\omega baaaa, pb, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.408}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-\omega^2 baaaa, pb, \text{ class } 5\rangle \ (p=1 \bmod 8) \tag{7.409}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-\omega^3 baaaa, pb, \text{ class } 5\rangle \ (p=1 \bmod 8) \tag{7.410}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-xbaaaa, pb-baaaa, \text{ class } 5\rangle \ (\text{all } x,\ x\sim -x) \tag{7.411}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-xbaaaa, pb-\omega baaaa, \text{ class } 5\rangle \ (\text{all } x,\ x\sim -x) \tag{7.412}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-xbaaaa, pb-\omega^2 baaaa, \text{ class } 5\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.413}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa, pa-xbaaaa, pb-\omega^3 baaaa, \text{ class } 5\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.414}$$

$$\langle a,b\,|\,baaab, baab-baaaa, babb+\omega baaa, pa-xbaaaa, pb-ybaaaa, \text{ class } 5\rangle \ (\text{all } x,\ y, x\sim -x) \tag{7.415}$$

$$\langle a,b\,|\,baaab, baab, babb+\omega baaa-baaaa, pa-xbaaaa, pb-ybaaaa, \text{ class } 5\rangle \ (\text{all } x,\ y, x\sim -x) \tag{7.416}$$

$$\langle a,b\,|\,baaab, baab-xbaaaa, babb+\omega baaa-baaaa, pa-ybaaaa, pb-zbaaaa, \text{ class } 5\rangle \ (x\neq 0,\ \text{all } y, z, x\sim -x) \tag{7.417}$$

8.30 Descendants of 6.408

$(1/2)(p+1)(p-1+\gcd(p-1,3))$ algebras

$$\langle a,b \,|\, pa-bab-xbaaaa, pb-baaa-ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.418}$$

$$\langle a,b \,|\, pa-bab-xbaaaa, pb-\omega baaa-ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.419}$$

$$\langle a,b \,|\, pa-bab-xbaaaa, pb-\omega^2 baaa-ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.420}$$

$$\langle a,b \,|\, pa-bab-xbaaaa, pb-baaa-ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ p=2 \bmod 3) \tag{7.421}$$

8.31 Descendants of 6.411

$2+2\gcd(p-1,3)+\gcd(p-1,4)$ algebras

$$\langle a,b \,|\, baaaa, pa-bab, pb, \text{ class } 5\rangle \tag{7.422}$$

$$\langle a,b \,|\, baaaa, pa-bab, pb-baaab, \text{ class } 5\rangle \tag{7.423}$$

$$\langle a,b \,|\, baaaa, pa-bab, pb-\omega baaab, \text{ class } 5\rangle \ (p=1 \bmod 3) \tag{7.424}$$

$$\langle a,b \,|\, baaaa, pa-bab, pb-\omega^2 baaab, \text{ class } 5\rangle \ (p=1 \bmod 3) \tag{7.425}$$

$$\langle a,b \,|\, baaab, pa-bab, pb, \text{ class } 5\rangle \tag{7.426}$$

$$\langle a,b \,|\, baaab, pa-bab-baaaa, pb, \text{ class } 5\rangle \tag{7.427}$$

$$\langle a,b \,|\, baaab, pa-bab-\omega baaaa, pb, \text{ class } 5\rangle \ (p=1 \bmod 3) \tag{7.428}$$

$$\langle a,b \,|\, baaab, pa-bab-\omega^2 baaaa, pb, \text{ class } 5\rangle \ (p=1 \bmod 3) \tag{7.429}$$

$$\langle a,b \,|\, baaab, pa-bab, pb-baaaa, \text{ class } 5\rangle \tag{7.430}$$

$$\langle a,b \,|\, baaab, pa-bab, pb-\omega baaaa, \text{ class } 5\rangle \tag{7.431}$$

$$\langle a,b \,|\, baaab, pa-bab, pb-\omega^2 baaaa, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.432}$$

$$\langle a,b \,|\, baaab, pa-bab, pb-\omega^3 baaaa, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.433}$$

8.32 Descendants of 6.412

$p-1+(1/2)(p+1)\gcd(p-1,4)$ algebras

$$\langle a,b\,|\,baaaa, pa-bab-baaa, pb-xbaaab, \text{ class } 5\rangle \text{ (all } x,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.434}$$

$$\langle a,b\,|\,baaaa, pa-bab-\omega baaa, pb-xbaaab, \text{ class } 5\rangle \text{ (all } x,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.435}$$

$$\langle a,b\,|\,baaaa, pa-bab-baaa, pb-xbaaab, \text{ class } 5\rangle \text{ (all } x,\ x\sim -x,\ p=3 \bmod 4) \tag{7.436}$$

$$\langle a,b\,|\,baaab, pa-bab-baaa, pb-xbaaaa, \text{ class } 5\rangle \text{ (all } x) \tag{7.437}$$

$$\langle a,b\,|\,baaab, pa-bab-\omega baaa, pb-xbaaaa, \text{ class } 5\rangle \text{ (all } x,\ p=1 \bmod 4) \tag{7.438}$$

$$\langle a,b\,|\,baaab, pa-bab-baaa-xbaaaa, pb+2baaaa, \text{ class } 5\rangle \text{ (} x\neq 0, x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.439}$$

$$\langle a,b\,|\,baaab, pa-bab-\omega baaa-xbaaaa, pb+(2*\omega^2)baaaa, \text{ class } 5\rangle \text{ (} x\neq 0, x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.440}$$

$$\langle a,b\,|\,baaab, pa-bab-baaa-xbaaaa, pb+2baaaa, \text{ class } 5\rangle \text{ (} x\neq 0, x\sim -x,\ p=3 \bmod 4) \tag{7.441}$$

8.33 Descendants of 6.414

$(1/2)(p+1)(p-1+\gcd(p-1,3))$ algebras

$$\langle a,b\,|\,pa-\omega bab-xbaaaa, pb-baaa-ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.442}$$

$$\langle a,b\,|\,pa-\omega bab-xbaaaa, pb-\omega baaa-ybaaaa, babb+baaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.443}$$

$$\langle a,b\,|\,pa-\omega bab-xbaaaa, pb-\omega^2 baaa-ybaaaa, babb+\omega baaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.444}$$

$$\langle a,b\,|\,pa-\omega bab-xbaaaa, pb-baaa-ybaaaa, \text{ class } 5\rangle \text{ (all } x,\ y, x\sim -x,\ p=2 \bmod 3) \tag{7.445}$$

8.34 Descendants of 6.417

$2+2\gcd(p-1,3)+\gcd(p-1,4)$ algebras

$$\langle a,b\,|\,baaaa, pa-\omega bab, pb, \text{ class } 5\rangle \tag{7.446}$$

$$\langle a,b\,|\,baaaa, pa-\omega bab, pb-baaab, \text{ class } 5\rangle \tag{7.447}$$

$$\langle a,b\,|\,baaaa, pa-\omega bab, pb-\omega baaab, \text{ class } 5\rangle \text{ (} p=1 \bmod 3) \tag{7.448}$$

$$\langle a,b\,|\,baaaa, pa-\omega bab, pb-\omega^2 baaab, \text{ class } 5\rangle \text{ (} p=1 \bmod 3) \tag{7.449}$$

$$\langle a, b \,|\, baaab, pa - \omega bab, pb, \text{ class } 5\rangle \tag{7.450}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - baaaa, pb, \text{ class } 5\rangle \tag{7.451}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - \omega baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.452}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - \omega^2 baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.453}$$

$$\langle a, b \,|\, baaab, pa - \omega bab, pb - baaaa, \text{ class } 5\rangle \tag{7.454}$$

$$\langle a, b \,|\, baaab, pa - \omega bab, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.455}$$

$$\langle a, b \,|\, baaab, pa - \omega bab, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.456}$$

$$\langle a, b \,|\, baaab, pa - \omega bab, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.457}$$

8.35 Descendants of 6.418

$p - 1 + (1/2)(p + 1)\gcd(p - 1, 4)$ algebras

$$\langle a, b \,|\, baaaa, pa - \omega bab - baaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.458}$$

$$\langle a, b \,|\, baaaa, pa - \omega bab - \omega baaa, pb - xbaaab, babb + baaab, \text{ class } 5\rangle \ (\text{all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.459}$$

$$\langle a, b \,|\, baaaa, pa - \omega bab - baaa, pb - xbaaab, \text{ class } 5\rangle \ (\text{all } x,\ x \sim -x,\ p = 3 \bmod 4) \tag{7.460}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - baaa, pb - xbaaaa, \text{ class } 5\rangle \ (\text{all } x) \tag{7.461}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - \omega baaa, pb - xbaaaa, baab + baaaa, \text{ class } 5\rangle \ (\text{all } x,\ p = 1 \bmod 4) \tag{7.462}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - baaa - xbaaaa, pb + 2\omega^{-1} baaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.463}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - \omega baaa - xbaaaa, pb + 2\omega baaaa, baab + baaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.464}$$

$$\langle a, b \,|\, baaab, pa - \omega bab - baaa - xbaaaa, pb + 2\omega^{-1} baaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim -x,\ p = 3 \bmod 4) \tag{7.465}$$

8.36 Descendants of 6.420

$5p + 1 + \gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, babba, pa - baa - xbabbb, pb - babbb, \text{ class } 5\rangle \text{ (all } x) \tag{7.466}$$

$$\langle a, b \,|\, babba, pa - baa - xbabbb, pb - \omega babbb, \text{ class } 5\rangle \text{ (all } x) \tag{7.467}$$

$$\langle a, b \,|\, babba, pa - baa, pb, \text{ class } 5\rangle \tag{7.468}$$

$$\langle a, b \,|\, babba, pa - baa - babbb, pb, \text{ class } 5\rangle \tag{7.469}$$

$$\langle a, b \,|\, babba, pa - baa - \omega babbb, pb, \text{ class } 5\rangle \tag{7.470}$$

$$\langle a, b \,|\, babba, pa - baa - \omega^2 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.471}$$

$$\langle a, b \,|\, babba, pa - baa - \omega^3 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.472}$$

$$\langle a, b \,|\, babbb, pa - baa, pb - xbabba, \text{ class } 5\rangle \text{ (all } x) \tag{7.473}$$

$$\langle a, b \,|\, babbb - babba, pa - baa, pb - xbabba, \text{ class } 5\rangle \text{ (all } x) \tag{7.474}$$

$$\langle a, b \,|\, babbb - \omega babba, pa - baa, pb - xbabba, \text{ class } 5\rangle \text{ (all } x) \tag{7.475}$$

8.37 Descendants of 6.421

$2p^2 - p - 1 + (p+1)\gcd(p-1, 3)$ algebras

$$\langle a, b \,|\, babba, pa - baa - babb - xbabbb, pb - ybabbb, \text{ class } 5\rangle \ (x \neq 0, \text{ all } y, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.476}$$

$$\langle a, b \,|\, babba, pa - baa - \omega babb - xbabbb, pb - ybabbb, \text{ class } 5\rangle \ (x \neq 0, \text{ all } y, x \sim x' \text{ if } x^3 = x'^3 \bmod p, \ p = 1 \bmod 3) \tag{7.477}$$

$$\langle a, b \,|\, babba, pa - baa - \omega^2 babb - xbabbb, pb - ybabbb, \text{ class } 5\rangle \ (x \neq 0, \text{ all } y, x \sim x' \text{ if } x^3 = x'^3 \bmod p, \ p = 1 \bmod 3) \tag{7.478}$$

$$\langle a, b \,|\, babba, pa - baa - babb, pb - xbabbb, \text{ class } 5\rangle \text{ (all } x, \ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.479}$$

$$\langle a, b \,|\, babba, pa - baa - \omega babb, pb - xbabbb, \text{ class } 5\rangle \text{ (all } x, \ x \sim x' \text{ if } x^3 = x'^3 \bmod p, \ p = 1 \bmod 3) \tag{7.480}$$

$$\langle a, b \,|\, babba, pa - baa - \omega^2 babb, pb - xbabbb, \text{ class } 5\rangle \text{ (all } x, \ x \sim x' \text{ if } x^3 = x'^3 \bmod p, \ p = 1 \bmod 3) \tag{7.481}$$

$$\langle a, b \,|\, babbb - xbabba, pa - baa - babb, pb - ybabba, \text{ class } 5\rangle \text{ (all } x, \ y, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.482}$$

$$\langle a, b \,|\, babbb - xbabba, pa - baa - \omega babb, pb - ybabba, \text{ class } 5\rangle \text{ (all } x, \ y, x \sim x' \text{ if } x^3 = x'^3 \bmod p, \ p = 1 \bmod 3) \tag{7.483}$$

$$\langle a, b \,|\, babbb - xbabba, pa - baa - \omega^2 babb, pb - ybabba, \text{ class } 5\rangle \text{ (all } x, \ y, x \sim x' \text{ if } x^3 = x'^3 \bmod p, \ p = 1 \bmod 3) \tag{7.484}$$

8.38 Descendants of 6.424

p^2 algebras

$$\langle a, b \,|\, pa - baa - xbabb, pb - babb - ybabba, \text{ class } 5\rangle \text{ (all } x,\, y) \tag{7.485}$$

8.39 Descendants of 6.425

3 algebras

$$\langle a, b \,|\, baabb, pa - baa, pb + bab, \text{ class } 5\rangle \tag{7.486}$$

$$\langle a, b \,|\, baabb - baaba, pa - baa, pb + bab, \text{ class } 5\rangle \tag{7.487}$$

$$\langle a, b \,|\, baabb - \omega baaba, pa - baa, pb + bab, \text{ class } 5\rangle \tag{7.488}$$

8.40 Descendants of 6.426

p algebras

$$\langle a, b \,|\, pa - baa - baab, pb + bab - xbaab, \text{ class } 5\rangle \text{ (all } x) \tag{7.489}$$

8.41 Descendants of 6.427

$p + 1$ algebras

$$\langle a, b \,|\, pa + bab, pb + \omega baa - xbaaa, baaab, \text{ class } 5\rangle \text{ (all } x,\, x \sim -x) \tag{7.490}$$

In the following two presentation, y is chosen so as to be any *one* integer such that $1 - \omega y^2$ is not a square $\bmod p$:

$$\langle a, b \,|\, pa + bab - xybaaa, pb + \omega baa - xbaaa, baaab - \omega ybaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim -x) \tag{7.491}$$

$$\langle a, b \,|\, pa + bab, pb + \omega baa, baaab + \omega ybaaaa, \text{ class } 5\rangle \tag{7.492}$$

8.42 Descendants of 6.428

2 algebras

$$\langle a, b \,|\, ba, p^2 b, \text{ class } 5\rangle \tag{7.493}$$

$$\langle a, b \,|\, ba - p^4 a, p^2 b, \text{ class } 5\rangle \tag{7.494}$$

8.43 Descendants of 6.429

1 algebra

$$\langle a, b \,|\, ba - p^3 a, p^2 b, \text{ class } 5\rangle \tag{7.495}$$

8.44 Descendants of 6.431

4 algebras

$$\langle a, b \,|\, baa, bab, pb, \text{ class } 5\rangle \tag{7.496}$$

$$\langle a, b \,|\, baa, bab - p^4 a, pb, \text{ class } 5\rangle \tag{7.497}$$

$$\langle a, b \,|\, baa, bab - \omega p^4 a, pb, \text{ class } 5\rangle \tag{7.498}$$

$$\langle a, b \,|\, baa - p^4 a, bab, pb, \text{ class } 5\rangle \tag{7.499}$$

8.45 Descendants of 6.435

$7 + 2\gcd(p-1, 3) + 2\gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, baaaa, bab, p^2 a, pb, \text{ class } 5\rangle \tag{7.500}$$

$$\langle a, b \,|\, baaaa, bab, p^2 a - baaab, pb, \text{ class } 5\rangle \tag{7.501}$$

$$\langle a, b \,|\, baaaa, bab, p^2 a - \omega baaab, pb, \text{ class } 5\rangle \tag{7.502}$$

$$\langle a, b \,|\, baaaa, bab, p^2 a, pb - baaab, \text{ class } 5\rangle \tag{7.503}$$

$$\langle a, b \,|\, baaab, bab, p^2 a - baaaa, pb, \text{ class } 5\rangle \tag{7.504}$$

$$\langle a, b \,|\, baaab, bab - baaaa, p^2 a - baaaa, pb, \text{ class } 5\rangle \tag{7.505}$$

$$\langle a, b \,|\, baaab, bab - \omega baaaa, p^2 a - baaaa, pb, \text{ class } 5\rangle \tag{7.506}$$

$$\langle a, b \,|\, baaab, bab - \omega^2 baaaa, p^2 a - baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.507}$$

$$\langle a, b \,|\, baaab, bab - \omega^3 baaaa, p^2 a - baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.508}$$

$$\langle a, b \,|\, baaab, bab - \omega^4 baaaa, p^2 a - baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.509}$$

$$\langle a, b \,|\, baaab, bab - \omega^5 baaaa, p^2 a - baaaa, pb, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.510}$$

$$\langle a, b \,|\, baaab, bab, p^2 a, pb, \text{ class } 5\rangle \tag{7.511}$$

$$\langle a, b \,|\, baaab, bab - baaaa, p^2 a, pb, \text{ class } 5\rangle \tag{7.512}$$

$$\langle a, b \,|\, baaab, bab, p^2 a, pb - baaaa, \text{ class } 5\rangle \tag{7.513}$$

$$\langle a, b \,|\, baaab, bab - baaaa, p^2 a, pb - baaaa, \text{ class } 5\rangle \tag{7.514}$$

$$\langle a, b \,|\, baaab, bab, p^2a, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.515}$$

$$\langle a, b \,|\, baaab, bab - baaaa, p^2a, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.516}$$

$$\langle a, b \,|\, baaab, bab, p^2a, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.517}$$

$$\langle a, b \,|\, baaab, bab - baaaa, p^2a, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.518}$$

$$\langle a, b \,|\, baaab, bab, p^2a, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.519}$$

$$\langle a, b \,|\, baaab, bab - baaaa, p^2a, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.520}$$

8.46 Descendants of 6.436

$2 + 2\gcd(p-1, 3) + \gcd(p-1, 4) + 2\gcd(p-1, 5)$ algebras

$$\langle a, b \,|\, baaaa, bab - baaa, p^2a, pb, \text{ class } 5\rangle \tag{7.521}$$

$$\langle a, b \,|\, baaaa, bab - baaa, p^2a - baaab, pb, \text{ class } 5\rangle \tag{7.522}$$

$$\langle a, b \,|\, baaaa, bab - baaa, p^2a - \omega baaab, pb, \text{ class } 5\rangle \tag{7.523}$$

$$\langle a, b \,|\, baaaa, bab - baaa, p^2a - xbaaab, pb, \text{ class } 5\rangle \ (x = \omega^2, \omega^3, \omega^4, \omega^5, \ p = 1 \bmod 3) \tag{7.524}$$

$$\langle a, b \,|\, baaaa, bab - baaa, p^2a, pb - baaab, \text{ class } 5\rangle \tag{7.525}$$

$$\langle a, b \,|\, baaaa, bab - baaa, p^2a, pb - xbaaab, \text{ class } 5\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4, \ p = 1 \bmod 5) \tag{7.526}$$

$$\langle a, b \,|\, baaab, bab - baaa, p^2a, pb, \text{ class } 5\rangle \tag{7.527}$$

$$\langle a, b \,|\, baaab, bab - baaa, p^2a - baaaa, pb, \text{ class } 5\rangle \tag{7.528}$$

$$\langle a, b \,|\, baaab, bab - baaa, p^2a - xbaaaa, pb, \text{ class } 5\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4, \ p = 1 \bmod 5) \tag{7.529}$$

$$\langle a, b \,|\, baaab, bab - baaa, p^2a, pb - baaaa, \text{ class } 5\rangle \tag{7.530}$$

$$\langle a, b \,|\, baaab, bab - baaa, p^2a, pb - \omega baaaa, \text{ class } 5\rangle \tag{7.531}$$

$$\langle a, b \,|\, baaab, bab - baaa, p^2a, pb - \omega^2 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.532}$$

$$\langle a, b \,|\, baaab, bab - baaa, p^2a, pb - \omega^3 baaaa, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.533}$$

8.47 Descendants of 6.442

$2p - 2 + (p+2)\gcd(p-1,3)$ algebras

$$\langle a, b \,|\, bab, p^2a, pb - baaa, \text{ class } 5\rangle \tag{7.534}$$

$$\langle a, b \,|\, bab, p^2a - baaaa, pb - baaa, \text{ class } 5\rangle \tag{7.535}$$

$$\langle a, b \,|\, bab - baaaa, p^2a - xbaaaa, pb - baaa, \text{ class } 5\rangle \text{ (all } x) \tag{7.536}$$

$$\langle a, b \,|\, bab, p^2a, pb - \omega baaa, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.537}$$

$$\langle a, b \,|\, bab, p^2a - baaaa, pb - \omega baaa, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.538}$$

$$\langle a, b \,|\, bab - baaaa, p^2a - xbaaaa, pb - \omega baaa, \text{ class } 5\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.539}$$

$$\langle a, b \,|\, bab, p^2a, pb - \omega^2 baaa, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.540}$$

$$\langle a, b \,|\, bab, p^2a - baaaa, pb - \omega^2 baaa, \text{ class } 5\rangle \ (p = 1 \bmod 3) \tag{7.541}$$

$$\langle a, b \,|\, bab - baaaa, p^2a - xbaaaa, pb - \omega^2 baaa, \text{ class } 5\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.542}$$

$$\langle a, b \,|\, bab, p^2a, pb - baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0,\ p = 2 \bmod 3) \tag{7.543}$$

$$\langle a, b \,|\, bab - baaaa, p^2a, pb - baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0,\ p = 2 \bmod 3) \tag{7.544}$$

$$\langle a, b \,|\, bab, p^2a, pb - baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.545}$$

$$\langle a, b \,|\, bab - baaaa, p^2a, pb - baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.546}$$

$$\langle a, b \,|\, bab, p^2a, pb - \omega baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.547}$$

$$\langle a, b \,|\, bab - baaaa, p^2a, pb - \omega baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.548}$$

$$\langle a, b \,|\, bab, p^2a, pb - \omega^2 baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.549}$$

$$\langle a, b \,|\, bab - baaaa, p^2a, pb - \omega^2 baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.550}$$

8.48 Descendants of 6.445

$2p - 2 + \gcd(p-1, 3)$ algebras

$$\langle a, b \,|\, bab - baaa, p^2a - xbaaaa, pb - baaa, \text{ class } 5\rangle \text{ (all } x,\ p = 2 \bmod 3) \tag{7.551}$$

$$\langle a, b \,|\, bab - baaa, p^2a, pb - baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0,\ p = 2 \bmod 3) \tag{7.552}$$

$$\langle a, b \,|\, bab - baaa, p^2a - xbaaaa, pb - baaa, \text{ class } 5\rangle \text{ (all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.553}$$

$$\langle a, b \,|\, bab - baaa, p^2a, pb - baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.554}$$

$$\langle a, b \,|\, bab - baaa, p^2a - xbaaaa, pb - \omega baaa, \text{ class } 5\rangle \text{ (all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.555}$$

$$\langle a, b \,|\, bab - baaa, p^2a, pb - \omega baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.556}$$

$$\langle a, b \,|\, bab - baaa, p^2a - xbaaaa, pb - \omega^2 baaa, \text{ class } 5\rangle \text{ (all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.557}$$

$$\langle a, b \,|\, bab - baaa, p^2a, pb - \omega^2 baaa - xbaaaa, \text{ class } 5\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.558}$$

8.49 Descendants of 6.448

$(p+1)/2$ algebras

$$\langle a, b \,|\, bab, p^2a, pb - baa - xbaaa, \text{ class } 5\rangle \text{ (all } x,\ x \sim -x) \tag{7.559}$$

8.50 Descendants of 6.451

$(p+1)/2$ algebras

$$\langle a, b \,|\, bab, p^2a, pb - \omega baa - xbaaa, \text{ class } 5\rangle \text{ (all } x,\ x \sim -x) \tag{7.560}$$

8.51 Descendants of 6.454

$8 + 2\gcd(p-1, 3) + 4\gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, babba, baa, p^2a, pb, \text{ class } 5\rangle \tag{7.561}$$

$$\langle a, b \,|\, babba, baa, p^2a, pb - babbb, \text{ class } 5\rangle \tag{7.562}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a, pb, \text{ class } 5\rangle \tag{7.563}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a, pb - babbb, \text{ class } 5\rangle \tag{7.564}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a, pb - \omega babbb, \text{ class } 5\rangle \tag{7.565}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a, pb - xbabbb, \text{ class } 5\rangle \ (x = \omega^2, \omega^3, \omega^4, \omega^5,\ p = 1 \bmod 3) \tag{7.566}$$

$$\langle a, b \,|\, babba, baa, p^2a - babbb, pb, \text{ class } 5\rangle \tag{7.567}$$

$$\langle a, b \,|\, babba, baa, p^2a - \omega babbb, pb, \text{ class } 5\rangle \tag{7.568}$$

$$\langle a, b \,|\, babba, baa, p^2a - \omega^2 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.569}$$

$$\langle a, b \,|\, babba, baa, p^2a - \omega^3 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.570}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a - babbb, pb, \text{ class } 5\rangle \tag{7.571}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a - \omega babbb, pb, \text{ class } 5\rangle \tag{7.572}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a - \omega^2 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.573}$$

$$\langle a, b \,|\, babba, baa - babbb, p^2a - \omega^3 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.574}$$

$$\langle a, b \,|\, babbb, baa, p^2a, pb, \text{ class } 5\rangle \tag{7.575}$$

$$\langle a, b \,|\, babbb, baa, p^2a - babba, pb, \text{ class } 5\rangle \tag{7.576}$$

$$\langle a, b \,|\, babbb, baa, p^2a, pb - babba, \text{ class } 5\rangle \tag{7.577}$$

$$\langle a, b \,|\, babbb, baa, p^2a, pb - \omega babba, \text{ class } 5\rangle \tag{7.578}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a, pb, \text{ class } 5\rangle \tag{7.579}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a - babba, pb, \text{ class } 5\rangle \tag{7.580}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a - \omega babba, pb, \text{ class } 5\rangle \tag{7.581}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a - \omega^2 babba, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.582}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a - \omega^3 babba, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.583}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a, pb - babba, \text{ class } 5\rangle \tag{7.584}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a, pb - \omega babba, \text{ class } 5\rangle \tag{7.585}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a, pb - \omega^2 babba, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.586}$$

$$\langle a, b \,|\, babbb - babba, baa, p^2a, pb - \omega^3 babba, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.587}$$

8.52 Descendants of 6.455

4 algebras

$$\langle a, b \,|\, baa, p^2a, pb - babb, \text{ class } 5\rangle \tag{7.588}$$

$$\langle a, b \,|\, baa, p^2a - babba, pb - babb, \text{ class } 5\rangle \tag{7.589}$$

$$\langle a, b \,|\, baa, p^2a, pb - babb - babba, \text{ class } 5\rangle \tag{7.590}$$

$$\langle a, b \,|\, baa, p^2a, pb - babb - \omega babba, \text{ class } 5\rangle \tag{7.591}$$

8.53 Descendants of 6.459

$4p + 2\gcd(p-1,3) + \gcd(p-1,4) + 2\gcd(p-1,5)$ algebras

$$\langle a, b \,|\, babba, baa - babb, p^2a, pb, \text{ class } 5\rangle \tag{7.592}$$

$$\langle a, b \,|\, babba, baa - babb, p^2a - babbb, pb, \text{ class } 5\rangle \tag{7.593}$$

$$\langle a, b \,|\, babba, baa - babb, p^2a - \omega babbb, pb, \text{ class } 5\rangle \tag{7.594}$$

$$\langle a, b \,|\, babba, baa - babb, p^2a - \omega^2 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.595}$$

$$\langle a, b \,|\, babba, baa - babb, p^2a - \omega^3 babbb, pb, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.596}$$

$$\langle a, b \,|\, babba, baa - babb, p^2a, pb - babbb, \text{ class } 5\rangle \tag{7.597}$$

$$\langle a, b \,|\, babba, baa - babb, p^2a, pb - xbabbb, \text{ class } 5\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.598}$$

$$\langle a, b \,|\, babba, baa - babb - babbb, p^2a - xbabbb, pb, \text{ class } 5\rangle \ (\text{all } x) \tag{7.599}$$

$$\langle a, b \,|\, babba, baa - babb - babbb, p^2a, pb - xbabbb, \text{ class } 5\rangle \ (x \neq 0) \tag{7.600}$$

$$\langle a, b \,|\, babbb, baa - babb, p^2a, pb, \text{ class } 5\rangle \tag{7.601}$$

$$\langle a, b \,|\, babbb, baa - babb, p^2a - babba, pb, \text{ class } 5\rangle \tag{7.602}$$

$$\langle a, b \,|\, babbb, baa - babb, p^2a - xbabba, pb, \text{ class } 5\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.603}$$

$$\langle a, b \,|\, babbb, baa - babb, p^2a, pb - babba, \text{ class } 5\rangle \tag{7.604}$$

$$\langle a, b \,|\, babbb, baa - babb, p^2a, pb - \omega babba, \text{ class } 5\rangle \tag{7.605}$$

$$\langle a, b \,|\, babbb, baa - babb, p^2a, pb - xbabba, \text{ class } 5\rangle \ (x = \omega^2, \omega^3, \omega^4, \omega^5,\ p = 1 \bmod 3) \tag{7.606}$$

$$\langle a, b \,|\, babbb - babba, baa - babb, p^2a - xbabba, pb, \text{ class } 5\rangle \ (\text{all } x) \tag{7.607}$$

$$\langle a, b \,|\, babbb - babba, baa - babb, p^2a, pb - xbabba, \text{ class } 5\rangle \ (x \neq 0) \tag{7.608}$$

8.54 Descendants of 6.460

$3p - 3 + \gcd(p-1, 4)$ algebras

$$\langle a, b \,|\, baa - babb, p^2a - xbabba, pb - babb, \text{ class } 5\rangle \text{ (all } x,\ x \sim -x,\ p = 3 \bmod 4) \tag{7.609}$$

$$\langle a, b \,|\, baa - babb, p^2a - xbabba, pb - \omega babb, \text{ class } 5\rangle \text{ (all } x,\ x \sim -x,\ p = 3 \bmod 4) \tag{7.610}$$

$$\langle a, b \,|\, baa - babb, p^2a, pb - babb - xbabba, \text{ class } 5\rangle \ (x \neq 0,\ p = 3 \bmod 4) \tag{7.611}$$

$$\langle a, b \,|\, baa - babb, p^2a, pb - \omega babb - xbabba, \text{ class } 5\rangle \ (x \neq 0,\ p = 3 \bmod 4) \tag{7.612}$$

$$\langle a, b \,|\, baa - babb, p^2a - xbabba, pb - babb, \text{ class } 5\rangle \text{ (all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.613}$$

$$\langle a, b \,|\, baa - babb, p^2a, pb - babb - xbabba, \text{ class } 5\rangle \ (x \neq 0, x \sim -x,\ p = 1 \bmod 4) \tag{7.614}$$

$$\langle a, b \,|\, baa - babb, p^2a - xbabba, pb - \omega babb, \text{ class } 5\rangle \text{ (all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.615}$$

$$\langle a, b \,|\, baa - babb, p^2a, pb - \omega babb - xbabba, \text{ class } 5\rangle \ (x \neq 0, x \sim -x,\ p = 1 \bmod 4) \tag{7.616}$$

$$\langle a, b \,|\, baa - babb, p^2a - xbabba, pb - \omega^2 babb, \text{ class } 5\rangle \text{ (all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.617}$$

$$\langle a, b \,|\, baa - babb, p^2a, pb - \omega^2 babb - xbabba, \text{ class } 5\rangle \ (x \neq 0, x \sim -x,\ p = 1 \bmod 4) \tag{7.618}$$

$$\langle a, b \,|\, baa - babb, p^2a - xbabba, pb - \omega^3 babb, \text{ class } 5\rangle \text{ (all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.619}$$

$$\langle a, b \,|\, baa - babb, p^2a, pb - \omega^3 babb - xbabba, \text{ class } 5\rangle \ (x \neq 0, x \sim -x,\ p = 1 \bmod 4) \tag{7.620}$$

8.55 Descendants of 6.467

2 algebras

$$\langle a, b \,|\, baa, pb - ba, \text{ class } 5\rangle \tag{7.621}$$

$$\langle a, b \,|\, baa - p^4a, pb - ba, \text{ class } 5\rangle \tag{7.622}$$

8.56 Descendants of 6.469

$4 + 2\gcd(p-1, 3) + 3\gcd(p-1, 5) + \gcd(p-1, 8)$ algebras

$$\langle a, b \,|\, bab, baaab, pa, pb, \text{ class } 6\rangle \tag{7.623}$$

$$\langle a, b \,|\, bab, baaab, pa, pb - baaaaa, \text{ class } 6\rangle \tag{7.624}$$

$$\langle a, b \,|\, bab, baaab, pa, pb - xbaaaaa, \text{ class } 6\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.625}$$

$$\langle a, b \,|\, bab, baaab, pa - baaaaa, pb, \text{ class } 6\rangle \tag{7.626}$$

$$\langle a, b \,|\, bab, baaab - baaaaa, pa, pb, \text{ class } 6\rangle \tag{7.627}$$

$$\langle a, b \,|\, bab, baaab - baaaaa, pa, pb - baaaaa, \text{ class } 6\rangle \tag{7.628}$$

$$\langle a, b \,|\, bab, baaab - baaaaa, pa, pb - xbaaaaa, \text{ class } 6\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.629}$$

$$\langle a, b \,|\, bab, baaab - baaaaa, pa - baaaaa, pb, \text{ class } 6\rangle \tag{7.630}$$

$$\langle a, b \,|\, bab, baaab - baaaaa, pa - \omega baaaaa, pb, \text{ class } 6\rangle \tag{7.631}$$

$$\langle a, b \,|\, bab, baaab - baaaaa, pa - xbaaaaa, pb, \text{ class } 6\rangle \ (x = \omega^2, \omega^3, \omega^4, \omega^5,\ p = 1 \bmod 3) \tag{7.632}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa, pb, \text{ class } 6\rangle \tag{7.633}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa, pb - baaaaa, \text{ class } 6\rangle \tag{7.634}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa, pb - xbaaaaa, \text{ class } 6\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.635}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa - baaaaa, pb, \text{ class } 6\rangle \tag{7.636}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa - \omega baaaaa, pb, \text{ class } 6\rangle \tag{7.637}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa - \omega^2 baaaaa, pb, \text{ class } 6\rangle \ (p = 1 \bmod 4) \tag{7.638}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa - \omega^3 baaaaa, pb, \text{ class } 6\rangle \ (p = 1 \bmod 4) \tag{7.639}$$

$$\langle a, b \,|\, bab - baaaaa, baaab, pa - xbaaaaa, pb, \text{ class } 6\rangle \ (x = \omega^4, \omega^5, \omega^6, \omega^7,\ p = 1 \bmod 8) \tag{7.640}$$

8.57 Descendants of 6.475

$4p - 1 + \gcd(p-1, 5) + \gcd(p-1, 7)$ algebras

$$\langle a, b \,|\, bab - baaaa, baaab, pa, pb, \text{ class } 6\rangle \tag{7.641}$$

$$\langle a, b \,|\, bab - baaaa, baaab, pa, pb - baaaaa, \text{ class } 6\rangle \tag{7.642}$$

$$\langle a, b \,|\, bab - baaaa, baaab, pa, pb - xbaaaaa, \text{ class } 6\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.643}$$

$$\langle a, b \,|\, bab - baaaa, baaab, pa - baaaaa, pb, \text{ class } 6\rangle \tag{7.644}$$

$$\langle a, b \,|\, bab - baaaa, baaab, pa - xbaaaaa, pb, \text{ class } 6\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4, \omega^5, \omega^6,\ p = 1 \bmod 7) \tag{7.645}$$

$$\langle a, b \,|\, bab - baaaa, baaab - baaaaa, pa, pb - xbaaaaa, \text{ class } 6\rangle \ (\text{all } x) \tag{7.646}$$

$$\langle a, b \,|\, bab - baaaa, baaab - baaaaa, pa - xbaaaaa, pb, \text{ class } 6\rangle \ (x \neq 0) \tag{7.647}$$

$$\langle a, b \,|\, bab - baaaa - baaaaa, baaab, pa, pb - xbaaaaa, \text{ class } 6\rangle \ (\text{all } x) \tag{7.648}$$

$$\langle a, b \,|\, bab - baaaa - baaaaa, baaab, pa - xbaaaaa, pb, \text{ class } 6\rangle \ (x \neq 0) \tag{7.649}$$

8.58 Descendants of 6.507

$2p^2 + p + 2p\gcd(p-1,3) + p\gcd(p-1,5)$ algebras

$$\langle a, b \,|\, bab - baaa, baaab - xbaaaaa, pa, pb, \text{ class } 6\rangle \ (\text{all } x) \tag{7.650}$$

$$\langle a, b \,|\, bab - baaa, baaab - xbaaaaa, pa - baaaaa, pb, \text{ class } 6\rangle \ (\text{all } x) \tag{7.651}$$

$$\langle a, b \,|\, bab - baaa, baaab - xbaaaaa, pa - \omega baaaaa, pb, \text{ class } 6\rangle \ (\text{all } x) \tag{7.652}$$

$$\langle a, b \,|\, bab - baaa, baaab - xbaaaaa, pa - ybaaaaa, pb, \text{ class } 6\rangle \ (\text{all } x,\ y = \omega^2, \omega^3, \omega^4, \omega^5,\ p = 1 \bmod 3) \tag{7.653}$$

$$\langle a, b \,|\, bab - baaa, baaab - xbaaaaa, pa - ybaaaaa, pb - baaaaa, \text{ class } 6\rangle \ (\text{all } x,\ y,\ p \neq 1 \bmod 5) \tag{7.654}$$

In the presentation below we have all $x,\ y,\ z = 1, \omega, \omega^2, \omega^3, \omega^4,\ y \sim y'$ if $y^5 = y'^5 \bmod p,\ p = 1 \bmod 5$:

$$\langle a, b \,|\, bab - baaa, baaab - xbaaaaa, pa - ybaaaaa, pb - zbaaaaa, \text{ class } 6\rangle \tag{7.655}$$

$$\langle a, b \,|\, bab - baaa - baaaaa, baaab - baaaaa, pa - xbaaaaa, pb - ybaaaaa, \text{ class } 6\rangle \ (\text{all } x,\ y,\ y \sim -y) \tag{7.656}$$

$$\langle a, b \,|\, bab - baaa - \omega baaaaa, baaab - baaaaa, pa - xbaaaaa, pb - ybaaaaa, \text{ class } 6\rangle \ (\text{all } x,\ y,\ y \sim -y) \tag{7.657}$$

8.59 Descendants of 6.518

2 algebras

$$\langle a, b \,|\, ba, pb, \text{ class } 6\rangle \tag{7.658}$$

$$\langle a, b \,|\, ba - p^5 a, pb, \text{ class } 6\rangle \tag{7.659}$$

8.60 Descendants of 3.1

$p + 14$ algebras

$$\langle a, b, c \,|\, ca, cb, \text{ class } 2\rangle \tag{7.660}$$

$$\langle a, b, c \,|\, cb, pa, \text{ class } 2\rangle \tag{7.661}$$

$$\langle a, b, c \,|\, cb, pa - ba, \text{ class } 2\rangle \tag{7.662}$$

$$\langle a, b, c \,|\, cb, pb, \text{ class } 2\rangle \tag{7.663}$$

$$\langle a, b, c \,|\, cb, pb - ba, \text{ class } 2\rangle \tag{7.664}$$

$$\langle a, b, c \,|\, cb, pb - ca, \text{ class } 2\rangle \tag{7.665}$$

$$\langle a, b, c \,|\, pb, pc, \text{ class } 2\rangle \tag{7.666}$$

$$\langle a, b, c \,|\, pb - ba, pc, \text{ class } 2\rangle \tag{7.667}$$

$$\langle a, b, c \,|\, pb - ca, pc, \text{ class } 2\rangle \tag{7.668}$$

$$\langle a, b, c \,|\, pb - cb, pc, \text{ class } 2\rangle \tag{7.669}$$

$$\langle a, b, c \,|\, pb - ba, pc - cb, \text{ class } 2\rangle \tag{7.670}$$

$$\langle a, b, c \,|\, pb - ca, pc - cb, \text{ class } 2\rangle \tag{7.671}$$

$$\langle a, b, c \,|\, pb - ba, pc - ca, \text{ class } 2\rangle \tag{7.672}$$

$$\langle a, b, c \,|\, pb - ca, pc - ba, \text{ class } 2\rangle \tag{7.673}$$

$$\langle a, b, c \,|\, pb - \omega ca, pc - ba, \text{ class } 2\rangle \tag{7.674}$$

$$\langle a, b, c \,|\, pb - xca, pc - ba - ca, \text{ class } 2\rangle \ (x \neq 0) \tag{7.675}$$

8.61 Descendants of 5.10

$2p + 7$ algebras

$$\langle a, b, c \,|\, ca, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.676}$$

$$\langle a, b, c \,|\, ca - baa, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.677}$$

$$\langle a, b, c \,|\, ca - p^2a, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.678}$$

$$\langle a, b, c \,|\, ca - xp^2a, cb - baa, pb - ba, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.679}$$

$$\langle a, b, c \,|\, ca - baa - p^2a, cb - baa, pb - ba, pc, \text{ class } 3\rangle \tag{7.680}$$

$$\langle a, b, c \,|\, ca - xbaa - p^2a, cb - p^2a, pb - ba, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.681}$$

$$\langle a, b, c \,|\, ca, cb - p^2a, pb - ba, pc, \text{ class } 3\rangle \tag{7.682}$$

$$\langle a, b, c \,|\, ca - baa, cb - p^2a, pb - ba, pc, \text{ class } 3\rangle \tag{7.683}$$

$$\langle a, b, c \,|\, ca - \omega baa, cb - p^2a, pb - ba, pc, \text{ class } 3\rangle \tag{7.684}$$

8.62 Descendants of 5.12

$3p^2 + 17p + 53 + \gcd(p - 1, 3) + \gcd(p - 1, 4)$ algebras

$$\langle a, b, c \,|\, bab, ca, cb, pa, pb, \text{ class } 3\rangle \tag{7.685}$$

$$\langle a, b, c \,|\, bab, ca, cb, pa - baa, pb, \text{ class } 3\rangle \tag{7.686}$$

$$\langle a, b, c \,|\, bab, ca, cb, pa, pb - baa, \text{ class } 3\rangle \tag{7.687}$$

$$\langle a, b, c \,|\, bab, ca, cb, pa, pb - \omega baa, \text{ class } 3\rangle \tag{7.688}$$

$$\langle a, b, c \,|\, bab, ca - p^2c, cb, pa, pb - xbaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.689}$$

$$\langle a, b, c \,|\, bab, ca - p^2c, cb, pa - baa, pb, \text{ class } 3\rangle \tag{7.690}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa, pb, \text{ class } 3\rangle \tag{7.691}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa - baa, pb, \text{ class } 3\rangle \tag{7.692}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa, pb - baa, \text{ class } 3\rangle \tag{7.693}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa, pb - \omega baa, \text{ class } 3\rangle \tag{7.694}$$

$$\langle a,b,c\,|\,bab, ca-p^2c, cb-baa, pa, pb-xbaa, \text{ class 3}\rangle \text{ (all } x) \tag{7.695}$$

$$\langle a,b,c\,|\,bab, ca-p^2c, cb-baa, pa-baa, pb, \text{ class 3}\rangle \tag{7.696}$$

$$\langle a,b,c\,|\,bab, ca, cb-p^2c, pa, pb, \text{ class 3}\rangle \tag{7.697}$$

$$\langle a,b,c\,|\,bab, ca, cb-p^2c, pa, pb-baa, \text{ class 3}\rangle \tag{7.698}$$

$$\langle a,b,c\,|\,bab, ca, cb-p^2c, pa, pb-\omega baa, \text{ class 3}\rangle \tag{7.699}$$

$$\langle a,b,c\,|\,bab, ca, cb-p^2c, pa-baa, pb-xbaa, \text{ class 3}\rangle \text{ (all } x) \tag{7.700}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa-p^2c, pa, pb, \text{ class 3}\rangle \tag{7.701}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa-p^2c, pa, pb-baa, \text{ class 3}\rangle \tag{7.702}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa-p^2c, pa, pb-\omega baa, \text{ class 3}\rangle \tag{7.703}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa-p^2c, pa-baa, pb-xbaa, \text{ class 3}\rangle \text{ (all } x) \tag{7.704}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-baa, pb, \text{ class 3}\rangle \tag{7.705}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-baa, pb-xbab, \text{ class 3}\rangle \ (x\neq 0, x\sim x^{-1}) \tag{7.706}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-baa-bab, pb-bab, \text{ class 3}\rangle \tag{7.707}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-baa-\omega bab, pb-bab, \text{ class 3}\rangle \tag{7.708}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa, pb, \text{ class 3}\rangle \tag{7.709}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-bab, pb, \text{ class 3}\rangle \tag{7.710}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-\omega bab, pb, \text{ class 3}\rangle \tag{7.711}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-\omega bab, pb-baa, \text{ class 3}\rangle \tag{7.712}$$

$$\langle a,b,c\,|\,p^2c, ca, cb, pa-xbab, pb-baa-bab, \text{ class 3}\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.713}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa, pb, \text{ class 3}\rangle \tag{7.714}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa, pb-baa, \text{ class 3}\rangle \tag{7.715}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa, pb-\omega baa, \text{ class 3}\rangle \tag{7.716}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa, pb-bab, \text{ class 3}\rangle \tag{7.717}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-baa, pb-xbab, \text{ class 3}\rangle \text{ (all } x) \tag{7.718}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-baa, pb-baa-bab, \text{ class 3}\rangle \tag{7.719}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-baa, pb-\omega baa-bab, \text{ class 3}\rangle \tag{7.720}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-bab, pb-xbaa-bab, \text{ class 3}\rangle \text{ (all } x) \tag{7.721}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-bab, pb, \text{ class 3}\rangle \tag{7.722}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-bab, pb-baa, \text{ class 3}\rangle \tag{7.723}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-bab, pb-\omega baa, \text{ class 3}\rangle \tag{7.724}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-\omega bab, pb-xbaa-bab, \text{ class 3}\rangle \text{ (all } x) \tag{7.725}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-\omega bab, pb, \text{ class 3}\rangle \tag{7.726}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-\omega bab, pb-baa, \text{ class 3}\rangle \tag{7.727}$$

$$\langle a,b,c\,|\,p^2c, ca, cb-baa, pa-\omega bab, pb-\omega baa, \text{ class 3}\rangle \tag{7.728}$$

$$\langle a,b,c\,|\,p^2c, ca-bab, cb-baa, pa-xbaa-ybab, pb-zbaa-tbab, \text{ class 3}\rangle \text{ (See Notes5.12)} \tag{7.729}$$

$$\langle a,b,c\,|\,p^2c, ca-\omega bab, cb-baa, pa-xbaa-ybab, pb-zbaa-tbab, \text{ class 3}\rangle \text{ (See Notes5.12)} \tag{7.730}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb, pa, pb, \text{ class 3}\rangle \tag{7.731}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-baa, pa, pb, \text{ class 3}\rangle \tag{7.732}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-bab, pa, pb, \text{ class 3}\rangle \tag{7.733}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb, pa, pb, \text{ class 3}\rangle \tag{7.734}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-baa, pa, pb, \text{ class 3}\rangle \tag{7.735}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-\omega baa, pa, pb, \text{ class 3}\rangle \tag{7.736}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb, pa, pb-bab, \text{ class 3}\rangle \tag{7.737}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-baa, pa, pb-bab, \text{ class 3}\rangle \tag{7.738}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-bab, pa, pb-bab, \text{ class } 3\rangle \tag{7.739}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-xbaa, pa, pb-bab, \text{ class } 3\rangle \text{ (all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p) \tag{7.740}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-\omega bab, cb-xbaa, pa, pb-bab, \text{ class } 3\rangle \text{ (all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.741}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-\omega^2 bab, cb-xbaa, pa, pb-bab, \text{ class } 3\rangle \text{ (all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.742}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-xbaa, pa-bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x) \tag{7.743}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-xbaa-bab, pa-bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x) \tag{7.744}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-xbaa-ybab, pa-bab, pb, \text{ class } 3\rangle \text{ (all } x,\ y,\ y\sim -y,\ p=3 \bmod 4) \tag{7.745}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-xbaa, pa-bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.746}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-\omega bab, cb-xbaa, pa-bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.747}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-xbaa-ybab, pa-bab, pb, \text{ class } 3\rangle \text{ (all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p,\ p=1 \bmod 4) \tag{7.748}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-\omega bab, cb-xbaa-ybab, pa-bab, pb, \text{ class } 3\rangle \text{ (all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p,\ p=1 \bmod 4) \tag{7.749}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-xbaa, pa-\omega bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x) \tag{7.750}$$

$$\langle a,b,c\,|\,p^2c-baa, ca, cb-xbaa-bab, pa-\omega bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x) \tag{7.751}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-xbaa-ybab, pa-\omega bab, pb, \text{ class } 3\rangle \text{ (all } x,\ y,\ y\sim -y,\ p=3 \bmod 4) \tag{7.752}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-xbaa, pa-\omega bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.753}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-\omega bab, cb-xbaa, pa-\omega bab, pb, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.754}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-bab, cb-xbaa-ybab, pa-\omega bab, pb, \text{ class } 3\rangle \text{ (all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p,\ p=1 \bmod 4) \tag{7.755}$$

$$\langle a,b,c\,|\,p^2c-baa, ca-\omega bab, cb-xbaa-ybab, pa-\omega bab, pb, \text{ class } 3\rangle \text{ (all } x,\ y\neq 0, y\sim y' \text{ if } y^4=y'^4 \bmod p,\ p=1 \bmod 4) \tag{7.756}$$

8.63 Descendants of 5.14

Algebra 5.14 has

$$2p^5 + 7p^4 + 19p^3 + 49p^2 + 128p + 256 + (p^2 + 7p + 29)\gcd(p-1,3)$$
$$+(p^2 + 7p + 24)\gcd(p-1,4) + (p+3)\gcd(p-1,5)$$

immediate descendants of order p^7 and p-class 3.

Algebra 5.14 has presentation

$$\langle a, b, c \,|\, cb,\, pa,\, pb,\, pc,\, \text{class } 2\rangle,$$

and so if L is an immediate descendant of 5.14 of order p^7 then L_2 is generated by ba, ca modulo L_3, and L_3 has order p^2 and is generated by baa, bab, bac, caa, cab. And cb, pa, pb, $pc \in L_3$. The commutator structure is given by the classification of nilpotent Lie algebras over $\mathbb{Z}_p$. So we may assume that one of the following holds:

$$cb = caa = cab = cac = 0,$$
$$caa = cab = cac = 0,\ cb = baa,$$
$$cb = bab = bac = cab = cac = 0,$$
$$cb = baa,\ bab = bac = cab = cac = 0,$$
$$cb = bac = cac = 0,\ caa = bab,$$
$$cb = baa,\ bac = cac = 0,\ caa = bab,$$
$$cb = baa = bac = cac = 0,$$
$$baa = bac = cac = 0,\ cb = caa,$$
$$cb = bac = caa = 0,\ cac = bab,$$
$$cb = bac = caa = 0,\ cac = \omega bab,$$
$$bac = caa = 0,\ cb = baa,\ cac = bab,$$
$$bac = caa = 0,\ cb = baa,\ cac = \omega bab,$$
$$cb = bac = 0,\ caa = baa,\ cac = -bab,$$
$$bac = 0,\ cb = caa = baa,\ cac = -bab,$$
$$cb = baa = bac = caa = 0,$$
$$cb = bac = caa = 0,\ baa = cac,$$
$$cb = bac = 0,\ baa = cac,\ caa = bab,$$
$$cb = bac = 0,\ baa = cac,\ caa = \omega bab,\ (p = 1 \bmod 3)$$
$$cb = baa = caa = cac = 0,$$
$$cb = baa = cac = 0,\ caa = bab,$$
$$cb = caa = cac = 0,\ baa = bab,$$
$$cb = baa = caa = 0,\ cac = \omega bab,$$
$$cb = baa = 0,\ caa = bac,\ cac = \omega bab,$$
$$cb = baa = 0,\ caa = kbab + bac,\ cac = \omega bab,\ (p = 2 \bmod 3),$$

where k is any (one) integer which is not a value of

$$\frac{\lambda(\lambda^2 + 3\omega\mu^2)}{\mu(3\lambda^2 + \omega\mu^2)} \bmod p.$$

Since the total number of descendants of 5.14 of order p^7 is of order $2p^5$, we need presentations with at least 5 parameters in some of these cases. In each case the commutator structure is determined, and so to give a presentation for the Lie rings we only need to specify pa, pb, pc. These powers take values in L_3, which has order p^2, so we need 2 coefficients for each of pa, pb, pc. For the sake of simplicity I give a single presentation with 6 parameters for each of the 24 different commutator structures defined above, and I give the conditions for two sets of parameters to define isomorphic Lie rings.

8.63.1 Case 1

$$\langle a, b, c \,|\, cb, caa, cab, cac, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab \rangle. \tag{7.757}$$

Here L_3 is generated by baa and bab, and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & 0 & \xi \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

There are $3p + 22$ algebras in all in this case.

8.63.2 Case 2

$$\langle a, b, c \,|\, cb - baa, caa, cab, cac, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab \rangle. \tag{7.758}$$

Here L_3 is generated by baa and bab, and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & 0 & \alpha^2 \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $5p + 13 + \gcd(p-1, 3) + \gcd(p-1, 4)$.

8.63.3 Case 3

$$\langle a, b, c \,|\, cb, bab, bac, cab, cac, pa - x_1 baa - x_2 caa, pb - x_3 baa - x_4 caa, pc - x_5 baa - x_6 caa \rangle. \tag{7.759}$$

L_3 is generated by baa and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & \nu & \xi \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha^2\mu \\ \alpha^2\nu & \alpha^2\xi \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $2p + 8 + \gcd(p-1, 4)$.

8.63.4 Case 4

$$\langle a, b, c \,|\, cb - baa, bab, bac, cab, cac, pa - x_1 baa - x_2 caa, pb - x_3 baa - x_4 caa, pc - x_5 baa - x_6 caa \rangle. \tag{7.760}$$

L_3 is generated by baa and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & 0 \\ 0 & \nu & \alpha^2 \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & 0 \\ \alpha^2\nu & \alpha^4 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $6p + 8 + 2\gcd(p-1,3) + \gcd(p-1,4) + \gcd(p-1,5)$.

8.63.5 Case 5

$$\langle a, b, c \,|\, cb, bac, caa - bab, cac, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab \rangle. \tag{7.761}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & \gamma \\ 0 & \lambda & \mu \\ 0 & 0 & \alpha^{-1}\lambda^2 \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha^2\mu + \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $5p + 13 + 2\gcd(p-1,3) + \gcd(p-1,4)$.

8.63.6 Case 6

$$\langle a, b, c \,|\, cb - baa, bac, caa - bab, cac, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab \rangle. \tag{7.762}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha^2 & \beta & \gamma \\ 0 & \pm\alpha^3 & \mu \\ 0 & 0 & \alpha^4 \end{pmatrix} A \begin{pmatrix} \pm\alpha^7 & \alpha^4\mu \pm \alpha^5\beta \\ 0 & \alpha^8 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$p^2 + 3p - 3 + (p+2)\gcd(p-1,3) + (p+1)\gcd(p-1,4) + (p+1)\gcd(p-1,5).$$

8.63.7 Case 7

$$\langle a,b,c \,|\, cb, baa, bac, cac, pa - x_1bab - x_2caa, pb - x_3bab - x_4caa, pc - x_5bab - x_6caa\rangle. \tag{7.763}$$

L_3 is generated by bab and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \lambda & 0 \\ 0 & 0 & \xi \end{pmatrix} A \begin{pmatrix} \alpha\lambda^2 & 0 \\ 0 & \alpha^2\xi \end{pmatrix}^{-1}.$$

The total number of algebras in this case is $2p^2 + 11p + 43 + \gcd(p-1, 4)$.

8.63.8 Case 8

$$\langle a,b,c \,|\, cb - caa, baa, bac, cac, pa - x_1bab - x_2caa, pb - x_3bab - x_4caa, pc - x_5bab - x_6caa\rangle. \tag{7.764}$$

L_3 is generated by bab and caa and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ caa \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \alpha^2 & 0 \\ 0 & 0 & \xi \end{pmatrix} A \begin{pmatrix} \alpha^5 & 0 \\ 0 & \alpha^2\xi \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$p^3 + 4p^2 + 6p + (p+5)\gcd(p-1, 3) + 3\gcd(p-1, 4) + \gcd(p-1, 5).$$

8.63.9 Cases 9 and 10

$$\langle a,b,c \,|\, cb, bac, caa, cac - bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab\rangle. \tag{7.765}$$

$$\langle a,b,c \,|\, cb, bac, caa, cac - \omega bab, pa - x_1baa - x_2bab, pb - x_3baa - x_4bab, pc - x_5baa - x_6bab\rangle. \tag{7.766}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \pm\lambda \end{pmatrix} A \begin{pmatrix} \alpha^2\lambda & \alpha\beta\lambda \\ 0 & \alpha\lambda^2 \end{pmatrix}^{-1}.$$

The total number of algebras in both cases is

$$p^3 + \frac{5}{2}p^2 + 7p + \frac{19}{2} + \frac{p+4}{2}\gcd(p-1, 4).$$

8.63.10 Cases 11 and 12

$$\langle a,b,c \,|\, cb-baa, bac, caa, cac-bab, pa-x_1baa-x_2bab, pb-x_3baa-x_4bab, pc-x_5baa-x_6bab\rangle. \tag{7.767}$$

$$\langle a,b,c \,|\, cb-baa, bac, caa, cac-\omega bab, pa-x_1baa-x_2bab, pb-x_3baa-x_4bab, pc-x_5baa-x_6bab\rangle. \tag{7.768}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & 0 \\ 0 & \pm\alpha^2 & 0 \\ 0 & 0 & \alpha^2 \end{pmatrix} A \begin{pmatrix} \pm\alpha^4 & \pm\alpha^3\beta \\ 0 & \alpha^5 \end{pmatrix}^{-1}.$$

The total number of algebras in both cases is

$$(p^4+p^3+4p^2+p-1+(p^2+2p+3)\gcd(p-1,3)+(p+2)\gcd(p-1,4))/2$$

8.63.11 Case 13

$$\langle a,b,c \,|\, cb, bac, caa-baa, cac+bab, pa-x_1baa-x_2bab, pb-x_3baa-x_4bab, pc-x_5baa-x_6bab\rangle. \tag{7.769}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & -\beta \\ 0 & \lambda & \mu \\ 0 & \mu & \lambda \end{pmatrix} A \begin{pmatrix} \alpha^2(\lambda+\mu) & \alpha\beta(\lambda+\mu) \\ 0 & \alpha(\lambda^2-\mu^2) \end{pmatrix}^{-1}.$$

In this case there are $2p^2+11p+27+\gcd(p-1,4)$ immediate descendants of order p^7 and p-class 3.

8.63.12 Case 14

$$\langle a,b,c \,|\, cb-baa, bac, caa-baa, cac+bab, pa-x_1baa-x_2bab, pb-x_3baa-x_4bab, pc-x_5baa-x_6bab\rangle. \tag{7.770}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & \beta & -\beta \\ 0 & \lambda & \lambda-\alpha^2 \\ 0 & \lambda-\alpha^2 & \lambda \end{pmatrix} A \begin{pmatrix} 2\alpha^2\lambda-\alpha^4 & 2\alpha\beta\lambda-\alpha^3\beta \\ 0 & 2\alpha^3\lambda-\alpha^5 \end{pmatrix}^{-1}.$$

In this case there are $p^3+2p^2+6p+10+(p+4)\gcd(p-1,3)$ algebras.

8.63.13 Case 15

$$\langle a,b,c \,|\, cb, baa, bac, caa, pa - x_1 bab - x_2 cac, pb - x_3 bab - x_4 cac, pc - x_5 bab - x_6 cac\rangle. \tag{7.771}$$

L_3 is generated by bab and cac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ cac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\beta^2 & 0 \\ 0 & \alpha\gamma^2 \end{pmatrix}^{-1}$$

and

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & \beta \\ 0 & \gamma & 0 \end{pmatrix} A \begin{pmatrix} 0 & \alpha\beta^2 \\ \alpha\gamma^2 & 0 \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$p^3 + \frac{7}{2}p^2 + \frac{17}{2}p + \frac{59}{2} + \frac{5}{2}\gcd(p-1,3) + \frac{p+1}{2}\gcd(p-1,4).$$

8.63.14 Case 16

$$\langle a,b,c \,|\, cb, bac, caa, cac - baa, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab\rangle. \tag{7.772}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-1}\gamma^2 & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\gamma^2 & 0 \\ 0 & \alpha^{-1}\gamma^4 \end{pmatrix}^{-1}.$$

The total number of algebras here is

$$2p^4 + 4p^3 + 8p^2 + 14p + 11 + 4\gcd(p-1,3) + 3\gcd(p-1,4).$$

8.63.15 Case 17

$$\langle a,b,c \,|\, cb, bac, caa - bab, cac - baa, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab\rangle. \tag{7.773}$$

L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-1}\gamma^2 & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\gamma^2 & 0 \\ 0 & \alpha^2\gamma \end{pmatrix}^{-1}$$

or

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & 0 & \alpha^{-1}\gamma^2 \\ 0 & \gamma & 0 \end{pmatrix} A \begin{pmatrix} 0 & \alpha\gamma^2 \\ \alpha^2\gamma & 0 \end{pmatrix}^{-1}$$

with $\alpha^3 = \gamma^3$.

If $p \neq 1 \bmod 3$ then $\alpha = \gamma$ and the number of orbits is

$$p^5 + p^4 + p^3 + p^2 + p + 2 + (p^2 + p + 1)\gcd(p-1, 4)/2.$$

If $p = 1 \bmod 3$ then $\alpha = \gamma$ or $\xi\gamma$ or $\xi^2\gamma$ where $\xi^3 = 1$. The number of orbits is then

$$(p^5 + p^4 + p^3 + p^2 + 7p + 10)/3 + (p^2 + p + 1)\gcd(p-1, 4)/2.$$

So in general the number of orbits is

$$(p^4 + 2p^3 + 3p^2 + 4p + 2)\frac{p-1}{\gcd(p-1, 3)} + 3p + 4 + (p^2 + p + 1)\gcd(p-1, 4)/2.$$

8.63.16 Case 18

$$\langle a, b, c \,|\, cb, bac, caa - \omega bab, cac - baa, pa - x_1 baa - x_2 bab, pb - x_3 baa - x_4 bab, pc - x_5 baa - x_6 bab\rangle \ (p = 1 \bmod 3). \quad (7.774)$$

This case is very similar to Case 17, though we do not have as many automorphisms. L_3 is generated by baa and bab and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha^{-1}\gamma^2 & 0 \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha\gamma^2 & 0 \\ 0 & \alpha^2\gamma \end{pmatrix}^{-1}$$

with $\alpha^3 = \gamma^3$.

The number of algebras is

$$(2p^5 + 2p^4 + 2p^3 + 2p^2 + 14p + 17)/3.$$

Combining Case 17 and Case 18, the total number of algebras in the two cases is

$$p^5 + p^4 + p^3 + p^2 - 2p - \frac{3}{2} + (3p + \frac{7}{2})\gcd(p-1, 3) + (p^2 + p + 1)\gcd(p-1, 4)/2.$$

8.63.17 Case 19

$$\langle a, b, c \,|\, cb, baa, caa, cac, pa - x_1 bab - x_2 bac, pb - x_3 bab - x_4 bac, pc - x_5 bab - x_6 bac\rangle. \quad (7.775)$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & \gamma \\ 0 & 0 & \delta \end{pmatrix} A \begin{pmatrix} \alpha\beta^2 & 2\alpha\beta\gamma \\ 0 & \alpha\beta\delta \end{pmatrix}^{-1}$$

The total number of algebras in this case is $2p^2 + 11p + 27 + \gcd(p-1, 4)$.

8.63.18 Case 20

$$\langle a,b,c \,|\, cb, baa, caa-bab, cac, pa-x_1bab-x_2bac, pb-x_3bab-x_4bac, pc-x_5bab-x_6bac\rangle. \tag{7.776}$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \alpha^{-1}\beta^2 \end{pmatrix} A \begin{pmatrix} \alpha\beta^2 & 0 \\ 0 & \beta^3 \end{pmatrix}^{-1}$$

The total number of algebras here is

$$2p^4+4p^3+6p^2+11p+11+2\gcd(p-1,3)+(p+1)\gcd(p-1,4).$$

8.63.19 Case 21

$$\langle a,b,c \,|\, cb, bab-baa, caa, cac, pa-x_1baa-x_2bac, pb-x_3baa-x_4bac, pc-x_5baa-x_6bac\rangle. \tag{7.777}$$

L_3 is generated by baa and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} baa \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 2\beta \\ 0 & \alpha & \beta \\ 0 & 0 & \gamma \end{pmatrix} A \begin{pmatrix} \alpha^3 & 2\alpha^2\beta \\ 0 & \alpha^2\gamma \end{pmatrix}^{-1}.$$

The total number of algebras in this case is

$$2p^3+6p^2+7p+7+(p+1)\gcd(p-1,4).$$

8.63.20 Case 22

$$\langle a,b,c \,|\, cb, baa, caa, cac-\omega bab, pa-x_1bab-x_2bac, pb-x_3bab-x_4bac, pc-x_5bab-x_6bac\rangle. \tag{7.778}$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \omega\beta & \pm\gamma \\ 0 & \omega\gamma & \pm\omega\beta \end{pmatrix} A \begin{pmatrix} \omega\alpha(\omega\beta^2+\gamma^2) & \pm 2\omega\alpha\beta\gamma \\ 2\omega^2\alpha\beta\gamma & \pm\omega\alpha(\omega\beta^2+\gamma^2) \end{pmatrix}^{-1}.$$

The total number of algebras in Case 22 is

$$(2p^3+3p^2+3p+13-\gcd(p-1,3)+(p+1)\gcd(p-1,4))/2.$$

8.63.21 Case 23

$$\langle a,b,c \,|\, cb, baa, caa-bac, cac-\omega bab, pa-x_1bab-x_2bac, pb-x_3bab-x_4bac, pc-x_5bab-x_6bac\rangle. \tag{7.779}$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \pm\alpha \end{pmatrix} A \begin{pmatrix} \alpha^3 & 0 \\ 0 & \pm\alpha^3 \end{pmatrix}^{-1}$$

or when $p = 2 \bmod 3$ and $12\omega\beta^2 = -1$,

$$A \to \begin{pmatrix} 4\omega\alpha\beta & -3\omega\alpha\beta & \frac{\alpha}{2} \\ 0 & -2\omega\alpha\beta & \alpha \\ 0 & \pm\omega\alpha & \mp 2\omega\alpha\beta \end{pmatrix} A \begin{pmatrix} \frac{8}{3}\omega^2\alpha^3\beta & \frac{4}{3}\omega\alpha^3 \\ \pm\frac{4}{3}\omega^2\alpha^3 & \pm\frac{8}{3}\omega^2\alpha^3\beta \end{pmatrix}^{-1}.$$

If $p = 1 \bmod 3$ there are $p^5+p^4+p^3+p^2+p+2+(p^2+p+1)\gcd(p-1,4)/2$ algebras.

When $p = 2 \bmod 3$ the number of algebras here is

$$\frac{1}{3}p^5 + \frac{1}{3}p^4 + \frac{1}{3}p^3 + \frac{1}{3}p^2 + p + 2 + (p^2+p+1)\gcd(p-1,4)/2.$$

8.63.22 Case 24

$$\langle a,b,c \,|\, cb, baa, caa-xbab-bac, cac-\omega bab, pa-x_1bab-x_2bac, pb-x_3bab-x_4bac, pc-x_5bab-x_6bac\rangle \ (p = 2 \bmod 3). \tag{7.780}$$

where x is any (fixed) integer which is not a value of

$$\frac{\lambda(\lambda^2+3\omega\mu^2)}{\mu(3\lambda^2+\omega\mu^2)} \bmod p.$$

L_3 is generated by bab and bac and if we let

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = A \begin{pmatrix} bab \\ bac \end{pmatrix}$$

then the isomorphism classes of algebras satisfying these commutator relations correspond to the orbits of 3×2 matrices A under the action

$$A \to \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{pmatrix} A \begin{pmatrix} \alpha^3 & 0 \\ 0 & \alpha^3 \end{pmatrix}^{-1}$$

and

$$A \to \begin{pmatrix} -4\alpha & k\alpha\beta+3\alpha & 3k\omega^{-1}\alpha+\alpha\beta \\ 0 & 2\alpha & 2\alpha\beta \\ 0 & 2\omega\alpha\beta & 2\alpha \end{pmatrix} A \begin{pmatrix} 32\alpha^3 & -32\alpha^3\beta \\ -32\omega\alpha^3\beta & 32\alpha^3 \end{pmatrix}^{-1}$$

where $\omega\beta^2 = -3$.

The number of orbits is

$$\frac{2}{3}p^5 + \frac{2}{3}p^4 + \frac{2}{3}p^3 + \frac{2}{3}p^2 + 2p + 3.$$

The total number of algebras from Case 23 and Case 24 is

$$p^5+p^4+p^3+p^2+4p+\frac{13}{2}-(p+\frac{3}{2})\gcd(p-1,3)+(p^2+p+1)\gcd(p-1,4)/2.$$

The total number of algebras from cases 17, 18, 23 and 24 is

$$p^5+p^4+p^3+p^2+2p+5+(2p+2)\gcd(p-1,3)+(p^2+p+1)\gcd(p-1,4).$$

8.64 Descendants of 5.15

$3p^2 + 12p + 14 + (p+2)\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, cb, pa - ba, pb, pc, \text{ class } 3\rangle \tag{7.781}$$

$$\langle a,b,c \,|\, cb, pa - ba - caa, pb, pc, \text{ class } 3\rangle \tag{7.782}$$

$$\langle a,b,c \,|\, cb, pa - ba - cac, pb, pc, \text{ class } 3\rangle \tag{7.783}$$

$$\langle a,b,c \,|\, cb, pa - ba - \omega cac, pb, pc, \text{ class } 3\rangle \tag{7.784}$$

$$\langle a,b,c \,|\, cb, pa - ba, pb, pc - caa, \text{ class } 3\rangle \tag{7.785}$$

$$\langle a,b,c \,|\, cb, pa - ba - cac, pb, pc - caa, \text{ class } 3\rangle \tag{7.786}$$

$$\langle a,b,c \,|\, cb, pa - ba - \omega cac, pb, pc - caa, \text{ class } 3\rangle \tag{7.787}$$

$$\langle a,b,c \,|\, cb, pa - ba - caa - xcac, pb, pc - caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.788}$$

$$\langle a,b,c \,|\, cb, pa - ba, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.789}$$

$$\langle a,b,c \,|\, cb, pa - ba - cac, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.790}$$

$$\langle a,b,c \,|\, cb, pa - ba - \omega cac, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.791}$$

$$\langle a,b,c \,|\, cb, pa - ba - caa - xcac, pb, pc - \omega caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.792}$$

$$\langle a,b,c \,|\, cb, pa - ba - xcaa, pb, pc - cac, \text{ class } 3\rangle \text{ (all } x) \tag{7.793}$$

$$\langle a,b,c \,|\, cb, pa - ba - caa - cac, pb, pc - cac, \text{ class } 3\rangle \tag{7.794}$$

$$\langle a,b,c \,|\, cb, pa - ba - caa - \omega cac, pb, pc - cac, \text{ class } 3\rangle \tag{7.795}$$

$$\langle a,b,c \,|\, cb, pa - ba, pb - caa, pc, \text{ class } 3\rangle \tag{7.796}$$

$$\langle a,b,c \,|\, cb, pa - ba - cac, pb - caa, pc, \text{ class } 3\rangle \tag{7.797}$$

$$\langle a,b,c \,|\, cb, pa - ba - \omega cac, pb - caa, pc, \text{ class } 3\rangle \tag{7.798}$$

$$\langle a,b,c \,|\, cb, pa - ba - \omega^2 cac, pb - caa, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.799}$$

$$\langle a,b,c \,|\, cb, pa - ba - \omega^3 cac, pb - caa, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.800}$$

$$\langle a,b,c \,|\, cb, pa - ba - xcac, pb - caa, pc - cac, \text{ class } 3\rangle \text{ (all } x) \tag{7.801}$$

$$\langle a,b,c\,|\,cb, pa-ba, pb-cac, pc, \text{ class } 3\rangle \tag{7.802}$$

$$\langle a,b,c\,|\,cb, pa-ba-caa, pb-cac, pc, \text{ class } 3\rangle \tag{7.803}$$

$$\langle a,b,c\,|\,cb, pa-ba, pb-cac, pc-caa, \text{ class } 3\rangle \tag{7.804}$$

$$\langle a,b,c\,|\,cb, pa-ba, pb-cac, pc-\omega caa, \text{ class } 3\rangle \tag{7.805}$$

$$\langle a,b,c\,|\,cb, pa-ba, pb-cac, pc-\omega^2 caa, \text{ class } 3\rangle\ (p=1 \bmod 4) \tag{7.806}$$

$$\langle a,b,c\,|\,cb, pa-ba, pb-cac, pc-\omega^3 caa, \text{ class } 3\rangle\ (p=1 \bmod 4) \tag{7.807}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba, pb, pc, \text{ class } 3\rangle \tag{7.808}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-cac, pb, pc, \text{ class } 3\rangle \tag{7.809}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-\omega cac, pb, pc, \text{ class } 3\rangle \tag{7.810}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0) \tag{7.811}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-cac, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0) \tag{7.812}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-\omega cac, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0) \tag{7.813}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-caa-ycac, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0,\ \text{all } y) \tag{7.814}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-xcac, pb, pc-cac, \text{ class } 3\rangle\ (\text{all } x) \tag{7.815}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-ycac, pb-caa, pc-xcac, \text{ class } 3\rangle\ (\text{all } x,\ y, x\sim -x) \tag{7.816}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba, pb-cac, pc-xcaa, \text{ class } 3\rangle\ (\text{all } x) \tag{7.817}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-xcaa, pb-cac, pc+2caa, \text{ class } 3\rangle\ (x\neq 0, x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.818}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba, pb-\omega cac, pc-xcaa, \text{ class } 3\rangle\ (\text{all } x,\ p=1 \bmod 4) \tag{7.819}$$

$$\langle a,b,c\,|\,cb-caa, pa-ba-xcaa, pb-\omega cac, pc+2caa, \text{ class } 3\rangle\ (x\neq 0, x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.820}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba, pb, pc, \text{ class } 3\rangle \tag{7.821}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-cac, pb, pc, \text{ class } 3\rangle \tag{7.822}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-\omega cac, pb, pc, \text{ class } 3\rangle \tag{7.823}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0) \tag{7.824}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-cac, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0) \tag{7.825}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-\omega cac, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0) \tag{7.826}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-caa-ycac, pb, pc-xcaa, \text{ class } 3\rangle\ (x\neq 0,\ \text{all } y) \tag{7.827}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-xcac, pb, pc-cac, \text{ class } 3\rangle\ (\text{all } x) \tag{7.828}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-ycac, pb-caa, pc-xcac, \text{ class } 3\rangle\ (\text{all } x,\ y,\ x\sim -x) \tag{7.829}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba, pb-cac, pc-xcaa, \text{ class } 3\rangle\ (\text{all } x) \tag{7.830}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-xcaa, pb-cac, pc+(2*\omega)caa, \text{ class } 3\rangle\ (x\neq 0, x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.831}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba, pb-\omega cac, pc-xcaa, \text{ class } 3\rangle\ (\text{all } x,\ p=1 \bmod 4) \tag{7.832}$$

$$\langle a,b,c\,|\,cb-\omega caa, pa-ba-xcaa, pb-\omega cac, pc+(2*\omega)caa, \text{ class } 3\rangle\ (x\neq 0, x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.833}$$

8.65 Descendants of 5.16

$p^4+2p^3+5p^2+14p$ algebras

$$\langle a,b,c\,|\,cb, pa, pb-ba, pc, \text{ class } 3\rangle \tag{7.834}$$

$$\langle a,b,c\,|\,cb, pa, pb-ba-caa, pc, \text{ class } 3\rangle \tag{7.835}$$

$$\langle a,b,c\,|\,cb, pa, pb-ba-cac, pc, \text{ class } 3\rangle \tag{7.836}$$

$$\langle a,b,c\,|\,cb, pa-caa, pb-ba, pc, \text{ class } 3\rangle \tag{7.837}$$

$$\langle a,b,c\,|\,cb, pa-caa, pb-ba-cac, pc, \text{ class } 3\rangle \tag{7.838}$$

$$\langle a,b,c\,|\,cb, pa-caa, pb-ba-caa-xcac, pc, \text{ class } 3\rangle\ (\text{all } x) \tag{7.839}$$

$$\langle a,b,c\,|\,cb, pa-cac, pb-ba, pc, \text{ class } 3\rangle \tag{7.840}$$

$$\langle a,b,c\,|\,cb, pa-cac, pb-ba-caa, pc, \text{ class } 3\rangle \tag{7.841}$$

$$\langle a,b,c\,|\,cb, pa-cac, pb-ba-cac, pc, \text{ class } 3\rangle \tag{7.842}$$

$$\langle a, b, c \,|\, cb, pa - \omega cac, pb - ba, pc, \text{ class } 3\rangle \tag{7.843}$$

$$\langle a, b, c \,|\, cb, pa - \omega cac, pb - ba - caa, pc, \text{ class } 3\rangle \tag{7.844}$$

$$\langle a, b, c \,|\, cb, pa - \omega cac, pb - ba - cac, pc, \text{ class } 3\rangle \tag{7.845}$$

$$\langle a, b, c \,|\, cb, pa, pb - ba, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.846}$$

$$\langle a, b, c \,|\, cb, pa, pb - ba - cac, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.847}$$

$$\langle a, b, c \,|\, cb, pa, pb - ba - caa, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.848}$$

$$\langle a, b, c \,|\, cb, pa, pb - ba - caa - cac, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.849}$$

$$\langle a, b, c \,|\, cb, pa - cac, pb - ba, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.850}$$

$$\langle a, b, c \,|\, cb, pa - cac, pb - ba - caa, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.851}$$

$$\langle a, b, c \,|\, cb, pa - cac, pb - ba - ycaa - cac, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0, \text{ all } y, \ y \sim -y) \tag{7.852}$$

$$\langle a, b, c \,|\, cb, pa - \omega cac, pb - ba, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.853}$$

$$\langle a, b, c \,|\, cb, pa - \omega cac, pb - ba - caa, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.854}$$

$$\langle a, b, c \,|\, cb, pa - \omega cac, pb - ba - ycaa - cac, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0, \text{ all } y, \ y \sim -y) \tag{7.855}$$

$$\langle a, b, c \,|\, cb, pa - caa - zcac, pb - ba, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0, \text{ all } z) \tag{7.856}$$

$$\langle a, b, c \,|\, cb, pa - caa - zcac, pb - ba - cac, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0, \text{ all } z) \tag{7.857}$$

$$\langle a, b, c \,|\, cb, pa - caa - zcac, pb - ba - caa - ycac, pc - xcaa, \text{ class } 3\rangle \ (x \neq 0, \text{ all } y, z) \tag{7.858}$$

$$\langle a, b, c \,|\, cb, pa - xcaa, pb - ba, pc - cac, \text{ class } 3\rangle \ (x \neq 1) \tag{7.859}$$

$$\langle a, b, c \,|\, cb, pa - xcaa, pb - ba - cac, pc - cac, \text{ class } 3\rangle \ (x \neq 1) \tag{7.860}$$

$$\langle a, b, c \,|\, cb, pa - xcaa, pb - ba - caa - ycac, pc - cac, \text{ class } 3\rangle \ (x \neq 1, \text{ all } y) \tag{7.861}$$

$$\langle a, b, c \,|\, cb, pa - caa - xcac, pb - ba, pc - cac, \text{ class } 3\rangle \ (\text{all } x) \tag{7.862}$$

$$\langle a, b, c \,|\, cb, pa - caa - xcac, pb - ba - cac, pc - cac, \text{ class } 3\rangle \ (\text{all } x) \tag{7.863}$$

$$\langle a, b, c \,|\, cb, pa - caa - xcac, pb - ba - caa, pc - cac, \text{ class } 3\rangle \ (\text{all } x) \tag{7.864}$$

$$\langle a,b,c\,|\,cb-caa, pa, pb-ba, pc-xcaa,\text{ class 3}\rangle\ (\text{all } x) \tag{7.865}$$

$$\langle a,b,c\,|\,cb-caa, pa-cac, pb-ba, pc-xcaa,\text{ class 3}\rangle\ (\text{all } x) \tag{7.866}$$

$$\langle a,b,c\,|\,cb-caa, pa-\omega cac, pb-ba, pc-xcaa,\text{ class 3}\rangle\ (\text{all } x) \tag{7.867}$$

$$\langle a,b,c\,|\,cb-caa, pa-ycac, pb-ba, pc-xcaa-cac,\text{ class 3}\rangle\ (\text{all } x,\ y) \tag{7.868}$$

$$\langle a,b,c\,|\,cb-caa, pa-caa-ycac, pb-ba, pc-xcaa-zcac,\text{ class 3}\rangle\ (\text{all } x,\ y, z) \tag{7.869}$$

$$\langle a,b,c\,|\,cb-caa, pa-ycac, pb-ba-cac, pc-zcaa-tcac,\text{ class 3}\rangle\ (\text{all } y, z, t,\ t\sim -t) \tag{7.870}$$

$$\langle a,b,c\,|\,cb-caa, pa-xcaa-ycac, pb-ba-cac, pc-zcaa-tcac,\text{ class 3}\rangle\ (x\neq 0, x\sim -x,\ \text{all } y, z, t) \tag{7.871}$$

$$\langle a,b,c\,|\,cb-caa, pa-ycac, pb-ba-\omega cac, pc-zcaa-tcac,\text{ class 3}\rangle\ (\text{all } y, z, t,\ t\sim -t) \tag{7.872}$$

$$\langle a,b,c\,|\,cb-caa, pa-xcaa-ycac, pb-ba-\omega cac, pc-zcaa-tcac,\text{ class 3}\rangle\ (x\neq 0,\ x\sim -x,\ \text{all } y, z, t) \tag{7.873}$$

8.66 Descendants of 5.18

$3p^3+6p^2+6p+11+(p+7)\gcd(p-1,3)+(p+1)\gcd(p-1,4)+\gcd(p-1,5)$ algebras

$$\langle a,b,c\,|\,cb, pa, pb-ca, pc,\text{ class 3}\rangle \tag{7.874}$$

$$\langle a,b,c\,|\,cb, pa, pb-ca-baa, pc,\text{ class 3}\rangle \tag{7.875}$$

$$\langle a,b,c\,|\,cb, pa, pb-ca-\omega baa, pc,\text{ class 3}\rangle \tag{7.876}$$

$$\langle a,b,c\,|\,cb, pa-bab, pb-ca, pc,\text{ class 3}\rangle \tag{7.877}$$

$$\langle a,b,c\,|\,cb, pa-bab, pb-ca-baa, pc,\text{ class 3}\rangle \tag{7.878}$$

$$\langle a,b,c\,|\,cb, pa-bab, pb-ca-\omega baa, pc,\text{ class 3}\rangle \tag{7.879}$$

$$\langle a,b,c\,|\,cb, pa-\omega bab, pb-ca, pc,\text{ class 3}\rangle \tag{7.880}$$

$$\langle a,b,c\,|\,cb, pa-\omega bab, pb-ca-baa, pc,\text{ class 3}\rangle \tag{7.881}$$

$$\langle a,b,c\,|\,cb, pa-\omega bab, pb-ca-\omega baa, pc,\text{ class 3}\rangle \tag{7.882}$$

$$\langle a,b,c\,|\,cb, pa, pb-ca-bab, pc,\text{ class 3}\rangle \tag{7.883}$$

$$\langle a,b,c\,|\,cb, pa, pb-ca-baa-bab, pc,\text{ class 3}\rangle \tag{7.884}$$

$$\langle a,b,c \,|\, cb, pa, pb - ca - \omega baa - bab, pc, \text{ class } 3\rangle \tag{7.885}$$

$$\langle a,b,c \,|\, cb, pa - bab, pb - ca - xbaa - bab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.886}$$

$$\langle a,b,c \,|\, cb, pa - \omega bab, pb - ca - xbaa - bab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.887}$$

$$\langle a,b,c \,|\, cb, pa - baa, pb - ca - xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.888}$$

$$\langle a,b,c \,|\, cb, pa - baa, pb - ca - baa - xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.889}$$

$$\langle a,b,c \,|\, cb, pa - baa, pb - ca - \omega baa - xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.890}$$

$$\langle a,b,c \,|\, cb, pa - baa - bab, pb - ca - xbaa - ybab, pc, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.891}$$

$$\langle a,b,c \,|\, cb, pa - baa - \omega bab, pb - ca - xbaa - ybab, pc, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.892}$$

$$\langle a,b,c \,|\, cb, pa, pb - ca, pc - baa, \text{ class } 3\rangle \tag{7.893}$$

$$\langle a,b,c \,|\, cb, pa - bab, pb - ca, pc - baa, \text{ class } 3\rangle \tag{7.894}$$

$$\langle a,b,c \,|\, cb, pa - \omega bab, pb - ca, pc - baa, \text{ class } 3\rangle \tag{7.895}$$

$$\langle a,b,c \,|\, cb, pa, pb - ca - bab, pc - baa, \text{ class } 3\rangle \tag{7.896}$$

$$\langle a,b,c \,|\, cb, pa - xbab, pb - ca - bab, pc - baa, \text{ class } 3\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.897}$$

$$\langle a,b,c \,|\, cb, pa, pb - ca, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.898}$$

$$\langle a,b,c \,|\, cb, pa - bab, pb - ca, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.899}$$

$$\langle a,b,c \,|\, cb, pa - \omega bab, pb - ca, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.900}$$

$$\langle a,b,c \,|\, cb, pa, pb - ca - bab, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.901}$$

$$\langle a,b,c \,|\, cb, pa - xbab, pb - ca - bab, pc - \omega baa, \text{ class } 3\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.902}$$

$$\langle a,b,c \,|\, cb, pa, pb - ca, pc - \omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.903}$$

$$\langle a,b,c \,|\, cb, pa - bab, pb - ca, pc - \omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.904}$$

$$\langle a,b,c \,|\, cb, pa - \omega bab, pb - ca, pc - \omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.905}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca-bab, pc-\omega^2 baa, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.906}$$

$$\langle a,b,c \,|\, cb, pa-xbab, pb-ca-bab, pc-\omega^2 baa, \text{ class } 3\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.907}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca, pc-bab, \text{ class } 3\rangle \tag{7.908}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca-baa, pc-bab, \text{ class } 3\rangle \tag{7.909}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca-\omega baa, pc-bab, \text{ class } 3\rangle \tag{7.910}$$

$$\langle a,b,c \,|\, cb, pa-baa, pb-ca-xbaa, pc-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.911}$$

$$\langle a,b,c \,|\, cb, pa-xbaa, pb-ca-ybaa, pc-baa-bab, \text{ class } 3\rangle \ (\text{all } x,\ y,\ p=2 \bmod 3) \tag{7.912}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca, pc-baa-bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.913}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca, pc-\omega baa-bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.914}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca, pc-\omega^2 baa-bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.915}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca-xbaa, pc-baa-bab, \text{ class } 3\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.916}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca-xbaa, pc-\omega baa-bab, \text{ class } 3\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.917}$$

$$\langle a,b,c \,|\, cb, pa, pb-ca-xbaa, pc-\omega^2 baa-bab, \text{ class } 3\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.918}$$

$$\langle a,b,c \,|\, cb, pa-xbaa, pb-ca-ybaa, pc-baa-bab, \text{ class } 3\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ \text{all } y,\ p=1 \bmod 3) \tag{7.919}$$

$$\langle a,b,c \,|\, cb, pa-xbaa, pb-ca-ybaa, pc-\omega baa-bab, \text{ class } 3\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ \text{all } y,\ p=1 \bmod 3) \tag{7.920}$$

$$\langle a,b,c \,|\, cb, pa-xbaa, pb-ca-ybaa, pc-\omega^2 baa-bab, \text{ class } 3\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ \text{all } y,\ p=1 \bmod 3) \tag{7.921}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca, pc, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.922}$$

$$\langle a,b,c \,|\, cb-baa, pa-bab, pb-ca, pc, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.923}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega bab, pb-ca, pc, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.924}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^2 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=1 \bmod 4,\ p=1 \bmod 3) \tag{7.925}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^3 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=1 \bmod 4,\ p=1 \bmod 3) \tag{7.926}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^4 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=1 \bmod 4,\ p=1 \bmod 3) \tag{7.927}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^5 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=1 \bmod 4,\ p=1 \bmod 3) \tag{7.928}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbab, pb-ca-bab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.929}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbab, pb-ca-\omega bab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.930}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbab, pb-ca-\omega^2 bab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.931}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbab, pb-ca-\omega^3 bab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.932}$$

$$\langle a,b,c \,|\, cb-baa, pa-baa-xbab, pb-ca-ybab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x,\ \text{all } y,\ p=1 \bmod 4) \tag{7.933}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega baa-xbab, pb-ca-ybab, pc, \text{ class } 3\rangle \ (\text{all } x,\ y, x\sim -x,\ p=1 \bmod 4) \tag{7.934}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^2 baa-xbab, pb-ca-ybab, pc, \text{ class } 3\rangle \ (\text{all } x,\ y, x\sim -x,\ p=1 \bmod 4) \tag{7.935}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^3 baa-xbab, pb-ca-ybab, pc, \text{ class } 3\rangle \ (\text{all } x,\ y, x\sim -x,\ p=1 \bmod 4) \tag{7.936}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa-ybab, pb-ca-baa-zbab, pc, \text{ class } 3\rangle \ (\text{all } x,\ y, z,\ p=1 \bmod 4) \tag{7.937}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa-ybab, pb-ca-\omega baa-zbab, pc, \text{ class } 3\rangle \ (\text{all } x,\ y, z,\ p=1 \bmod 4) \tag{7.938}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca, pc, \text{ class } 3\rangle \ (p=3 \bmod 4) \tag{7.939}$$

$$\langle a,b,c \,|\, cb-baa, pa-bab, pb-ca, pc, \text{ class } 3\rangle \ (p=3 \bmod 4) \tag{7.940}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega bab, pb-ca, pc, \text{ class } 3\rangle \ (p=3 \bmod 4) \tag{7.941}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^2 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=3 \bmod 4,\ p=1 \bmod 3) \tag{7.942}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^3 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=3 \bmod 4,\ p=1 \bmod 3) \tag{7.943}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^4 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=3 \bmod 4,\ p=1 \bmod 3) \tag{7.944}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega^5 bab, pb-ca, pc, \text{ class } 3\rangle \ (p=3 \bmod 4,\ p=1 \bmod 3) \tag{7.945}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbab, pb-ca-bab, pc, \text{ class } 3\rangle \ (\text{all } x,\ p=3 \bmod 4) \tag{7.946}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbab, pb-ca-\omega bab, pc, \text{ class } 3\rangle \text{ (all } x,\ p=3 \bmod 4) \tag{7.947}$$

$$\langle a,b,c \,|\, cb-baa, pa-baa-xbab, pb-ca-ybab, pc, \text{ class } 3\rangle \text{ (all } x,\ y,\ p=3 \bmod 4) \tag{7.948}$$

$$\langle a,b,c \,|\, cb-baa, pa-\omega baa-xbab, pb-ca-ybab, pc, \text{ class } 3\rangle \text{ (all } x,\ y,\ p=3 \bmod 4) \tag{7.949}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa-ybab, pb-ca-baa-zbab, pc, \text{ class } 3\rangle \text{ (all } x,\ y,z,\ p=3 \bmod 4) \tag{7.950}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa-ybab, pb-ca-\omega baa-zbab, pc, \text{ class } 3\rangle \text{ (all } x,\ y,z,\ p=3 \bmod 4) \tag{7.951}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca, pc-bab, \text{ class } 3\rangle \tag{7.952}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca, pc-\omega bab, \text{ class } 3\rangle \ (p=1 \bmod 5) \tag{7.953}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca, pc-\omega^2 bab, \text{ class } 3\rangle \ (p=1 \bmod 5) \tag{7.954}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca, pc-\omega^3 bab, \text{ class } 3\rangle \ (p=1 \bmod 5) \tag{7.955}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca, pc-\omega^4 bab, \text{ class } 3\rangle \ (p=1 \bmod 5) \tag{7.956}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca-xbaa, pc-bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p) \tag{7.957}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca-xbaa, pc-\omega bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ p=1 \bmod 5) \tag{7.958}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca-xbaa, pc-\omega^2 bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ p=1 \bmod 5) \tag{7.959}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca-xbaa, pc-\omega^3 bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ p=1 \bmod 5) \tag{7.960}$$

$$\langle a,b,c \,|\, cb-baa, pa, pb-ca-xbaa, pc-\omega^4 bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ p=1 \bmod 5) \tag{7.961}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa, pb-ca-ybaa, pc-bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ \text{all } y) \tag{7.962}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa, pb-ca-ybaa, pc-\omega bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ \text{all } y,\ p=1 \bmod 5) \tag{7.963}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa, pb-ca-ybaa, pc-\omega^2 bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ \text{all } y,\ p=1 \bmod 5) \tag{7.964}$$

$$\langle a,b,c \,|\, cb-baa, pa-xbaa, pb-ca-ybaa, pc-\omega^3 bab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ \text{all } y,\ p=1 \bmod 5) \tag{7.965}$$

$$\langle a,b,c\,|\,cb-baa, pa-xbaa, pb-ca-ybaa, pc-\omega^4 bab, \text{ class } 3\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p,\ \text{all } y,\ p=1 \bmod 5) \tag{7.966}$$

$$\langle a,b,c\,|\,cb-baa, pa-xbab, pb-ca, pc-baa, \text{ class } 3\rangle\ (\text{all } x) \tag{7.967}$$

$$\langle a,b,c\,|\,cb-baa, pa-xbab, pb-ca-ybab, pc-baa, \text{ class } 3\rangle\ (y\neq 0,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ \text{all } x) \tag{7.968}$$

$$\langle a,b,c\,|\,cb-baa, pa-xbab, pb-ca, pc-\omega baa, \text{ class } 3\rangle\ (\text{all } x,\ p=1 \bmod 3) \tag{7.969}$$

$$\langle a,b,c\,|\,cb-baa, pa-xbab, pb-ca-ybab, pc-\omega baa, \text{ class } 3\rangle\ (y\neq 0,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ \text{all } x,\ p=1 \bmod 3) \tag{7.970}$$

$$\langle a,b,c\,|\,cb-baa, pa-xbab, pb-ca, pc-\omega^2 baa, \text{ class } 3\rangle\ (\text{all } x,\ p=1 \bmod 3) \tag{7.971}$$

$$\langle a,b,c\,|\,cb-baa, pa-xbab, pb-ca-ybab, pc-\omega^2 baa, \text{ class } 3\rangle\ (y\neq 0,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ \text{all } x,\ p=1 \bmod 3) \tag{7.972}$$

$$\langle a,b,c\,|\,cb-baa, pa-ybab, pb-ca-zbab, pc-baa-xbab, \text{ class } 3\rangle\ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ \text{all } y,z) \tag{7.973}$$

$$\langle a,b,c\,|\,cb-baa, pa-ybab, pb-ca-zbab, pc-\omega baa-xbab, \text{ class } 3\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ \text{all } y,z,\ p=1 \bmod 3) \tag{7.974}$$

$$\langle a,b,c\,|\,cb-baa, pa-ybab, pb-ca-zbab, pc-\omega^2 baa-xbab, \text{ class } 3\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ \text{all } y,z,\ p=1 \bmod 3) \tag{7.975}$$

8.67 Descendants of 5.8

$p+8$ algebras

$$\langle a,b,c\,|\,ba, ca-p^2a, cb-p^2b, pc, \text{ class } 3\rangle \tag{7.976}$$

$$\langle a,b,c\,|\,ba, ca-p^2b, cb-p^2a, pc, \text{ class } 3\rangle \tag{7.977}$$

$$\langle a,b,c\,|\,ba, ca-\omega p^2b, cb-p^2a, pc, \text{ class } 3\rangle \tag{7.978}$$

$$\langle a,b,c\,|\,ba, ca-xp^2b, cb-p^2a-p^2b, pc, \text{ class } 3\rangle\ (x\neq 0) \tag{7.979}$$

$$\langle a,b,c\,|\,ba, ca, cb-p^2a, pc, \text{ class } 3\rangle \tag{7.980}$$

$$\langle a,b,c\,|\,ba-p^2b, ca, cb-p^2a, pc, \text{ class } 3\rangle \tag{7.981}$$

$$\langle a,b,c\,|\,ba, ca, cb-p^2b, pc, \text{ class } 3\rangle \tag{7.982}$$

$$\langle a, b, c \,|\, ba - p^2 a, ca, cb - p^2 b, pc, \text{ class } 3\rangle \tag{7.983}$$

$$\langle a, b, c \,|\, ba, ca, cb, pc, \text{ class } 3\rangle \tag{7.984}$$

$$\langle a, b, c \,|\, ba - p^2 a, ca, cb, pc, \text{ class } 3\rangle \tag{7.985}$$

8.68 Descendants of 5.9

$4p^2 + 26p + 107 + 5\gcd(p-1, 3) + (p+4)\gcd(p-1, 4)$ algebras

$$\langle a, b, c \,|\, baa, ca, cb, pb, pc, \text{ class } 3\rangle \tag{7.986}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, pb, pc, \text{ class } 3\rangle \tag{7.987}$$

$$\langle a, b, c \,|\, baa, ca - p^2 a, cb, pb, pc, \text{ class } 3\rangle \tag{7.988}$$

$$\langle a, b, c \,|\, baa, ca - bab - p^2 a, cb, pb, pc, \text{ class } 3\rangle \tag{7.989}$$

$$\langle a, b, c \,|\, baa, ca - \omega bab - p^2 a, cb, pb, pc, \text{ class } 3\rangle \tag{7.990}$$

$$\langle a, b, c \,|\, baa, ca, cb - p^2 a, pb, pc, \text{ class } 3\rangle \tag{7.991}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb - p^2 a, pb, pc, \text{ class } 3\rangle \tag{7.992}$$

$$\langle a, b, c \,|\, baa, ca - p^2 a, cb - p^2 a, pb, pc, \text{ class } 3\rangle \tag{7.993}$$

$$\langle a, b, c \,|\, baa, ca - bab - p^2 a, cb - p^2 a, pb, pc, \text{ class } 3\rangle \tag{7.994}$$

$$\langle a, b, c \,|\, baa, ca - \omega bab - p^2 a, cb - p^2 a, pb, pc, \text{ class } 3\rangle \tag{7.995}$$

$$\langle a, b, c \,|\, baa, ca, cb, pb, pc - bab, \text{ class } 3\rangle \tag{7.996}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, pb, pc - bab, \text{ class } 3\rangle \tag{7.997}$$

$$\langle a, b, c \,|\, baa, ca - p^2 a, cb, pb, pc - bab, \text{ class } 3\rangle \tag{7.998}$$

$$\langle a, b, c \,|\, baa, ca - bab - p^2 a, cb, pb, pc - bab, \text{ class } 3\rangle \tag{7.999}$$

$$\langle a, b, c \,|\, baa, ca - \omega bab - p^2 a, cb, pb, pc - bab, \text{ class } 3\rangle \tag{7.1000}$$

$$\langle a, b, c \,|\, baa, ca, cb - p^2 a, pb, pc - bab, \text{ class } 3\rangle \tag{7.1001}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb - p^2 a, pb, pc - bab, \text{ class } 3\rangle \tag{7.1002}$$

$$\langle a,b,c \,|\, baa, ca - xbab - p^2a, cb - p^2a, pb, pc - bab, \text{ class } 3\rangle \text{ (all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.1003}$$

$$\langle a,b,c \,|\, baa, ca, cb - \omega p^2a, pb, pc - bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1004}$$

$$\langle a,b,c \,|\, baa, ca - bab, cb - \omega p^2a, pb, pc - bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1005}$$

$$\langle a,b,c \,|\, baa, ca - xbab - p^2a, cb - \omega p^2a, pb, pc - bab, \text{ class } 3\rangle \text{ (all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.1006}$$

$$\langle a,b,c \,|\, baa, ca, cb - \omega^2 p^2a, pb, pc - bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1007}$$

$$\langle a,b,c \,|\, baa, ca - bab, cb - \omega^2 p^2a, pb, pc - bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1008}$$

$$\langle a,b,c \,|\, baa, ca - xbab - p^2a, cb - \omega^2 p^2a, pb, pc - bab, \text{ class } 3\rangle \text{ (all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.1009}$$

$$\langle a,b,c \,|\, baa, ca, cb, pb - bab, pc, \text{ class } 3\rangle \tag{7.1010}$$

$$\langle a,b,c \,|\, baa, ca - bab, cb, pb - bab, pc, \text{ class } 3\rangle \tag{7.1011}$$

$$\langle a,b,c \,|\, baa, ca - p^2a, cb, pb - bab, pc, \text{ class } 3\rangle \tag{7.1012}$$

$$\langle a,b,c \,|\, baa, ca - bab - p^2a, cb, pb - bab, pc, \text{ class } 3\rangle \tag{7.1013}$$

$$\langle a,b,c \,|\, baa, ca - \omega bab - p^2a, cb, pb - bab, pc, \text{ class } 3\rangle \tag{7.1014}$$

$$\langle a,b,c \,|\, baa, ca, cb - p^2a, pb - bab, pc, \text{ class } 3\rangle \tag{7.1015}$$

$$\langle a,b,c \,|\, baa, ca - bab, cb - p^2a, pb - bab, pc, \text{ class } 3\rangle \tag{7.1016}$$

$$\langle a,b,c \,|\, baa, ca - \omega bab, cb - p^2a, pb - bab, pc, \text{ class } 3\rangle \tag{7.1017}$$

$$\langle a,b,c \,|\, baa, ca - \omega^2 bab, cb - p^2a, pb - bab, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.1018}$$

$$\langle a,b,c \,|\, baa, ca - \omega^3 bab, cb - p^2a, pb - bab, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.1019}$$

$$\langle a,b,c \,|\, baa, ca - xbab - p^2a, cb - p^2a, pb - bab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1020}$$

$$\langle a,b,c \,|\, baa, ca - xbab - \omega p^2a, cb - p^2a, pb - bab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1021}$$

$$\langle a,b,c \,|\, bab, ca, cb, pb, pc, \text{ class } 3\rangle \tag{7.1022}$$

$$\langle a,b,c \,|\, bab, ca - p^2a, cb, pb, pc, \text{ class } 3\rangle \tag{7.1023}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pb, pc, \text{ class } 3\rangle \tag{7.1024}$$

$$\langle a, b, c \,|\, bab, ca - p^2a, cb - baa, pb, pc, \text{ class } 3\rangle \tag{7.1025}$$

$$\langle a, b, c \,|\, bab, ca - \omega p^2a, cb - baa, pb, pc, \text{ class } 3\rangle \tag{7.1026}$$

$$\langle a, b, c \,|\, bab, ca, cb - p^2a, pb, pc, \text{ class } 3\rangle \tag{7.1027}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa - p^2a, pb, pc, \text{ class } 3\rangle \tag{7.1028}$$

$$\langle a, b, c \,|\, bab, ca, cb, pb, pc - baa, \text{ class } 3\rangle \tag{7.1029}$$

$$\langle a, b, c \,|\, bab, ca - p^2a, cb, pb, pc - baa, \text{ class } 3\rangle \tag{7.1030}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pb, pc - baa, \text{ class } 3\rangle \tag{7.1031}$$

$$\langle a, b, c \,|\, bab, ca - p^2a, cb - baa, pb, pc - baa, \text{ class } 3\rangle \tag{7.1032}$$

$$\langle a, b, c \,|\, bab, ca - \omega p^2a, cb - baa, pb, pc - baa, \text{ class } 3\rangle \tag{7.1033}$$

$$\langle a, b, c \,|\, bab, ca, cb - p^2a, pb, pc - baa, \text{ class } 3\rangle \tag{7.1034}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa - p^2a, pb, pc - baa, \text{ class } 3\rangle \tag{7.1035}$$

$$\langle a, b, c \,|\, bab, ca, cb, pb - baa, pc, \text{ class } 3\rangle \tag{7.1036}$$

$$\langle a, b, c \,|\, bab, ca - p^2a, cb, pb - baa, pc, \text{ class } 3\rangle \tag{7.1037}$$

$$\langle a, b, c \,|\, bab, ca - xp^2a, cb - baa, pb - baa, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1038}$$

$$\langle a, b, c \,|\, bab, ca, cb - p^2a, pb - baa, pc, \text{ class } 3\rangle \tag{7.1039}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa - p^2a, pb - baa, pc, \text{ class } 3\rangle \tag{7.1040}$$

$$\langle a, b, c \,|\, bab, ca, cb, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1041}$$

$$\langle a, b, c \,|\, bab, ca - p^2a, cb, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1042}$$

$$\langle a, b, c \,|\, bab, ca - xp^2a, cb - baa, pb - \omega baa, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1043}$$

$$\langle a, b, c \,|\, bab, ca, cb - p^2a, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1044}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa - p^2a, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1045}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb, pc, \text{ class } 3\rangle \tag{7.1046}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb - baa, pc, \text{ class } 3\rangle \tag{7.1047}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1048}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb - bab, pc, \text{ class } 3\rangle \tag{7.1049}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb, pc - baa, \text{ class } 3\rangle \tag{7.1050}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb - bab, pc - baa, \text{ class } 3\rangle \tag{7.1051}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb, pc - bab, \text{ class } 3\rangle \tag{7.1052}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb - baa, pc - bab, \text{ class } 3\rangle \tag{7.1053}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb, pb - \omega baa, pc - bab, \text{ class } 3\rangle \tag{7.1054}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb, pc, \text{ class } 3\rangle \tag{7.1055}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - baa, pc, \text{ class } 3\rangle \tag{7.1056}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1057}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - bab, pc, \text{ class } 3\rangle \tag{7.1058}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - baa - bab, pc, \text{ class } 3\rangle \tag{7.1059}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - \omega baa - bab, pc, \text{ class } 3\rangle \tag{7.1060}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb, pc - bab, \text{ class } 3\rangle \tag{7.1061}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - baa, pc - bab, \text{ class } 3\rangle \tag{7.1062}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - \omega baa, pc - bab, \text{ class } 3\rangle \tag{7.1063}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb, pc - baa, \text{ class } 3\rangle \tag{7.1064}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - bab, pc - baa, \text{ class } 3\rangle \tag{7.1065}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - bab, pb - xbab, pc - baa - bab, \text{ class } 3\rangle \text{ (all } x) \tag{7.1066}$$

$$\langle a, b, c \,|\, p^2 a, ca, cb - baa, pb, pc, \text{ class } 3\rangle \tag{7.1067}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-baa, pc, \text{ class } 3\rangle \tag{7.1068}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-\omega baa, pc, \text{ class } 3\rangle \tag{7.1069}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-bab, pc, \text{ class } 3\rangle \tag{7.1070}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-baa-bab, pc, \text{ class } 3\rangle \tag{7.1071}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-\omega baa-bab, pc, \text{ class } 3\rangle \tag{7.1072}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb, pc-bab, \text{ class } 3\rangle \tag{7.1073}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-baa, pc-bab, \text{ class } 3\rangle \tag{7.1074}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-\omega baa, pc-bab, \text{ class } 3\rangle \tag{7.1075}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-\omega^2 baa, pc-bab, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.1076}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-\omega^3 baa, pc-bab, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.1077}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb, pc-baa, \text{ class } 3\rangle \tag{7.1078}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-bab, pc-baa, \text{ class } 3\rangle \tag{7.1079}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa, pb-xbab, pc-baa-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1080}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb, pc, \text{ class } 3\rangle \tag{7.1081}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb-bab, pc, \text{ class } 3\rangle \tag{7.1082}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb-\omega bab, pc, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.1083}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb-baa-xbab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim 1-x) \tag{7.1084}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb-\omega baa-xbab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim \omega-x) \tag{7.1085}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb-xbaa, pc-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1086}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb-xbab, pc-baa-ybab, \text{ class } 3\rangle \ (\text{all } x,\ y\neq \frac{1}{2},\ y\sim 1-y) \tag{7.1087}$$

$$\langle a,b,c\,|\,p^2a, ca, cb-baa-bab, pb-xbab, pc-baa-\frac{1}{2}bab, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x) \tag{7.1088}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb, pc, \text{ class } 3\rangle \tag{7.1089}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb-baa, pc, \text{ class } 3\rangle \tag{7.1090}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb-\omega baa, pc, \text{ class } 3\rangle \tag{7.1091}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb-bab, pc, \text{ class } 3\rangle \tag{7.1092}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb, pc-baa, \text{ class } 3\rangle \tag{7.1093}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb-bab, pc-baa, \text{ class } 3\rangle \tag{7.1094}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb-\omega bab, pc-baa, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.1095}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb-\omega^2 bab, pc-baa, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.1096}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb, pb-xbaa, pc-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1097}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb, pc, \text{ class } 3\rangle \tag{7.1098}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb-bab, pc, \text{ class } 3\rangle \tag{7.1099}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb-\omega bab, pc, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.1100}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb-baa-xbab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x) \tag{7.1101}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb-\omega baa-xbab, pc, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x) \tag{7.1102}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb-xbaa, pc-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1103}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb-xbab, pc-baa, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x) \tag{7.1104}$$

$$\langle a,b,c\,|\,p^2a, ca-bab, cb-\omega baa, pb-ybab, pc-xbaa-bab, \text{ class } 3\rangle \ (x\neq 0,\ \text{all } y,\ x\sim -x) \tag{7.1105}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb, pb, pc-bab, \text{ class } 3\rangle \tag{7.1106}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb, pb-xbab, pc, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1107}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-baa, pb, pc-bab, \text{ class } 3\rangle \tag{7.1108}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-baa, pb, pc-\omega bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.1109}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-baa, pb, pc-\omega^2 bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.1110}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-baa, pb-xbab, pc, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1111}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-bab, pb, pc-bab, \text{ class } 3\rangle \tag{7.1112}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-bab, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1113}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-baa-bab, pb, pc-xbab, \text{ class } 3\rangle \ (x\neq 0, x\sim -x) \tag{7.1114}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-baa-bab, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1115}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-\omega baa-bab, pb, pc-xbab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.1116}$$

$$\langle a,b,c\,|\,p^2a-baa, ca, cb-\omega baa-bab, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1117}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb, pb, pc-bab, \text{ class } 3\rangle \tag{7.1118}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1119}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-baa, pb, pc-xbab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.1120}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-\omega baa, pb, pc-xbab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.1121}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-\omega^2 baa, pb, pc-xbab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.1122}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-\omega^3 baa, pb, pc-xbab, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.1123}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-baa, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1124}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-\omega baa, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1125}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-\omega^2 baa, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x,\ p=1 \bmod 4) \tag{7.1126}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-\omega^3 baa, pb-xbab, pc, \text{ class } 3\rangle \text{ (all } x,\ p=1 \bmod 4) \tag{7.1127}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-xbaa-bab, pb, pc-ybab, \text{ class } 3\rangle \text{ (all } x,\ y\neq 0,\ y\sim -y) \tag{7.1128}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-xbaa-bab, pb-ybab, pc, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.1129}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-xbaa-\omega bab, pb, pc-ybab, \text{ class } 3\rangle \text{ (all } x,\ y\neq 0,\ y\sim -y) \tag{7.1130}$$

$$\langle a,b,c\,|\,p^2a-baa, ca-bab, cb-xbaa-\omega bab, pb-ybab, pc, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.1131}$$

$$\langle a,b,c\,|\,p^2a-bab, ca, cb, pb, pc, \text{ class } 3\rangle \tag{7.1132}$$

$$\langle a,b,c\,|\,p^2a-bab,ca,cb,pb-baa,pc,\text{ class 3}\rangle \tag{7.1133}$$

$$\langle a,b,c\,|\,p^2a-bab,ca,cb,pb-\omega baa,pc,\text{ class 3}\rangle \tag{7.1134}$$

$$\langle a,b,c\,|\,p^2a-bab,ca,cb,pb,pc-baa,\text{ class 3}\rangle \tag{7.1135}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-baa,cb,pb,pc,\text{ class 3}\rangle \tag{7.1136}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-baa,cb,pb-baa,pc,\text{ class 3}\rangle \tag{7.1137}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-baa,cb,pb-\omega baa,pc,\text{ class 3}\rangle \tag{7.1138}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-baa,cb,pb,pc-baa,\text{ class 3}\rangle \tag{7.1139}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-bab,cb,pb,pc,\text{ class 3}\rangle \tag{7.1140}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-bab,cb,pb-baa,pc,\text{ class 3}\rangle \tag{7.1141}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-bab,cb,pb-\omega baa,pc,\text{ class 3}\rangle \tag{7.1142}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-bab,cb,pb,pc-baa,\text{ class 3}\rangle \tag{7.1143}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-bab,cb,pb,pc-\omega baa,\text{ class 3}\rangle\ (p=1\bmod 4) \tag{7.1144}$$

$$\langle a,b,c\,|\,p^2a-bab,ca,cb-baa,pb,pc,\text{ class 3}\rangle \tag{7.1145}$$

$$\langle a,b,c\,|\,p^2a-bab,ca,cb-baa,pb-baa,pc,\text{ class 3}\rangle \tag{7.1146}$$

$$\langle a,b,c\,|\,p^2a-bab,ca,cb-baa,pb-\omega baa,pc,\text{ class 3}\rangle \tag{7.1147}$$

$$\langle a,b,c\,|\,p^2a-bab,ca,cb-baa,pb,pc-xbaa,\text{ class 3}\rangle\ (x\neq 0,\ x\sim -x) \tag{7.1148}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-bab,cb-baa,pb-xbaa,pc,\text{ class 3}\rangle\ (\text{all } x) \tag{7.1149}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-bab,cb-baa,pb,pc-xbaa,\text{ class 3}\rangle\ (x\neq 0, x\sim -x) \tag{7.1150}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-\omega bab,cb-baa,pb-xbaa,pc,\text{ class 3}\rangle\ (\text{all } x) \tag{7.1151}$$

$$\langle a,b,c\,|\,p^2a-bab,ca-\omega bab,cb-baa,pb,pc-xbaa,\text{ class 3}\rangle\ (x\neq 0,\ x\sim -x) \tag{7.1152}$$

$$\langle a,b,c\,|\,p^2a-\omega bab,ca,cb,pb,pc,\text{ class 3}\rangle \tag{7.1153}$$

$$\langle a,b,c\,|\,p^2a-\omega bab,ca,cb,pb-baa,pc,\text{ class 3}\rangle \tag{7.1154}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca, cb, pb-\omega baa, pc, \text{ class } 3\rangle \tag{7.1155}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca, cb, pb, pc-baa, \text{ class } 3\rangle \tag{7.1156}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-baa, cb, pb, pc, \text{ class } 3\rangle \tag{7.1157}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-baa, cb, pb-baa, pc, \text{ class } 3\rangle \tag{7.1158}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-baa, cb, pb-\omega baa, pc, \text{ class } 3\rangle \tag{7.1159}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-baa, cb, pb, pc-baa, \text{ class } 3\rangle \tag{7.1160}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-bab, cb, pb, pc, \text{ class } 3\rangle \tag{7.1161}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-bab, cb, pb-baa, pc, \text{ class } 3\rangle \tag{7.1162}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-bab, cb, pb-\omega baa, pc, \text{ class } 3\rangle \tag{7.1163}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-bab, cb, pb, pc-baa, \text{ class } 3\rangle \tag{7.1164}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-bab, cb, pb, pc-\omega baa, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.1165}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca, cb-baa, pb, pc, \text{ class } 3\rangle \tag{7.1166}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca, cb-baa, pb-baa, pc, \text{ class } 3\rangle \tag{7.1167}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca, cb-baa, pb-\omega baa, pc, \text{ class } 3\rangle \tag{7.1168}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca, cb-baa, pb, pc-xbaa, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.1169}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-bab, cb-baa, pb-xbaa, pc, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1170}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-bab, cb-baa, pb, pc-xbaa, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.1171}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-\omega bab, cb-baa, pb-xbaa, pc, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1172}$$

$$\langle a,b,c\,|\,p^2a-\omega bab, ca-\omega bab, cb-baa, pb, pc-xbaa, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.1173}$$

8.69 Descendants of 6.100

3 algebras

$$\langle a, b, c \,|\, baa, caa, cb, pb - \omega ca, pc - ba, \text{ class } 3\rangle \tag{7.1174}$$

$$\langle a, b, c \,|\, baa, caa, cb - p^2 a, pb - \omega ca, pc - ba, \text{ class } 3\rangle \tag{7.1175}$$

$$\langle a, b, c \,|\, baa, p^2 a, cb, pb - \omega ca, pc - ba, \text{ class } 3\rangle \tag{7.1176}$$

8.70 Descendants of 6.101

$3(p-1)/2$ algebras

$$\langle a, b, c \,|\, baa, caa, cb, pb - xca, pc - ba - ca, \text{ class } 3\rangle \text{ (all } x \text{ with } 1 + 4x \text{ not a square } \bmod p) \tag{7.1177}$$

$$\langle a, b, c \,|\, baa, caa, cb - p^2 a, pb - xca, pc - ba - ca, \text{ class } 3\rangle \text{ (all } x \text{ with } 1 + 4x \text{ not a square } \bmod p) \tag{7.1178}$$

$$\langle a, b, c \,|\, baa, p^2 a, cb, pb - xca, pc - ba - ca, \text{ class } 3\rangle \text{ (all } x \text{ with } 1 + 4x \text{ not a square } \bmod p) \tag{7.1179}$$

8.71 Descendants of 6.102

$p + 3$ algebras

$$\langle a, b, c \,|\, baa, cb - xp^2 b, pa - ca, pc - ba, \text{ class } 3\rangle \text{ (all } x) \tag{7.1180}$$

$$\langle a, b, c \,|\, p^2 b, cb, pa - ca, pc - ba, \text{ class } 3\rangle \tag{7.1181}$$

$$\langle a, b, c \,|\, p^2 b - baa, cb, pa - ca, pc - ba, \text{ class } 3\rangle \tag{7.1182}$$

$$\langle a, b, c \,|\, p^2 b - \omega baa, cb, pa - ca, pc - ba, \text{ class } 3\rangle \tag{7.1183}$$

8.72 Descendants of 6.103

$p + 3$ algebras

$$\langle a, b, c \,|\, bab, cb, pa - ba, pc - ca, \text{ class } 3\rangle \tag{7.1184}$$

$$\langle a, b, c \,|\, bab, cb - p^2 b, pa - ba, pc - ca, \text{ class } 3\rangle \tag{7.1185}$$

$$\langle a, b, c \,|\, p^2 b - xbab, cb, pa - ba, pc - ca, \text{ class } 3\rangle \text{ (all } x) \tag{7.1186}$$

$$\langle a, b, c \,|\, p^2 b + bab, cb, pa - ba, pc - ca - bab, \text{ class } 3\rangle \tag{7.1187}$$

8.73 Descendants of 6.104

$5p + 24 + 3\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, bab, bac, caa, cab, cac, cbb, cbc, pa - baa, pb, pc, \text{ class } 3\rangle \tag{7.1188}$$

$$\langle a,b,c \,|\, baa, bab, caa, cab + bac, cac, cbb, cbc, pa - bac, pb, pc, \text{ class } 3\rangle \tag{7.1189}$$

$$\langle a,b,c \,|\, baa, bac, caa, cab, cac + bab, cbb, cbc, pa - bab, pb, pc, \text{ class } 3\rangle \tag{7.1190}$$

$$\langle a,b,c \,|\, baa, bac, caa, cab, cac + \omega bab, cbb, cbc, pa - bab, pb, pc, \text{ class } 3\rangle \tag{7.1191}$$

$$\langle a,b,c \,|\, baa, bac, caa, cab, cac, cbb, cbc, pa - bab, pb, pc, \text{ class } 3\rangle \tag{7.1192}$$

$$\langle a,b,c \,|\, baa, bac, caa, cab, cac, cbb, cbc, pa - \omega bab, pb, pc, \text{ class } 3\rangle \tag{7.1193}$$

$$\langle a,b,c \,|\, baa, bac, caa - bab, cab, cac, cbb, cbc, pa - bab, pb, pc, \text{ class } 3\rangle \tag{7.1194}$$

$$\langle a,b,c \,|\, baa, bac, caa - bab, cab, cac, cbb, cbc, pa - \omega bab, pb, pc, \text{ class } 3\rangle \tag{7.1195}$$

$$\langle a,b,c \,|\, baa, bac, caa, cab - bab, cac - xbab, cbb, cbc, pa - bab, pb, pc, \text{ class } 3\rangle \ (x \neq -2) \tag{7.1196}$$

$$\langle a,b,c \,|\, baa, bac, caa, cab - bab, cac - \frac{1}{4}bab, cbb, cbc, pa - \omega bab, pb, pc, \text{ class } 3\rangle \tag{7.1197}$$

$$\langle a,b,c \,|\, baa, bac, caa, cab - bab, cac + 2bab, cbb, cbc, pa - bab, pb, pc, \text{ class } 3\rangle \tag{7.1198}$$

$$\langle a,b,c \,|\, baa, bac, caa - bab, cab - bab, cac + 2bab, cbb, cbc, pa - bab, pb, pc, \text{ class } 3\rangle \tag{7.1199}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, cab, cac, cbc, pa - cbb, pb, pc, \text{ class } 3\rangle \tag{7.1200}$$

$$\langle a,b,c \,|\, bab, bac, caa, cab, cac, cbb - baa, cbc, pa - baa, pb, pc, \text{ class } 3\rangle \tag{7.1201}$$

$$\langle a,b,c \,|\, baa, bab, bac, cab, cac, cbb - caa, cbc, pa - caa, pb, pc, \text{ class } 3\rangle \tag{7.1202}$$

$$\langle a,b,c \,|\, baa, bab, bac, cab, cac, cbb - \omega caa, cbc, pa - caa, pb, pc, \text{ class } 3\rangle \tag{7.1203}$$

$$\langle a,b,c \,|\, baa, bab, caa - xbac, cab, cac, cbb - bac, cbc, pa - bac, pb, pc, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1204}$$

$$\langle a,b,c \,|\, bab, bac - baa, caa + 2baa, cab, cac, cbb - baa, cbc, pa - baa, pb, pc, \text{ class } 3\rangle \tag{7.1205}$$

$$\langle a,b,c \,|\, bab, bac - baa, caa + 2baa, cab, cac, cbb - baa, cbc, pa - \omega baa, pb, pc, \text{ class } 3\rangle \tag{7.1206}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, cab, cbb - cac, cbc, pa - cac, pb, pc, \text{ class } 3\rangle \tag{7.1207}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, cab, cbb - cac, cbc, pa - \omega cac, pb, pc, \text{ class } 3\rangle \tag{7.1208}$$

$$\langle a,b,c\,|\,baa,bac,caa,cab,cac+bab,cbb+bab,cbc,pa-bab,pb,pc,\text{ class 3}\rangle \tag{7.1209}$$

$$\langle a,b,c\,|\,baa,bac,caa,cab,cac+bab,cbb+bab,cbc,pa-\omega bab,pb,pc,\text{ class 3}\rangle \tag{7.1210}$$

$$\langle a,b,c\,|\,baa,bac,caa,cab,cac+\omega bab,cbb+\omega bab,cbc,pa-bab,pb,pc,\text{ class 3}\rangle \tag{7.1211}$$

$$\langle a,b,c\,|\,baa,bac,caa,cab,cac+\omega bab,cbb+\omega bab,cbc,pa-\omega bab,pb,pc,\text{ class 3}\rangle \tag{7.1212}$$

$$\langle a,b,c\,|\,bab-xbaa,bac,caa,cab,cac-baa,cbb-baa,cbc,pa-baa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p) \tag{7.1213}$$

$$\langle a,b,c\,|\,bab-xbaa,bac,caa,cab,cac-\omega baa,cbb-\omega baa,cbc,pa-baa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1214}$$

$$\langle a,b,c\,|\,bab-xbaa,bac,caa,cab,cac-\omega^2 baa,cbb-\omega^2 baa,cbc,pa-baa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1215}$$

$$\langle a,b,c\,|\,bab-xbaa,bac,caa,cab,cac-baa,cbb-baa,cbc,pa-\omega baa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p) \tag{7.1216}$$

$$\langle a,b,c\,|\,bab-xbaa,bac,caa,cab,cac-\omega baa,cbb-\omega baa,cbc,pa-\omega baa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1217}$$

$$\langle a,b,c\,|\,bab-xbaa,bac,caa,cab,cac-\omega^2 baa,cbb-\omega^2 baa,cbc,pa-\omega baa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1218}$$

$$\langle a,b,c\,|\,bab,bac,caa,cab,cac,cbb,cbc,pa,pb,pc,\text{ class 3}\rangle \tag{7.1219}$$

$$\langle a,b,c\,|\,baa,bac,caa,cab,cac-bab,cbb,cbc,pa,pb,pc,\text{ class 3}\rangle \tag{7.1220}$$

$$\langle a,b,c\,|\,baa,bac,caa,cab,cac-\omega bab,cbb,cbc,pa,pb,pc,\text{ class 3}\rangle \tag{7.1221}$$

$$\langle a,b,c\,|\,baa,bac,caa-bab,cab,cac,cbb,cbc,pa,pb,pc,\text{ class 3}\rangle \tag{7.1222}$$

$$\langle a,b,c\,|\,bab,bac,caa,cab,cac-baa,cbb-baa,cbc-xbaa,pa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p) \tag{7.1223}$$

$$\langle a,b,c\,|\,bab,bac,caa,cab,cac-baa,cbb-\omega baa,cbc-xbaa,pa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1224}$$

$$\langle a,b,c\,|\,bab,bac,caa,cab,cac-baa,cbb-\omega^2 baa,cbc-xbaa,pa,pb,pc,\text{ class 3}\rangle\ (\text{all } x,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1225}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa-baa,cab,cac+baa,cbb+baa,cbc-baa,pa,pb,pc,\text{ class 3}\rangle \tag{7.1226}$$

8.74 Descendants of 6.105

$11 + 5\gcd(p-1,3) + \gcd(p-1,5)$ algebras

$$\langle a,b,c \,|\, baa, bab, bac, cac, pa - cb, pb, pc, \text{ class } 3\rangle \tag{7.1227}$$

$$\langle a,b,c \,|\, baa, bab, bac, cac, pa - cb - caa, pb, pc, \text{ class } 3\rangle \tag{7.1228}$$

$$\langle a,b,c \,|\, baa, bab, bac, cac, pa - cb, pb, pc - caa, \text{ class } 3\rangle \tag{7.1229}$$

$$\langle a,b,c \,|\, baa, bab, bac, cac, pa - cb, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.1230}$$

$$\langle a,b,c \,|\, baa, bab, bac, cac, pa - cb, pb - caa, pc, \text{ class } 3\rangle \tag{7.1231}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, pa - cb, pb, pc, \text{ class } 3\rangle \tag{7.1232}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, pa - cb, pb, pc - cac, \text{ class } 3\rangle \tag{7.1233}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, pa - cb, pb - cac, pc, \text{ class } 3\rangle \tag{7.1234}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, pa - cb, pb - \omega cac, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1235}$$

$$\langle a,b,c \,|\, baa, bab, bac, caa, pa - cb, pb - \omega^2 cac, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1236}$$

$$\langle a,b,c \,|\, bab, bac, caa, cac - baa, pa - cb, pb, pc, \text{ class } 3\rangle \tag{7.1237}$$

$$\langle a,b,c \,|\, bab, bac, caa, cac - baa, pa - cb, pb, pc - baa, \text{ class } 3\rangle \tag{7.1238}$$

$$\langle a,b,c \,|\, bab, bac, caa, cac - baa, pa - cb, pb, pc - xbaa, \text{ class } 3\rangle \ (x = \omega, \omega^2, \omega^3, \omega^4,\ p = 1 \bmod 5) \tag{7.1239}$$

$$\langle a,b,c \,|\, bab, bac, caa, cac - baa, pa - cb, pb - baa, pc, \text{ class } 3\rangle \tag{7.1240}$$

$$\langle a,b,c \,|\, bab, bac, caa, cac - baa, pa - cb, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1241}$$

$$\langle a,b,c \,|\, bab, bac, caa, cac - baa, pa - cb, pb - xbaa, pc, \text{ class } 3\rangle \ (x = \omega^2, \omega^3, \omega^4, \omega^5,\ p = 1 \bmod 3) \tag{7.1242}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac + bab, pa - cb, pb, pc, \text{ class } 3\rangle \tag{7.1243}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac + bab, pa - cb, pb, pc - bab, \text{ class } 3\rangle \tag{7.1244}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac + bab, pa - cb, pb, pc - \omega bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1245}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac + bab, pa - cb, pb, pc - \omega^2 bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1246}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac + bab, pa - cb, pb - bab, pc - bab, \text{ class } 3\rangle \tag{7.1247}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac+\omega bab, pa-cb, pb, pc, \text{ class } 3\rangle \tag{7.1248}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac+\omega bab, pa-cb, pb, pc-bab, \text{ class } 3\rangle \tag{7.1249}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac+\omega bab, pa-cb, pb, pc-\omega bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.1250}$$

$$\langle a,b,c \,|\, baa, bac, caa, cac+\omega bab, pa-cb, pb, pc-\omega^2 bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.1251}$$

8.75 Descendants of 6.106

$p^2+10p+32+\gcd(p-1,3)+\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, caa, cab, cac, cbc, pa-ba, pb, pc-xcbb, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1252}$$

$$\langle a,b,c \,|\, caa, cab, cac, cbc, pa-ba, pb-cbb, pc-xcbb, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1253}$$

$$\langle a,b,c \,|\, caa, cab, cbb-cac, cbc, pa-ba, pb, pc-xcac, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1254}$$

$$\langle a,b,c \,|\, caa, cab, cbb-cac, cbc, pa-ba, pb-cac, pc-xcac, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1255}$$

$$\langle a,b,c \,|\, cab, cac, cbb-caa, cbc, pa-ba, pb, pc-xcaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1256}$$

$$\langle a,b,c \,|\, cab, cac, cbb-caa, cbc, pa-ba, pb-caa, pc-xcaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1257}$$

$$\langle a,b,c \,|\, cab, cac, cbb-\omega caa, cbc, pa-ba, pb, pc-xcaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1258}$$

$$\langle a,b,c \,|\, cab, cac, cbb-\omega caa, cbc, pa-ba, pb-caa, pc-xcaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.1259}$$

$$\langle a,b,c \,|\, cab, cac-caa, cbb-caa, cbc, pa-ba, pb-ycaa, pc-xcaa, \text{ class } 3\rangle \ (\text{all } x,\ y,\ y\sim -y) \tag{7.1260}$$

$$\langle a,b,c \,|\, cab, cac-caa, cbb-\omega caa, cbc, pa-ba, pb-ycaa, pc-xcaa, \text{ class } 3\rangle \ (\text{all } x,\ y,\ y\sim -y) \tag{7.1261}$$

$$\langle a,b,c \,|\, caa, cac, cbb, cbc, pa-ba, pb, pc, \text{ class } 3\rangle \tag{7.1262}$$

$$\langle a,b,c \,|\, caa, cac, cbb, cbc, pa-ba, pb, pc-cab, \text{ class } 3\rangle \tag{7.1263}$$

$$\langle a,b,c \,|\, caa, cac, cbb, cbc, pa-ba, pb-cab, pc, \text{ class } 3\rangle \tag{7.1264}$$

$$\langle a,b,c \,|\, caa, cac, cbb, cbc, pa-ba, pb-cab, pc-cab, \text{ class } 3\rangle \tag{7.1265}$$

$$\langle a,b,c \,|\, caa, cac, cbb, cbc-cab, pa-ba, pb, pc, \text{ class } 3\rangle \tag{7.1266}$$

$$\langle a,b,c \,|\, caa, cac, cbb, cbc-cab, pa-ba, pb-cab, pc, \text{ class } 3\rangle \tag{7.1267}$$

$$\langle a, b, c \,|\, caa, cac, cbb, cbc - cab, pa - ba, pb - \omega cab, pc, \text{ class } 3\rangle \tag{7.1268}$$

$$\langle a, b, c \,|\, caa, cac, cbb, cbc - cab, pa - ba, pb - xcab, pc - cab, \text{ class } 3\rangle \text{ (all } x) \tag{7.1269}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba, pb, pc, \text{ class } 3\rangle \tag{7.1270}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba, pb, pc - cbc, \text{ class } 3\rangle \tag{7.1271}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba, pb - cbc, pc, \text{ class } 3\rangle \tag{7.1272}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba, pb - \omega cbc, pc, \text{ class } 3\rangle \tag{7.1273}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba - cbc, pb, pc, \text{ class } 3\rangle \tag{7.1274}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba - cbc, pb, pc - cbc, \text{ class } 3\rangle \tag{7.1275}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba - cbc, pb - cbc, pc, \text{ class } 3\rangle \tag{7.1276}$$

$$\langle a, b, c \,|\, caa, cab, cac, cbb, pa - ba - cbc, pb - \omega cbc, pc, \text{ class } 3\rangle \tag{7.1277}$$

$$\langle a, b, c \,|\, cab, cac, cbb, cbc - caa, pa - ba, pb, pc, \text{ class } 3\rangle \tag{7.1278}$$

$$\langle a, b, c \,|\, cab, cac, cbb, cbc - caa, pa - ba, pb, pc - caa, \text{ class } 3\rangle \tag{7.1279}$$

$$\langle a, b, c \,|\, cab, cac, cbb, cbc - caa, pa - ba, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.1280}$$

$$\langle a, b, c \,|\, cab, cac, cbb, cbc - caa, pa - ba, pb - caa, pc, \text{ class } 3\rangle \tag{7.1281}$$

$$\langle a, b, c \,|\, cab, cac, cbb, cbc - caa, pa - ba, pb - \omega caa, pc, \text{ class } 3\rangle \tag{7.1282}$$

$$\langle a, b, c \,|\, cab, cac, cbb, cbc - caa, pa - ba, pb - \omega^2 caa, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.1283}$$

$$\langle a, b, c \,|\, cab, cac, cbb, cbc - caa, pa - ba, pb - \omega^3 caa, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.1284}$$

$$\langle a, b, c \,|\, caa, cab, cbb, cbc, pa - ba, pb, pc, \text{ class } 3\rangle \tag{7.1285}$$

$$\langle a, b, c \,|\, caa, cab, cbb, cbc, pa - ba - cac, pb, pc, \text{ class } 3\rangle \tag{7.1286}$$

$$\langle a, b, c \,|\, caa, cab, cbb, cbc, pa - ba - \omega cac, pb, pc, \text{ class } 3\rangle \tag{7.1287}$$

$$\langle a, b, c \,|\, caa, cab, cbb, cbc, pa - ba, pb, pc - cac, \text{ class } 3\rangle \tag{7.1288}$$

$$\langle a, b, c \,|\, caa, cab, cbb, cbc, pa - ba - cac, pb, pc - cac, \text{ class } 3\rangle \tag{7.1289}$$

$$\langle a,b,c \,|\, caa, cab, cbb, cbc, pa - ba - \omega cac, pb, pc - cac, \text{ class } 3\rangle \tag{7.1290}$$

$$\langle a,b,c \,|\, caa, cab, cbb, cbc, pa - ba, pb - cac, pc, \text{ class } 3\rangle \tag{7.1291}$$

$$\langle a,b,c \,|\, cab, cac, cbb, cbc, pa - ba, pb, pc, \text{ class } 3\rangle \tag{7.1292}$$

$$\langle a,b,c \,|\, cab, cac, cbb, cbc, pa - ba, pb, pc - caa, \text{ class } 3\rangle \tag{7.1293}$$

$$\langle a,b,c \,|\, cab, cac, cbb, cbc, pa - ba, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.1294}$$

$$\langle a,b,c \,|\, cab, cac, cbb, cbc, pa - ba, pb - caa, pc, \text{ class } 3\rangle \tag{7.1295}$$

$$\langle a,b,c \,|\, cab, cac - caa, cbb, cbc, pa - ba, pb, pc, \text{ class } 3\rangle \tag{7.1296}$$

$$\langle a,b,c \,|\, cab, cac - caa, cbb, cbc, pa - ba, pb, pc - caa, \text{ class } 3\rangle \tag{7.1297}$$

$$\langle a,b,c \,|\, cab, cac - caa, cbb, cbc, pa - ba, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.1298}$$

$$\langle a,b,c \,|\, cab, cac - caa, cbb, cbc, pa - ba, pb - caa, pc, \text{ class } 3\rangle \tag{7.1299}$$

$$\langle a,b,c \,|\, cab, cac - caa, cbb, cbc, pa - ba, pb - \omega caa, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1300}$$

$$\langle a,b,c \,|\, cab, cac - caa, cbb, cbc, pa - ba, pb - \omega^2 caa, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1301}$$

8.76 Descendants of 6.108

$(p-1)(12 + \gcd(p-1,3))/2 + \gcd(p-1,4)/2$ algebras

$$\langle a,b,c \,|\, baa, bab, pa - ca, pb - cb, pc, \text{ class } 3\rangle \tag{7.1302}$$

$$\langle a,b,c \,|\, baa, bac, pa - ca, pb - cb, pc, \text{ class } 3\rangle \tag{7.1303}$$

$$\langle a,b,c \,|\, baa, bac, pa - ca - bab, pb - cb, pc, \text{ class } 3\rangle \tag{7.1304}$$

$$\langle a,b,c \,|\, baa, bac, pa - ca - \omega bab, pb - cb, pc, \text{ class } 3\rangle \tag{7.1305}$$

$$\langle a,b,c \,|\, baa, bac, pa - ca, pb - cb, pc - bab, \text{ class } 3\rangle \tag{7.1306}$$

$$\langle a,b,c \,|\, baa, bab, pa - ca, pb - xcb, pc, \text{ class } 3\rangle \ (x \neq 0, 1, -1, \ x \sim x^{-1}) \tag{7.1307}$$

$$\langle a,b,c \,|\, bab, bac - baa, pa - ca, pb + 2cb, pc, \text{ class } 3\rangle \tag{7.1308}$$

$$\langle a,b,c \,|\, baa, bac, pa - ca, pb - xcb, pc, \text{ class } 3\rangle \ (x \neq 0, 1, -1, \ x \sim x^{-1}) \tag{7.1309}$$

$$\langle a,b,c \,|\, baa, bac, pa-ca-bab, pb-xcb, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1310}$$

$$\langle a,b,c \,|\, baa, bac, pa-ca-\omega bab, pb-xcb, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1311}$$

$$\langle a,b,c \,|\, baa, bac, pa-ca, pb-xcb, pc-bab, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1312}$$

$$\langle a,b,c \,|\, bab, bac, pa-ca, pb-xcb, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1313}$$

$$\langle a,b,c \,|\, bab, bac, pa-ca, pb-xcb-baa, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1314}$$

$$\langle a,b,c \,|\, bab, bac, pa-ca, pb-xcb-\omega baa, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1315}$$

$$\langle a,b,c \,|\, bab, bac, pa-ca, pb-xcb, pc-baa, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1316}$$

$$\langle a,b,c \,|\, bab-baa, bac, pa-ca, pb-xcb, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1317}$$

$$\langle a,b,c \,|\, bab-baa, bac, pa-ca, pb-xcb-baa, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1318}$$

$$\langle a,b,c \,|\, bab-baa, bac, pa-ca, pb-xcb-\omega baa, pc, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1319}$$

$$\langle a,b,c \,|\, bab-baa, bac, pa-ca, pb-xcb, pc-baa, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.1320}$$

$$\langle a,b,c \,|\, bab-baa, bac, pa-ca, pb-xcb, pc-\omega baa, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1},\ p = 1 \bmod 3) \tag{7.1321}$$

$$\langle a,b,c \,|\, bab-baa, bac, pa-ca, pb-xcb, pc-\omega^2 baa, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1},\ p = 1 \bmod 3) \tag{7.1322}$$

$$\langle a,b,c \,|\, baa, bab, pa-ca, pb+cb, pc, \text{ class } 3\rangle \tag{7.1323}$$

$$\langle a,b,c \,|\, baa, cab, pa-ca, pb+cb, pc, \text{ class } 3\rangle \tag{7.1324}$$

$$\langle a,b,c \,|\, baa, cab, pa-ca-bab, pb+cb, pc, \text{ class } 3\rangle \tag{7.1325}$$

$$\langle a,b,c \,|\, baa, cab, pa-ca-\omega bab, pb+cb, pc, \text{ class } 3\rangle \tag{7.1326}$$

$$\langle a,b,c \,|\, baa, cab, pa-ca, pb+cb, pc-bab, \text{ class } 3\rangle \tag{7.1327}$$

$$\langle a,b,c \,|\, bab-baa, cab, pa-ca, pb+cb, pc, \text{ class } 3\rangle \tag{7.1328}$$

$$\langle a,b,c \,|\, bab-baa, cab, pa-ca, pb+cb-baa, pc, \text{ class } 3\rangle \tag{7.1329}$$

$$\langle a,b,c \,|\, bab-baa, cab, pa-ca, pb+cb-\omega baa, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.1330}$$

$$\langle a,b,c \,|\, bab-baa, cab, pa-ca, pb+cb, pc-baa, \text{ class } 3\rangle \tag{7.1331}$$

$$\langle a,b,c \,|\, bab-baa, cab, pa-ca, pb+cb, pc-\omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1332}$$

$$\langle a,b,c \,|\, bab-baa, cab, pa-ca, pb+cb, pc-\omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1333}$$

8.77 Descendants of 6.109

$7 + 2\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, baa, bab, pa - ca - cb, pb - cb, pc, \text{ class } 3\rangle \tag{7.1334}$$

$$\langle a,b,c \,|\, baa, caa, pa - ca - cb, pb - cb, pc, \text{ class } 3\rangle \tag{7.1335}$$

$$\langle a,b,c \,|\, baa, caa, pa - ca - cb, pb - cb - bab, pc, \text{ class } 3\rangle \tag{7.1336}$$

$$\langle a,b,c \,|\, baa, caa, pa - ca - cb, pb - cb - \omega bab, pc, \text{ class } 3\rangle \tag{7.1337}$$

$$\langle a,b,c \,|\, baa, caa, pa - ca - cb, pb - cb, pc - bab, \text{ class } 3\rangle \tag{7.1338}$$

$$\langle a,b,c \,|\, baa, caa, pa - ca - cb, pb - cb, pc - \omega bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1339}$$

$$\langle a,b,c \,|\, baa, caa, pa - ca - cb, pb - cb, pc - \omega^2 bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1340}$$

$$\langle a,b,c \,|\, bab, caa, pa - ca - cb, pb - cb, pc, \text{ class } 3\rangle \tag{7.1341}$$

$$\langle a,b,c \,|\, bab, caa, pa - ca - cb, pb - cb - baa, pc, \text{ class } 3\rangle \tag{7.1342}$$

$$\langle a,b,c \,|\, bab, caa, pa - ca - cb, pb - cb - \omega baa, pc, \text{ class } 3\rangle \tag{7.1343}$$

$$\langle a,b,c \,|\, bab, caa, pa - ca - cb, pb - cb, pc - baa, \text{ class } 3\rangle \tag{7.1344}$$

$$\langle a,b,c \,|\, bab, caa, pa - ca - cb, pb - cb, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1345}$$

$$\langle a,b,c \,|\, bab, caa, pa - ca - cb, pb - cb, pc - \omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1346}$$

8.78 Descendants of 6.110

$2 + \gcd(p-1,3) + \gcd(p-1,4)/2$ algebras

$$\langle a,b,c \,|\, baa, bab, pa - \omega cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.1347}$$

$$\langle a,b,c \,|\, baa, cbb, pa - \omega cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.1348}$$

$$\langle a,b,c \,|\, baa, cbb, pa - \omega cb, pb - ca, pc - bab, \text{ class } 3\rangle \tag{7.1349}$$

$$\langle a,b,c \,|\, baa, cbb, pa - \omega cb, pb - ca, pc - \omega bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1350}$$

$$\langle a,b,c \,|\, baa, cbb, pa - \omega cb, pb - ca, pc - \omega^2 bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1351}$$

$$\langle a,b,c \,|\, baa, cbb, pa - \omega cb, pb - ca - bab, pc, \text{ class } 3\rangle \tag{7.1352}$$

$$\langle a,b,c \,|\, baa, cbb, pa - \omega cb, pb - ca - \omega bab, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.1353}$$

8.79 Descendants of 6.111

$(p-1)(4+\gcd(p-1,3))/2$ algebras

$$\langle a,b,c \,|\, baa, bab, pa - xcb, pb - ca - cb, pc, \text{ class } 3\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.1354}$$

$$\langle a,b,c \,|\, baa, bac, pa - xcb, pb - ca - cb, pc, \text{ class } 3\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.1355}$$

$$\langle a,b,c \,|\, baa, bac, pa - xcb, pb - ca - cb - bab, pc, \text{ class } 3\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.1356}$$

$$\langle a,b,c \,|\, baa, bac, pa - xcb, pb - ca - cb - \omega bab, pc, \text{ class } 3\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.1357}$$

$$\langle a,b,c \,|\, baa, bac, pa - xcb, pb - ca - cb, pc - bab, \text{ class } 3\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.1358}$$

$$\langle a,b,c \,|\, baa, bac, pa - xcb, pb - ca - cb, pc - \omega bab, \text{ class } 3\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p,\ p = 1 \bmod 3) \tag{7.1359}$$

$$\langle a,b,c \,|\, baa, bac, pa - xcb, pb - ca - cb, pc - \omega^2 bab, \text{ class } 3\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p,\ p = 1 \bmod 3) \tag{7.1360}$$

8.80 Descendants of 6.112

$p^2 + 4p + 3 + 2\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, bab, cbb, pa - ba, pb - ca, pc, \text{ class } 3\rangle \tag{7.1361}$$

$$\langle a,b,c \,|\, bab, cbb, pa - ba - cbc, pb - ca, pc, \text{ class } 3\rangle \tag{7.1362}$$

$$\langle a,b,c \,|\, bab, cbb, pa - ba - \omega cbc, pb - ca, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1363}$$

$$\langle a,b,c \,|\, bab, cbb, pa - ba - \omega^2 cbc, pb - ca, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1364}$$

$$\langle a,b,c \,|\, bab, cbb, pa - ba - xcbc, pb - ca, pc - cbc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1365}$$

$$\langle a,b,c \,|\, bab, cbc, pa - ba, pb - ca, pc - xcbb, \text{ class } 3\rangle \text{ (all } x) \tag{7.1366}$$

$$\langle a,b,c \,|\, bab, cbc, pa - ba, pb - ca - cbb, pc - xcbb, \text{ class } 3\rangle \text{ (all } x) \tag{7.1367}$$

$$\langle a,b,c \,|\, bab, cbc - cbb, pa - ba, pb - ca - xcbb, pc - ycbb, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.1368}$$

$$\langle a,b,c \,|\, cbb, cbc, pa - ba, pb - ca, pc, \text{ class } 3\rangle \tag{7.1369}$$

$$\langle a,b,c \,|\, cbb, cbc - bab, pa - ba, pb - ca, pc, \text{ class } 3\rangle \tag{7.1370}$$

$$\langle a,b,c \,|\, cbb, cbc - \omega bab, pa - ba, pb - ca, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1371}$$

$$\langle a,b,c \,|\, cbb, cbc - \omega^2 bab, pa - ba, pb - ca, pc, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.1372}$$

$$\langle a,b,c \,|\, cbb-bab, cbc-xbab, pa-ba, pb-ca, pc, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x) \tag{7.1373}$$

$$\langle a,b,c \,|\, cbb-\omega bab, cbc-xbab, pa-ba, pb-ca, pc, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x) \tag{7.1374}$$

8.81 Descendants of 6.113

$5p+4$ algebras

$$\langle a,b,c \,|\, bab, cac, pa-ba, pb-cb, pc, \text{ class } 3\rangle \tag{7.1375}$$

$$\langle a,b,c \,|\, bab, cac, pa-ba, pb-cb, pc-caa, \text{ class } 3\rangle \tag{7.1376}$$

$$\langle a,b,c \,|\, bab, cac, pa-ba, pb-cb, pc-\omega caa, \text{ class } 3\rangle \tag{7.1377}$$

$$\langle a,b,c \,|\, bab, cac, pa-ba, pb-cb-caa, pc-xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.1378}$$

$$\langle a,b,c \,|\, bab, cac, pa-ba, pb-cb-\omega caa, pc-xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.1379}$$

$$\langle a,b,c \,|\, bab, caa, pa-ba-xcac, pb-cb, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.1380}$$

$$\langle a,b,c \,|\, bab, caa, pa-ba-xcac, pb-cb, pc-cac, \text{ class } 3\rangle \text{ (all } x) \tag{7.1381}$$

$$\langle a,b,c \,|\, caa, cac, pa-ba, pb-cb, pc, \text{ class } 3\rangle \tag{7.1382}$$

$$\langle a,b,c \,|\, caa, cac-xbab, pa-ba, pb-cb, pc, \text{ class } 3\rangle \ (x\neq 0) \tag{7.1383}$$

$$\langle a,b,c \,|\, caa-bab, cac+2bab, pa-ba, pb-cb, pc, \text{ class } 3\rangle \tag{7.1384}$$

8.82 Descendants of 6.114

$4p-4$ algebras

$$\langle a,b,c \,|\, bac, pa-ba, pb-cb, pc+ba-ca, \text{ class } 3\rangle \tag{7.1385}$$

$$\langle a,b,c \,|\, bac-bab, pa-ba, pb-cb, pc+ba-ca-xbab, \text{ class } 3\rangle \text{ (all } x) \tag{7.1386}$$

$$\langle a,b,c \,|\, bac, pa-ba, pb-cb, pc-3ba-ca, \text{ class } 3\rangle \tag{7.1387}$$

$$\langle a,b,c \,|\, bac+bab, pa-ba, pb-cb, pc-3ba-ca, \text{ class } 3\rangle \tag{7.1388}$$

$$\langle a,b,c \,|\, bac-zbab, pa-ba, pb-cb, pc-xba-ca, \text{ class } 3\rangle \ (x\neq -1,3) \tag{7.1389}$$

In algebra 7.1389, (for a given $x\neq -1,3$) z and z' define isomorphic algebras if the ratios $1:z$ and $1:z'$ are in the same orbit of ratios $\alpha:\beta$ under the action

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \to A \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

where A equals

$$\begin{pmatrix} x-1 & 1 \\ -1 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} x^2-2x & x-1 \\ 1-x & -1 \end{pmatrix} \text{ or } \begin{pmatrix} (1+\gamma x)(\gamma x-2\gamma+1) & \gamma(\gamma x+2-\gamma) \\ -\gamma(\gamma x+2-\gamma) & -(-1+\gamma)(\gamma+1) \end{pmatrix}$$

with $\gamma \neq -1$ and γ is not a root of $\gamma^2+(x-1)\gamma+1=0$. (Note that the ratio $1:0$ is in the same orbit as the ratio $0:1$.)

8.83 Descendants of 6.115

3 algebras

$$\langle a,b,c \,|\, bac, cac, pa-ba, pb-ca, pc-cb, \text{ class } 3\rangle \tag{7.1390}$$

$$\langle a,b,c \,|\, bac, cac-bab, pa-ba, pb-ca, pc-cb, \text{ class } 3\rangle \tag{7.1391}$$

$$\langle a,b,c \,|\, bac, cac-\omega bab, pa-ba, pb-ca, pc-cb, \text{ class } 3\rangle \tag{7.1392}$$

8.84 Descendants of 6.116

3 algebras

$$\langle a,b,c \,|\, bab, pa-ba, pb-ca, pc+cb, \text{ class } 3\rangle \tag{7.1393}$$

$$\langle a,b,c \,|\, bab, pa-ba-cac, pb-ca, pc+cb, \text{ class } 3\rangle \tag{7.1394}$$

$$\langle a,b,c \,|\, bab, pa-ba-\omega cac, pb-ca, pc+cb, \text{ class } 3\rangle \tag{7.1395}$$

8.85 Descendants of 6.117

$p+1$ algebras

$$\langle a,b,c \,|\, bab, pa-ba-xcac, pb-ca, pc-\omega ba+cb, \text{ class } 3\rangle \text{ (all } x) \tag{7.1396}$$

$$\langle a,b,c \,|\, cac, pa-ba, pb-ca, pc-\omega ba+cb, \text{ class } 3\rangle \tag{7.1397}$$

8.86 Descendants of 6.118

$11+4\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, ca, cb, baaa, baab, pa, pb, pc, \text{ class } 4\rangle \tag{7.1398}$$

$$\langle a,b,c \,|\, ca, cb, baaa, baab, pa, pb-babb, pc, \text{ class } 4\rangle \tag{7.1399}$$

$$\langle a,b,c \,|\, ca, cb, baaa, baab, pa-babb, pb, pc, \text{ class } 4\rangle \tag{7.1400}$$

$$\langle a,b,c \,|\, ca, cb, baaa, baab, pa-\omega babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1401}$$

$$\langle a,b,c \,|\, ca, cb, baaa, baab, pa-\omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1402}$$

$$\langle a,b,c \,|\, ca, cb, baaa, baab, pa, pb, pc-babb, \text{ class } 4\rangle \tag{7.1403}$$

$$\langle a, b, c \,|\, ca - babb, cb, baaa, baab, pa, pb, pc, \text{ class } 4\rangle \tag{7.1404}$$

$$\langle a, b, c \,|\, ca - babb, cb, baaa, baab, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.1405}$$

$$\langle a, b, c \,|\, ca - babb, cb, baaa, baab, pa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1406}$$

$$\langle a, b, c \,|\, ca - babb, cb, baaa, baab, pa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1407}$$

$$\langle a, b, c \,|\, ca - babb, cb, baaa, baab, pa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1408}$$

$$\langle a, b, c \,|\, ca - babb, cb, baaa, baab, pa, pb, pc - babb, \text{ class } 4\rangle \tag{7.1409}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + baaa, pa, pb, pc, \text{ class } 4\rangle \tag{7.1410}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + baaa, pa, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1411}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + baaa, pa, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1412}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + baaa, pa, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1413}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + baaa, pa - baaa, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1414}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + baaa, pa, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1415}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + \omega baaa, pa, pb, pc, \text{ class } 4\rangle \tag{7.1416}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + \omega baaa, pa, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1417}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + \omega baaa, pa, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1418}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + \omega baaa, pa, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1419}$$

$$\langle a, b, c \,|\, ca, cb, baab, babb + \omega baaa, pa, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1420}$$

8.87 Descendants of 6.119

$(p-1)/2+3+2\gcd(p-1,3)+\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, ca, cb, pa-bab, pb, pc, \text{ class } 4\rangle \tag{7.1421}$$

$$\langle a,b,c \,|\, ca, cb, pa-bab-baaa, pb, pc, \text{ class } 4\rangle \tag{7.1422}$$

$$\langle a,b,c \,|\, ca, cb, pa-bab-\omega baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1423}$$

$$\langle a,b,c \,|\, ca, cb, pa-bab, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1424}$$

$$\langle a,b,c \,|\, ca, cb, pa-bab, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1425}$$

$$\langle a,b,c \,|\, ca, cb, pa-bab, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1426}$$

$$\langle a,b,c \,|\, ca, cb, pa-bab, pb, pc-baaa, \text{ class } 4\rangle \tag{7.1427}$$

$$\langle a,b,c \,|\, ca, cb-baaa, pa-bab, pb, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x) \tag{7.1428}$$

$$\langle a,b,c \,|\, ca, cb-baaa, pa-bab-baaa, pb, pc, \text{ class } 4\rangle \tag{7.1429}$$

$$\langle a,b,c \,|\, ca, cb-baaa, pa-bab-\omega baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1430}$$

$$\langle a,b,c \,|\, ca, cb-baaa, pa-bab, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1431}$$

$$\langle a,b,c \,|\, ca, cb-baaa, pa-bab, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1432}$$

$$\langle a,b,c \,|\, ca, cb-baaa, pa-bab, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1433}$$

8.88 Descendants of 6.120

$(p-1)/2+3+2\gcd(p-1,3)+\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, ca, cb, pa-\omega bab, pb, pc, \text{ class } 4\rangle \tag{7.1434}$$

$$\langle a,b,c \,|\, ca, cb, pa-\omega bab-baaa, pb, pc, \text{ class } 4\rangle \tag{7.1435}$$

$$\langle a,b,c \,|\, ca, cb, pa-\omega bab-\omega baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1436}$$

$$\langle a,b,c \,|\, ca, cb, pa-\omega bab, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1437}$$

$$\langle a,b,c \,|\, ca, cb, pa-\omega bab, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1438}$$

$$\langle a,b,c \,|\, ca, cb, pa-\omega bab, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1439}$$

$$\langle a, b, c \,|\, ca, cb, pa - \omega bab, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1440}$$

$$\langle a, b, c \,|\, ca, cb - baaa, pa - \omega bab, pb, pc - xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x \sim -x) \tag{7.1441}$$

$$\langle a, b, c \,|\, ca, cb - baaa, pa - \omega bab - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1442}$$

$$\langle a, b, c \,|\, ca, cb - baaa, pa - \omega bab - \omega baaa, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1443}$$

$$\langle a, b, c \,|\, ca, cb - baaa, pa - \omega bab, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1444}$$

$$\langle a, b, c \,|\, ca, cb - baaa, pa - \omega bab, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1445}$$

$$\langle a, b, c \,|\, ca, cb - baaa, pa - \omega bab, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1446}$$

8.89 Descendants of 6.121

$2p + 4 + 2\gcd(p - 1, 3)$ algebras

$$\langle a, b, c \,|\, ca, cb, pa - baa, pb, pc, \text{ class } 4\rangle \tag{7.1447}$$

$$\langle a, b, c \,|\, ca, cb, pa - baa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1448}$$

$$\langle a, b, c \,|\, ca, cb, pa - baa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1449}$$

$$\langle a, b, c \,|\, ca, cb, pa - baa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1450}$$

$$\langle a, b, c \,|\, ca, cb, pa - baa - xbabb, pb - babb, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.1451}$$

$$\langle a, b, c \,|\, ca, cb, pa - baa, pb, pc - babb, \text{ class } 4\rangle \tag{7.1452}$$

$$\langle a, b, c \,|\, ca - babb, cb, pa - baa, pb, pc, \text{ class } 4\rangle \tag{7.1453}$$

$$\langle a, b, c \,|\, ca - babb, cb, pa - baa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1454}$$

$$\langle a, b, c \,|\, ca - babb, cb, pa - baa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1455}$$

$$\langle a, b, c \,|\, ca - babb, cb, pa - baa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1456}$$

$$\langle a, b, c \,|\, ca - babb, cb, pa - baa - xbabb, pb - babb, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.1457}$$

$$\langle a, b, c \,|\, ca - babb, cb, pa - baa, pb, pc - babb, \text{ class } 4\rangle \tag{7.1458}$$

8.90 Descendants of 6.122

$p+2$ algebras

$$\langle a,b,c \,|\, ca, cb, pa - baa, pb + bab, pc, \text{ class } 4\rangle \tag{7.1459}$$

$$\langle a,b,c \,|\, ca, cb, pa - baa, pb + bab, pc - baab, \text{ class } 4\rangle \tag{7.1460}$$

$$\langle a,b,c \,|\, ca, cb, pa - baa, pb + bab - baab, pc, \text{ class } 4\rangle \tag{7.1461}$$

$$\langle a,b,c \,|\, ca, cb, pa - baa - baab, pb + bab - xbaab, pc, \text{ class } 4\rangle \ (x \neq 0) \tag{7.1462}$$

8.91 Descendants of 6.125

$p+1$ algebras

$$\langle a,b,c \,|\, ca, cb, pa + bab, pb + \omega baa, pc - baaa, \text{ class } 4\rangle \tag{7.1463}$$

$$\langle a,b,c \,|\, ca, cb, pa + bab - xbaaa, pb + \omega baa - ybaaa, pc, \text{ class } 4\rangle \ (\text{all } x,\ y,\ (x,y) \sim (x',y') \text{ if } y^2 - \omega x^2 = y'^2 - \omega x'^2 \bmod p) \tag{7.1464}$$

8.92 Descendants of 6.127

$3p + 4 + 6\gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, ca, cb, pa, pb, pc - bab, \text{ class } 4\rangle \tag{7.1465}$$

$$\langle a,b,c \,|\, ca, cb, pa - baaa, pb, pc - bab, \text{ class } 4\rangle \tag{7.1466}$$

$$\langle a,b,c \,|\, ca, cb, pa, pb - baaa, pc - bab, \text{ class } 4\rangle \tag{7.1467}$$

$$\langle a,b,c \,|\, ca, cb, pa, pb - \omega baaa, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1468}$$

$$\langle a,b,c \,|\, ca, cb, pa, pb - \omega^2 baaa, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1469}$$

$$\langle a,b,c \,|\, ca, cb, pa, pb, pc - bab - baaa, \text{ class } 4\rangle \tag{7.1470}$$

$$\langle a,b,c \,|\, ca, cb, pa - baaa, pb, pc - bab - baaa, \text{ class } 4\rangle \tag{7.1471}$$

$$\langle a,b,c \,|\, ca, cb, pa - \omega baaa, pb, pc - bab - baaa, \text{ class } 4\rangle \tag{7.1472}$$

$$\langle a,b,c \,|\, ca, cb, pa - \omega^2 baaa, pb, pc - bab - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1473}$$

$$\langle a,b,c \,|\, ca, cb, pa - \omega^3 baaa, pb, pc - bab - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1474}$$

$$\langle a,b,c \,|\, ca, cb, pa, pb - baaa, pc - bab - baaa, \text{ class } 4\rangle \tag{7.1475}$$

$$\langle a,b,c \,|\, ca, cb, pa, pb - \omega baaa, pc - bab - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1476}$$

$$\langle a,b,c \,|\, ca, cb, pa, pb - \omega^2 baaa, pc - bab - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1477}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa, pb, pc - bab, \text{ class } 4\rangle \tag{7.1478}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa - baaa, pb, pc - bab, \text{ class } 4\rangle \tag{7.1479}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa - \omega baaa, pb, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1480}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa - \omega^2 baaa, pb, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1481}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa, pb - baaa, pc - bab, \text{ class } 4\rangle \tag{7.1482}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa, pb - \omega baaa, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1483}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa, pb - \omega^2 baaa, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1484}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa, pb - xbaaa, pc - bab - baaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1485}$$

$$\langle a,b,c \,|\, ca, cb - baaa, pa - xbaaa, pb, pc - bab - baaa, \text{ class } 4\rangle \ (x \neq 0) \tag{7.1486}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa, pb, pc - bab, \text{ class } 4\rangle \tag{7.1487}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa - baaa, pb, pc - bab, \text{ class } 4\rangle \tag{7.1488}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa - \omega baaa, pb, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1489}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa - \omega^2 baaa, pb, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1490}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa, pb - baaa, pc - bab, \text{ class } 4\rangle \tag{7.1491}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa, pb - \omega baaa, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1492}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa, pb - \omega^2 baaa, pc - bab, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1493}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa, pb - xbaaa, pc - bab - baaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1494}$$

$$\langle a,b,c \,|\, ca, cb - \omega baaa, pa - xbaaa, pb, pc - bab - baaa, \text{ class } 4\rangle \ (x \neq 0) \tag{7.1495}$$

8.93 Descendants of 6.131

$15 + (p + 10)\gcd(p - 1, 3) + \gcd(p - 1, 4) + \gcd(p - 1, 7)$ algebras

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1496}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa, pb, pc - babb, \text{ class } 4\rangle \tag{7.1497}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.1498}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa, pb - babb, pc - babb, \text{ class } 4\rangle \tag{7.1499}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1500}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1501}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1502}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa - babb, pb - babb, pc, \text{ class } 4\rangle \tag{7.1503}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa - \omega babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1504}$$

$$\langle a, b, c \,|\, baaa, baab, ca - bab, cb, pa - \omega^2 babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1505}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1506}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb, pa - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1507}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb, pa, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1508}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb, pa, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1509}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb, pa, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1510}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb, pa, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1511}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb - baaa, pa, pb, pc, \text{ class } 4\rangle \tag{7.1512}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb - baaa, pa - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1513}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb - baaa, pa - xbaaa, pb, pc, \text{ class } 4\rangle \ (x = \omega^i,\ i = 1, 2, 3, 4, 5, 6,\ p = 1 \bmod 7) \tag{7.1514}$$

$$\langle a, b, c \,|\, baab, babb, ca - bab, cb - baaa, pa, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1515}$$

$$\langle a,b,c \,|\, baab, babb, ca-bab, cb-baaa, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \tag{7.1516}$$

$$\langle a,b,c \,|\, baab, babb, ca-bab, cb-baaa, pa, pb-xbaaa, pc, \text{ class } 4\rangle \ (x=\omega^i,\ i=2,3,4,5,\ p=1 \bmod 3) \tag{7.1517}$$

$$\langle a,b,c \,|\, baab, babb, ca-bab, cb-baaa, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.1518}$$

$$\langle a,b,c \,|\, baab, babb, ca-bab, cb-baaa, pa, pb, pc-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1519}$$

$$\langle a,b,c \,|\, baab, babb, ca-bab, cb-baaa, pa, pb, pc-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1520}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1521}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1522}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1523}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1524}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa-baaa, pb-xbaaa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x) \tag{7.1525}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa-\omega baaa, pb-xbaaa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1526}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa-\omega^2 baaa, pb-xbaaa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1527}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.1528}$$

$$\langle a,b,c \,|\, baab, babb-baaa, ca-bab, cb, pa, pb, pc-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1529}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1530}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1531}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1532}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1533}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa-baaa, pb-xbaaa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x) \tag{7.1534}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa-\omega baaa, pb-xbaaa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1535}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa-\omega^2 baaa, pb-xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1536}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.1537}$$

$$\langle a,b,c \,|\, baab, babb-\omega baaa, ca-bab, cb, pa, pb, pc-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1538}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1539}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa, pb-baab, pc, \text{ class } 4\rangle \tag{7.1540}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa-baab, pb, pc, \text{ class } 4\rangle \tag{7.1541}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa-baab, pb-baab, pc, \text{ class } 4\rangle \tag{7.1542}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa-baab, pb-\omega baab, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1543}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa-baab, pb-\omega^2 baab, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1544}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa, pb, pc-baab, \text{ class } 4\rangle \tag{7.1545}$$

$$\langle a,b,c \,|\, baaa, babb, ca-bab, cb, pa, pb, pc-\omega baab, \text{ class } 4\rangle \tag{7.1546}$$

8.94 Descendants of 6.132

$(p+1+3(p+1)\gcd(p-1,3)+\gcd(p-1,4))/2$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb, pc, \text{ class } 4\rangle \tag{7.1547}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab-baaa, pb, pc, \text{ class } 4\rangle \tag{7.1548}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab-\omega baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1549}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1550}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1551}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1552}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb, pc-baaa, \text{ class } 4\rangle \tag{7.1553}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb, pc-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1554}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb, pc-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1555}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-bab, pb, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1556}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-bab, pb, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1557}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega^2 baaa, pa-bab, pb, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1558}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-bab, pb-xbaaa, pc, \text{ class } 4\rangle \ (x\neq 0) \tag{7.1559}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-bab, pb-xbaaa, pc, \text{ class } 4\rangle \ (x\neq 0,\ p=1 \bmod 3) \tag{7.1560}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega^2 baaa, pa-bab, pb-xbaaa, pc, \text{ class } 4\rangle \ (x\neq 0,\ p=1 \bmod 3) \tag{7.1561}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-bab-xbaaa, pb, pc, \text{ class } 4\rangle \ (x\neq 0, x\sim x' \text{ if } x^6=x'^6 \bmod p) \tag{7.1562}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-bab-xbaaa, pb, pc, \text{ class } 4\rangle \ (x\neq 0, x\sim x' \text{ if } x^6=x'^6 \bmod p,\ p=1 \bmod 3) \tag{7.1563}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega^2 baaa, pa-bab-xbaaa, pb, pc, \text{ class } 4\rangle \ (x\neq 0, x\sim x' \text{ if } x^6=x'^6 \bmod p,\ p=1 \bmod 3) \tag{7.1564}$$

8.95 Descendants of 6.133

$(p+1+3(p+1)\gcd(p-1,3)+\gcd(p-1,4))/2$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb, pc, \text{ class } 4\rangle \tag{7.1565}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab-baaa, pb, pc, \text{ class } 4\rangle \tag{7.1566}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab-\omega baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1567}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1568}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1569}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1570}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb, pc-baaa, \text{ class } 4\rangle \tag{7.1571}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb, pc-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1572}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb, pc-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1573}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-\omega bab, pb, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1574}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-\omega bab, pb, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1575}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega^2 baaa, pa-\omega bab, pb, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1576}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-\omega bab, pb-xbaaa, pc, \text{ class } 4\rangle \ (x\neq 0) \tag{7.1577}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-\omega bab, pb-xbaaa, pc, \text{ class } 4\rangle \ (x\neq 0,\ p=1 \bmod 3) \tag{7.1578}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega^2 baaa, pa-\omega bab, pb-xbaaa, pc, \text{ class } 4\rangle \ (x\neq 0,\ p=1 \bmod 3) \tag{7.1579}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-\omega bab-xbaaa, pb, pc, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p) \tag{7.1580}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-\omega bab-xbaaa, pb, pc, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p,\ p=1 \bmod 3) \tag{7.1581}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega^2 baaa, pa-\omega bab-xbaaa, pb, pc, \text{ class } 4\rangle \ (x\neq 0, x\sim x' \text{ if } x^6=x'^6 \bmod p,\ p=1 \bmod 3) \tag{7.1582}$$

8.96 Descendants of 6.134

$3p-1+\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa-baa, pb, pc-xbabb, \text{ class } 4\rangle \text{ (all } x) \tag{7.1583}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-baa, pb-babb, pc-xbabb, \text{ class } 4\rangle \text{ (all } x) \tag{7.1584}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-baa-babb, pb, pc-babb, \text{ class } 4\rangle \tag{7.1585}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-baa-\omega babb, pb, pc-babb, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1586}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-baa-\omega^2 babb, pb, pc-babb, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1587}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-baa-xbabb, pb-babb, pc-babb, \text{ class } 4\rangle \ (x\neq 0) \tag{7.1588}$$

8.97 Descendants of 6.135

$p^2+2p+3+\gcd(p-1,3)+\gcd(p-1,4)+\gcd(p-1,5)$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-bab, pc, \text{ class } 4\rangle \tag{7.1589}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-bab-baaa, pc, \text{ class } 4\rangle \tag{7.1590}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-bab-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1591}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-bab-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1592}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-baaa, pb-bab-xbaaa, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.1593}$$

$$\langle a,b,c\,|\,ca-bab,cb-baaa,pa,pb-bab,pc,\text{ class }4\rangle \tag{7.1594}$$

$$\langle a,b,c\,|\,ca-bab,cb-xbaaa,pa,pb-bab,pc,\text{ class }4\rangle\ (x=\omega^i,\ i=1,2,3,4,\ p=1\bmod 5) \tag{7.1595}$$

$$\langle a,b,c\,|\,ca-bab,cb-baaa,pa,pb-bab-xbaaa,pc,\text{ class }4\rangle\ (x\neq 0,\ x\sim x'\text{ if }x^5=x'^5\bmod p) \tag{7.1596}$$

$$\langle a,b,c\,|\,ca-bab,cb-zbaaa,pa,pb-bab-xbaaa,pc,\text{ class }4\rangle\ (z=\omega^i,\ i=1,2,3,4,\ x\neq 0,\ x\sim x'\text{ if }x^5=x'^5\bmod p,\ p=1\bmod 5) \tag{7.1597}$$

$$\langle a,b,c\,|\,ca-bab,cb-baaa,pa-xbaaa,pb-bab-ybaaa,pc,\text{ class }4\rangle\ (x\neq 0,\ x\sim x'\text{ if }x^5=x'^5\bmod p,\ \text{all }y) \tag{7.1598}$$

In the presentation below we have $z=\omega^i,\ i=1,2,3,4,\ x\neq 0,\ x\sim x'$ if $x^5=x'^5\bmod p$, all $y,\ p=1\bmod 5$:

$$\langle a,b,c\,|\,ca-bab,cb-zbaaa,pa-xbaaa,pb-bab-ybaaa,pc,\text{ class }4\rangle \tag{7.1599}$$

$$\langle a,b,c\,|\,ca-bab,cb-xbaaa,pa,pb-bab,pc-baaa,\text{ class }4\rangle\ (\text{all }x,\ x\sim x'\text{ if }x^4=x'^4\bmod p) \tag{7.1600}$$

$$\langle a,b,c\,|\,ca-bab,cb-xbaaa,pa,pb-bab,pc-\omega baaa,\text{ class }4\rangle\ (\text{all }x,\ x\sim x'\text{ if }x^4=x'^4\bmod p) \tag{7.1601}$$

$$\langle a,b,c\,|\,ca-bab,cb-xbaaa,pa,pb-bab,pc-\omega^2 baaa,\text{ class }4\rangle\ (\text{all }x,\ x\sim x'\text{ if }x^4=x'^4\bmod p,\ p=1\bmod 4) \tag{7.1602}$$

$$\langle a,b,c\,|\,ca-bab,cb-xbaaa,pa,pb-bab,pc-\omega^3 baaa,\text{ class }4\rangle\ (\text{all }x,\ x\sim x'\text{ if }x^4=x'^4\bmod p,\ p=1\bmod 4) \tag{7.1603}$$

$$\langle a,b,c\,|\,ca-bab,cb,pa+baa,pb-bab,pc,\text{ class }4\rangle \tag{7.1604}$$

$$\langle a,b,c\,|\,ca-bab,cb,pa+baa,pb-bab-baab,pc,\text{ class }4\rangle \tag{7.1605}$$

$$\langle a,b,c\,|\,ca-bab,cb,pa+baa,pb-bab,pc-baab,\text{ class }4\rangle \tag{7.1606}$$

$$\langle a,b,c\,|\,ca-bab,cb,pa+baa,pb-bab,pc-\omega baab,\text{ class }4\rangle \tag{7.1607}$$

8.98 Descendants of 6.138

$p(p+1)/2$ algebras

$$\langle a,b,c\,|\,ca-bab,cb,pa-baa-ybabb,pb-baa,pc-xbabb,\text{ class }4\rangle\ (\text{all }x,\ y,\ y\sim -y) \tag{7.1608}$$

8.99 Descendants of 6.139

$(p+3)/2 + \gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-baa, pc, \text{ class } 4\rangle \tag{7.1609}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-babb, pb-baa, pc, \text{ class } 4\rangle \tag{7.1610}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega babb, pb-baa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1611}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega^2 babb, pb-baa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1612}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-xbabb, pb-baa, pc-babb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1613}$$

8.100 Descendants of 6.140

$(3p+1)/2$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb-baa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1614}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb-baa-xbaaa, pc-ybaaa, \text{ class } 4\rangle \ (\text{If 2 is a square } \bmod p \text{ and } y^2 = 2,\ x \neq 0,\ x \sim -x,\ y \sim -y) \tag{7.1615}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb-baa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1616}$$

The following presentation only arises if 2 is not a square modulo p:

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb-baa-xbaaa, pc-ybaaa, \text{ class } 4\rangle \ (x \neq 0,\ x \sim -x,\ \omega y^2 = 2,\ y \sim -y) \tag{7.1617}$$

8.101 Descendants of 6.142

$p(p+1/2$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa-baa-ybabb, pb-\omega baa, pc-xbabb, \text{ class } 4\rangle \ (\text{all } x,\ y,\ y \sim -y) \tag{7.1618}$$

8.102 Descendants of 6.143

$(p+3)/2 + \gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-\omega baa, pc, \text{ class } 4\rangle \tag{7.1619}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-babb, pb-\omega baa, pc, \text{ class } 4\rangle \tag{7.1620}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega babb, pb-\omega baa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1621}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega^2 babb, pb-\omega baa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1622}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-xbabb, pb-\omega baa, pc-babb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1623}$$

8.103 Descendants of 6.144

$(3p+1)/2$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb-\omega baa, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1624}$$

The following presentation only arises if 2 is not a square modulo p:

$$\langle a,b,c \,|\, ca-bab, cb, pa-bab, pb-\omega baa-xbaaa, pc-ybaaa, \text{ class } 4\rangle \ (x\neq 0,\ x\sim -x,\ y^2=2\omega^3,\ y\sim -y) \tag{7.1625}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb-\omega baa, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1626}$$

The following presentation only arises if 2 is a square modulo p:

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega bab, pb-\omega baa-xbaaa, pc-ybaaa, \text{ class } 4\rangle \ (x\neq 0,\ x\sim -x,\ y^2=2\omega^2,\ y\sim -y) \tag{7.1627}$$

8.104 Descendants of 6.146

$2p^2+3p$ algebras

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-xbaaa, pc-bab, \text{ class } 4\rangle \text{ (all } x) \tag{7.1628}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa, pb-xbaaa, pc-bab-baaa, \text{ class } 4\rangle \text{ (all } x) \tag{7.1629}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-baaa, pb, pc-bab, \text{ class } 4\rangle \tag{7.1630}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-xbaaa, pb, pc-bab-baaa, \text{ class } 4\rangle \ (x\neq 0) \tag{7.1631}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa, pb-xbaaa, pc-bab-ybaaa, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim -y) \tag{7.1632}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-zbaaa, pb, pc-bab, \text{ class } 4\rangle \ (z\neq 0,\ z\sim -z) \tag{7.1633}$$

$$\langle a,b,c \,|\, ca-bab, cb-baaa, pa-zbaaa, pb, pc-bab-ybaaa, \text{ class } 4\rangle \ (y,z\neq 0,\ y\sim -y) \tag{7.1634}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa, pb-xbaaa, pc-bab-ybaaa, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim -y) \tag{7.1635}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-zbaaa, pb, pc-bab, \text{ class } 4\rangle \ (z\neq 0,\ z\sim -z) \tag{7.1636}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baaa, pa-zbaaa, pb, pc-bab-ybaaa, \text{ class } 4\rangle \ (y,z\neq 0,\ y\sim -y) \tag{7.1637}$$

8.105 Descendants of 6.148

$p^3 + p^2 + p + (p+2)\gcd(p-1,3) + \gcd(p-1,5)$ algebras

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb, pc - baa, \text{ class } 4\rangle \tag{7.1638}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb, pc - baa - babb, \text{ class } 4\rangle \tag{7.1639}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb, pc - baa - \omega babb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1640}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb, pc - baa - \omega^2 babb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1641}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - babb, pc - baa, \text{ class } 4\rangle \tag{7.1642}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega babb, pc - baa, \text{ class } 4\rangle \ (p = 1 \bmod 5) \tag{7.1643}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega^2 babb, pc - baa, \text{ class } 4\rangle \ (p = 1 \bmod 5) \tag{7.1644}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega^3 babb, pc - baa, \text{ class } 4\rangle \ (p = 1 \bmod 5) \tag{7.1645}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega^4 babb, pc - baa, \text{ class } 4\rangle \ (p = 1 \bmod 5) \tag{7.1646}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - babb, pc - baa - xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^5 = x'^5 \bmod p) \tag{7.1647}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega babb, pc - baa - xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.1648}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega^2 babb, pc - baa - xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.1649}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega^3 babb, pc - baa - xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.1650}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa, pb - \omega^4 babb, pc - baa - xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.1651}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa - babb, pb, pc - baa - xbabb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1652}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa - \omega babb, pb, pc - baa - xbabb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1653}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa - \omega^2 babb, pb, pc - baa - xbabb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x,\ p = 1 \bmod 3) \tag{7.1654}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa - \omega^3 babb, pb, pc - baa - xbabb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x,\ p = 1 \bmod 3) \tag{7.1655}$$

$$\langle a,b,c \,|\, ca - bab, cb, pa - \omega^4 babb, pb, pc - baa - xbabb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x,\ p = 1 \bmod 3) \tag{7.1656}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega^5 babb, pb, pc-baa-xbabb, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.1657}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-babb, pb-xbabb, pc-baa-ybabb, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p,\ \text{all } y) \tag{7.1658}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega babb, pb-xbabb, pc-baa-ybabb, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p,\ \text{all } y) \tag{7.1659}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega^2 babb, pb-xbabb, pc-baa-ybabb, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p,\ \text{all } y,\ p=1 \bmod 3) \tag{7.1660}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega^3 babb, pb-xbabb, pc-baa-ybabb, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p,\ \text{all } y,\ p=1 \bmod 3) \tag{7.1661}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega^4 babb, pb-xbabb, pc-baa-ybabb, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p,\ \text{all } y,\ p=1 \bmod 3) \tag{7.1662}$$

$$\langle a,b,c \,|\, ca-bab, cb, pa-\omega^5 babb, pb-xbabb, pc-baa-ybabb, \text{ class } 4\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^6=x'^6 \bmod p,\ \text{all } y,\ p=1 \bmod 3) \tag{7.1663}$$

$$\langle a,b,c \,|\, ca-bab-babb, cb, pa-xbabb, pb, pc-baa-ybabb, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim -y) \tag{7.1664}$$

$$\langle a,b,c \,|\, ca-bab-babb, cb, pa-xbabb, pb-ybabb, pc-baa-zbabb, \text{ class } 4\rangle \text{ (all } x,z,\ y\neq 0,\ y\sim -y) \tag{7.1665}$$

$$\langle a,b,c \,|\, ca-bab-\omega babb, cb, pa-xbabb, pb, pc-baa-ybabb, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim -y) \tag{7.1666}$$

$$\langle a,b,c \,|\, ca-bab-\omega babb, cb, pa-xbabb, pb-ybabb, pc-baa-zbabb, \text{ class } 4\rangle \text{ (all } x,z,\ y\neq 0,\ y\sim -y) \tag{7.1667}$$

8.106 Descendants of 6.150

Algebra 6.150 has $4p+14+(p^2/2+2p+13/2)\gcd(p-1,3)+\gcd(p-1,4)$ descendants of order p^7. It has presentation

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa,\ pb,\ pc, \text{ class } 3\rangle$$

and so if L is a descendant of 6.150 of order p^7 then the commutator structure of L is determined by the classification of nilpotent Lie algebras over $\mathbb{Z}_p$. So we can assume that one of the following sets of commutator relations holds.

$$\begin{aligned}
baaa &= baab = ca-baa = cb = 0,\\
baaa &= baab = ca-baa-babb = cb = 0,\\
baaa &= baab = babb,\ ca-baa = cb = 0,\\
babb &= -baaa,\ baab = ca-baa = cb = 0,\\
babb &= -\omega baaa,\ baab = ca-baa = cb = 0,\\
baaa &= babb = ca-baa = cb = 0,\\
babb &= baab,\ baaa = ca-baa = cb = 0,\\
baaa &= baab,\ babb = xbaaa,\ ca-baa = cb = 0\ (x\neq 0,1).
\end{aligned}$$

For each of these cases we obtain a generator d for L_4 (d equals one of $baaa$, $baab$, $babb$) and we write

$$\begin{pmatrix} pa \\ pb \\ pc \end{pmatrix} = Ad$$

where A is a 3×1 matrix over $\mathbb{Z}_p$. In each case the isomorphism classes of algebras are given by the orbits of the matrices A under a given action by a group of automorphisms. I was able to "solve" the problem in every case, providing presentations with fewer parameters, and explicit relatively simple equivalence relations on the parameter sets. However in four of the cases the equivalence classes are slightly more complex than usual. For example in one case the equivalence classes for a parameter y were $\{\pm y, \pm\frac{\omega}{y}\}$. These four cases are 3,4,5,8, and these are described below.

8.106.1 Case 3

If $baaa = baab = babb$, $ca - baa = cb = 0$ then L_4 is generated by $baaa$ and the action on A is

$$A \to \alpha^{-4} \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \alpha & -\gamma \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \to -\alpha^{-4} \begin{pmatrix} 0 & \alpha & \gamma \\ \alpha & 0 & -\gamma \\ 0 & 0 & \alpha^2 \end{pmatrix} A.$$

8.106.2 Case 4

If $babb + baaa = baab = ca - baa = cb = 0$ then L_4 is generated by $baaa$ and the action on A is

$$A \to \alpha^{-4} \begin{pmatrix} \pm\alpha & 0 & \gamma \\ 0 & \alpha & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \to \alpha^{-4} \begin{pmatrix} 0 & \pm\alpha & \gamma \\ \alpha & 0 & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A.$$

8.106.3 Case 5

If $babb + \omega baaa = baab = ca - baa = cb = 0$ then L_4 is generated by $baaa$ and the action on A is

$$A \to \alpha^{-4} \begin{pmatrix} \pm\alpha & 0 & \gamma \\ 0 & \alpha & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \to \omega^{-2}\alpha^{-4} \begin{pmatrix} 0 & \pm\alpha & \gamma \\ \omega\alpha & 0 & \varepsilon \\ 0 & 0 & \omega\alpha^2 \end{pmatrix} A.$$

8.106.4 Case 8

If $baaa = baab$, $babb = xbaaa$, $ca - baa = cb = 0$ where $x \neq 0, 1$ then L_4 is generated by $baaa$. (If we set $x = 0$ we have Case 7, and if we set $x = 1$ we have Case 3.) The action on A is

$$A \to \alpha^{-4} \begin{pmatrix} \alpha & 0 & \gamma \\ 0 & \alpha & \varepsilon \\ 0 & 0 & \alpha^2 \end{pmatrix} A$$

and

$$A \to x^{-2}\alpha^{-4}\begin{pmatrix} 0 & \alpha & \gamma \\ x\alpha & 0 & \varepsilon \\ 0 & 0 & -x\alpha^2 \end{pmatrix} A.$$

8.106.5 The presentations

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1668}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1669}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1670}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1671}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.1672}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa - babb, pb - babb, pc, \text{ class } 4\rangle \tag{7.1673}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa - \omega babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1674}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa - \omega^2 babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1675}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa, pb, pc - babb, \text{ class } 4\rangle \tag{7.1676}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa, cb, pa, pb, pc - \omega babb, \text{ class } 4\rangle \tag{7.1677}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1678}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.1679}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa, pb - \omega babb, pc, \text{ class } 4\rangle \tag{7.1680}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa, pb - \omega^2 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1681}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa, pb - \omega^3 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1682}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1683}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1684}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1685}$$

$$\langle a, b, c \,|\, baaa, baab, ca - baa - babb, cb, pa - babb, pb - xbabb, pc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.1686}$$

$$\langle a,b,c \,|\, baaa, baab, ca-baa-babb, cb, pa-\omega babb, pb-xbabb, pc, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.1687}$$

$$\langle a,b,c \,|\, baaa, baab, ca-baa-babb, cb, pa-\omega^2 babb, pb-xbabb, pc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.1688}$$

$$\langle a,b,c \,|\, baaa, baab, ca-baa-babb, cb, pa, pb, pc-babb, \text{ class } 4\rangle \tag{7.1689}$$

$$\langle a,b,c \,|\, baaa, baab, ca-baa-babb, cb, pa, pb, pc-\omega babb, \text{ class } 4\rangle \tag{7.1690}$$

$$\langle a,b,c \,|\, baab-baaa, babb-baaa, ca-baa, cb, pa-xbaaa, pb-ybaaa, pc-zbaaa, \text{ class } 4\rangle \ (\text{Case 3 above}) \tag{7.1691}$$

$$\langle a,b,c \,|\, babb+baaa, baab, ca-baa, cb, pa-xbaaa, pb-ybaaa, pc-zbaaa, \text{ class } 4\rangle \ (\text{Case 4 above}) \tag{7.1692}$$

$$\langle a,b,c \,|\, babb+\omega baaa, baab, ca-baa, cb, pa-xbaaa, pb-ybaaa, pc, \text{ class } 4\rangle \ (\text{Case 5 above}) \tag{7.1693}$$

$$\langle a,b,c \,|\, baaa, babb, ca-baa, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1694}$$

$$\langle a,b,c \,|\, baaa, babb, ca-baa, cb, pa, pb-baab, pc, \text{ class } 4\rangle \tag{7.1695}$$

$$\langle a,b,c \,|\, baaa, babb, ca-baa, cb, pa-baab, pb-baab, pc, \text{ class } 4\rangle \tag{7.1696}$$

$$\langle a,b,c \,|\, baaa, babb, ca-baa, cb, pa-baab, pb-\omega baab, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1697}$$

$$\langle a,b,c \,|\, baaa, babb, ca-baa, cb, pa-baab, pb-\omega^2 baab, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1698}$$

$$\langle a,b,c \,|\, baaa, babb, ca-baa, cb, pa, pb, pc-baab, \text{ class } 4\rangle \tag{7.1699}$$

$$\langle a,b,c \,|\, babb-baab, baaa, ca-baa, cb, pa, pb, pc, \text{ class } 4\rangle \tag{7.1700}$$

$$\langle a,b,c \,|\, babb-baab, baaa, ca-baa, cb, pa, pb-baab, pc, \text{ class } 4\rangle \tag{7.1701}$$

$$\langle a,b,c \,|\, babb-baab, baaa, ca-baa, cb, pa, pb-\omega baab, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1702}$$

$$\langle a,b,c \,|\, babb-baab, baaa, ca-baa, cb, pa, pb-\omega^2 baab, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1703}$$

$$\langle a,b,c \,|\, babb-baab, baaa, ca-baa, cb, pa-baab, pb-xbaab, pc, \text{ class } 4\rangle \ (\text{all } x) \tag{7.1704}$$

$$\langle a,b,c \,|\, babb-baab, baaa, ca-baa, cb, pa-\omega baab, pb-xbaab, pc, \text{ class } 4\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.1705}$$

$$\langle a,b,c \,|\, babb-baab, baaa, ca-baa, cb, pa-\omega^2 baab, pb-xbaab, pc, \text{ class } 4\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.1706}$$

$$\langle a,b,c \,|\, babb - baab, baaa, ca - baa, cb, pa, pb, pc - baab, \text{ class } 4\rangle \tag{7.1707}$$

$$\langle a,b,c \,|\, babb - baab, baaa, ca - baa, cb, pa, pb, pc - \omega baab, \text{ class } 4\rangle \tag{7.1708}$$

$$\langle a,b,c \,|\, baab - baaa, babb - xbaaa, ca - baa, cb, pa - ybaaa, pb - zbaaa, pc - tbaaa, \text{ class } 4\rangle \text{ (Case 8 above)} \tag{7.1709}$$

8.107 Descendants of 6.151

$p^2 + p + 2 + (p+1)\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, ca - baa, cb, pa - baa, pb, pc, \text{ class } 4\rangle \tag{7.1710}$$

$$\langle a,b,c \,|\, ca - baa, cb, pa - baa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1711}$$

$$\langle a,b,c \,|\, ca - baa, cb, pa - baa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1712}$$

$$\langle a,b,c \,|\, ca - baa, cb, pa - baa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1713}$$

$$\langle a,b,c \,|\, ca - baa - babb, cb, pa - baa - xbabb, pb, pc, \text{ class } 4\rangle \ (\text{all } x) \tag{7.1714}$$

$$\langle a,b,c \,|\, ca - baa - \omega babb, cb, pa - baa - xbabb, pb, pc, \text{ class } 4\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.1715}$$

$$\langle a,b,c \,|\, ca - baa - \omega^2 babb, cb, pa - baa - xbabb, pb, pc, \text{ class } 4\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.1716}$$

$$\langle a,b,c \,|\, ca - baa - xbabb, cb, pa - baa - ybabb, pb - babb, pc, \text{ class } 4\rangle \ (\text{all } x,\ y) \tag{7.1717}$$

$$\langle a,b,c \,|\, ca - baa - xbabb, cb, pa - baa, pb, pc - babb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1718}$$

$$\langle a,b,c \,|\, ca - baa - xbabb, cb, pa - baa, pb, pc - \omega babb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1719}$$

8.108 Descendants of 6.152

$(p^2 + 2p + 3 + 2\gcd(p-1,3) + (p+1)\gcd(p-1,4))/2$ algebras

$$\langle a,b,c \,|\, ca - baa, cb, pa - bab, pb, pc, \text{ class } 4\rangle \tag{7.1720}$$

$$\langle a,b,c \,|\, ca - baa, cb, pa - bab, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1721}$$

$$\langle a,b,c \,|\, ca - baa, cb, pa - bab, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1722}$$

$$\langle a,b,c \,|\, ca - baa, cb, pa - bab, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1723}$$

$$\langle a,b,c \,|\, ca - baa, cb, pa - bab - baaa, pb - xbaaa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim x' \text{ if } x^4 = x'^4 \bmod p) \tag{7.1724}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-bab-\omega baaa, pb-xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ x\sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.1725}$$

$$\langle a,b,c \,|\, ca-baa, cb-baaa, pa-bab-xbaaa, pb-ybaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p) \tag{7.1726}$$

$$\langle a,b,c \,|\, ca-baa, cb-\omega baaa, pa-bab-xbaaa, pb-ybaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.1727}$$

$$\langle a,b,c \,|\, ca-baa, cb-xbaaa, pa-bab, pb, pc-baaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1728}$$

$$\langle a,b,c \,|\, ca-baa, cb-xbaaa, pa-bab, pb, pc-\omega baaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1729}$$

8.109 Descendants of 6.153

$p(p+1)/2$ algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-bab-ybaaa, pb, pc-xbaaa, \text{ class } 4\rangle \text{ (} x\neq 0,\ \text{all } y,\ y\sim -y) \tag{7.1730}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-bab, pb-ybaaa, pc, \text{ class } 4\rangle \text{ (all } y,\ y\sim -y) \tag{7.1731}$$

8.110 Descendants of 6.154

$(p^2+2p+3+2\gcd(p-1,3)+(p+1)\gcd(p-1,4))/2$ algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab, pb, pc, \text{ class } 4\rangle \tag{7.1732}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab, pb-baaa, pc, \text{ class } 4\rangle \tag{7.1733}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab, pb-\omega baaa, pc, \text{ class } 4\rangle \text{ (} p = 1 \bmod 3) \tag{7.1734}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \text{ (} p = 1 \bmod 3) \tag{7.1735}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab-baaa, pb-xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ x\sim x' \text{ if } x^4 = x'^4 \bmod p) \tag{7.1736}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab-\omega baaa, pb-xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ x\sim x' \text{ if } x^4 = x'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.1737}$$

$$\langle a,b,c \,|\, ca-baa, cb-baaa, pa-\omega bab-xbaaa, pb-ybaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p) \tag{7.1738}$$

$$\langle a,b,c \,|\, ca-baa, cb-\omega baaa, pa-\omega bab-xbaaa, pb-ybaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ y,\ y\sim y' \text{ if } y^4 = y'^4 \bmod p,\ p = 1 \bmod 4) \tag{7.1739}$$

$$\langle a,b,c \,|\, ca-baa, cb-xbaaa, pa-\omega bab, pb, pc-baaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1740}$$

$$\langle a,b,c \,|\, ca-baa, cb-xbaaa, pa-\omega bab, pb, pc-\omega baaa, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1741}$$

8.111 Descendants of 6.155

$p(p+1)/2$ algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-\omega bab-ybaaa, pb, pc-xbaaa, \text{ class } 4\rangle \ (x \neq 0, \text{ all } y,\ y \sim -y) \tag{7.1742}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-\omega bab, pb-ybaaa, pc, \text{ class } 4\rangle \ (\text{all } y,\ y \sim -y) \tag{7.1743}$$

8.112 Descendants of 6.156

p algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-bab, pb+baa, pc-xbaaa, \text{ class } 4\rangle \ (x \neq 0,\ x \sim -x) \tag{7.1744}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-bab-xbaaa, pb+baa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1745}$$

8.113 Descendants of 6.157

$2p-1$ algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-bab, pb+\omega baa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x) \tag{7.1746}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-bab-xbaaa, pb+\omega baa, pc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim -x) \tag{7.1747}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-bab, pb+\omega baa-xbaaa, pc+\omega baaa, \text{ class } 4\rangle \ (x \neq 0,\ x \sim -x) \tag{7.1748}$$

8.114 Descendants of 6.158

p algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab, pb+\omega baa, pc-xbaaa, \text{ class } 4\rangle \ (x \neq 0, -\omega,\ x \sim -\omega - x) \tag{7.1749}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-\omega bab-xbaaa, pb+\omega baa, pc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.1750}$$

8.115 Descendants of 6.159

$(p^3+p^2)/2$ algebras

$$\langle a,b,c \,|\, ca-baa-xbabb, cb, pa-baa, pb-baa, pc-ybabb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x,\ y \neq 0) \tag{7.1751}$$

$$\langle a,b,c \,|\, ca-baa-xbabb, cb, pa-baa-ybabb, pb-baa-zbabb, pc, \text{ class } 4\rangle \ (\text{all } x,\ y,z,\ (x,y,z) \sim (-x,-y,-z)) \tag{7.1752}$$

8.116 Descendants of 6.160

$(p^3+p^2)/2$ algebras

$$\langle a,b,c \,|\, ca-baa-xbabb, cb, pa-baa, pb-\omega baa, pc-ybabb, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x,\ y \neq 0) \tag{7.1753}$$

$$\langle a,b,c \,|\, ca-baa-xbabb, cb, pa-baa-ybabb, pb-\omega baa-zbabb, pc, \text{ class } 4\rangle \ (\text{all } x,\ y,z,\ (x,y,z) \sim (-x,-y,-z)) \tag{7.1754}$$

8.117 Descendants of 6.160a

$(p+3)/2$ algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa, pb+bab, pc-xbaab, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1755}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-baab, pb+bab, pc-baab, \text{ class } 4\rangle \tag{7.1756}$$

8.118 Descendants of 6.161

$2p-1$ algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-bab, pb+bab, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x) \tag{7.1757}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-bab, pb+bab-xbaaa, pc+baaa, \text{ class } 4\rangle \ (x\neq 0,\ x\sim -x) \tag{7.1758}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-bab, pb+bab-xbaaa, pc-2baaa, \text{ class } 4\rangle \ (x\neq 0, x\sim -x) \tag{7.1759}$$

8.119 Descendants of 6.162

$2p-1$ algebras

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-\omega bab, pb+bab, pc-xbaaa, \text{ class } 4\rangle \text{ (all } x) \tag{7.1760}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-\omega bab, pb+bab-xbaaa, pc+\omega^{-1}baaa, \text{ class } 4\rangle \ (x\neq 0,\ x\sim -x) \tag{7.1761}$$

$$\langle a,b,c \,|\, ca-baa, cb, pa-baa-\omega bab, pb+bab-xbaaa, pc-2\omega^{-1}baaa, \text{ class } 4\rangle \ (x\neq 0, x\sim -x) \tag{7.1762}$$

8.120 Descendants of 6.163

Algebras 6.163 – 6.167 give a classification of algebras of order p^6 with presentations

$$\langle a,b,c \,|\, ca-baa,\ cb,\ pa-\lambda baa-\mu bab,\ pb+\nu baa+\xi bab,\ pc, \text{ class } 3\rangle$$

with $\lambda,\mu,\nu,\xi\neq 0$. Most of these algebras are terminal, and we need a slightly different classification of these algebras from that given in the classification of nilpotent Lie rings of order p^6, so as to classify the capable ones. It turns out that $\frac{5}{2}p-\frac{9}{2}+\frac{1}{2}\gcd(p-1,4)$ of these algebras are capable, and that they have a total of $\frac{1}{2}p^3+2p^2-5p+\frac{1}{2}+\frac{p}{2}\gcd(p-1,4)$ descendants of order p^7 and p-class 4.

If we write the parameters λ,μ,ν,ξ above in a matrix $A=\begin{pmatrix}\lambda & \mu\\ \nu & \xi\end{pmatrix}$ then A and B give isomorphic algebras if

$$B=\begin{pmatrix}\frac{\lambda}{\alpha\beta} & \frac{\mu}{\beta^2}\\ \frac{\nu}{\alpha^2} & \frac{\xi}{\alpha\beta}\end{pmatrix} \text{ or } \begin{pmatrix}\frac{\xi}{\alpha\beta} & \frac{\nu}{\beta^2}\\ \frac{\mu}{\alpha^2} & \frac{\lambda}{\alpha\beta}\end{pmatrix}.$$

So we can take $\lambda=1$ and $\mu=1$ or ω (or any other fixed integer which is not a square $\bmod p$). Given these values of λ,μ it turns out that the algebra is terminal unless $\xi=1$ or $\xi=\mu\nu$. We choose a set of representatives for the equivalence classes (as defined above) containing matrices of the form

$$\begin{pmatrix}1 & 1\\ \nu & 1\end{pmatrix}, \begin{pmatrix}1 & \omega\\ \nu & 1\end{pmatrix}, \begin{pmatrix}1 & 1\\ \nu & \nu\end{pmatrix}, \begin{pmatrix}1 & \omega\\ \nu & \omega\nu\end{pmatrix}.$$

Corresponding to these choices we obtain two families of descendants of order p^7 with the following presentations involving parameters y,z,t:

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-baa-\mu bab-ybaaa,\ pb+\nu baa+bab-zbaaa,\ pc-tbaaa,\ \text{class 3}\rangle\ (\mu=1,\omega), \tag{7.1763}$$

$$\langle a,b,c\,|\,ca-baa,\ cb,\ pa-baa-\mu bab-ybaaa,\ pb+\nu baa+\mu\nu bab-zbaaa,\ pc-tbaaa,\ \text{class 3}\rangle\ (\mu=1,\omega). \tag{7.1764}$$

For the family 7.1763, (y,z,t) and (y',z',t') gives isomorphic algebras if

$$(y',z',t')=(\pm y+\gamma t+\gamma\mu^{-1}+\varepsilon,\pm z+\varepsilon t-\nu\gamma\mu^{-1}+\nu\varepsilon-2\varepsilon\mu^{-1},t)$$

for some γ,ε, and where if $\mu\nu=1$ we need $\gamma=\mu\varepsilon$, or if

$$(y',z',t')=(-\alpha z-\gamma t+\varepsilon-\nu\gamma+2\gamma\mu^{-1},-\alpha^{-1}y-\varepsilon t-\gamma\mu^{-2}-\varepsilon\mu^{-1},-t-\nu+\mu^{-1})$$

for some γ,ε, where $\alpha^2\nu=\mu$, and where if $\mu\nu=1$ we need $\gamma=\mu\varepsilon$.

For the family 7.1764 we can assume that $\mu\nu\neq 1$. In this case (y,z,t) and (y',z',t') gives isomorphic algebras if

$$(y',z',t')=(\pm y+\mu\varepsilon t+2\mu,\pm z+\varepsilon t-2\mu\varepsilon,t),$$

for some ε, or (when $\mu\nu=-1$ and $p=1\bmod 4$, and $\alpha^2=-\mu^2$),

$$(y',z',t')=(-\alpha z-\mu\varepsilon t-2\varepsilon,\alpha^{-1}y-\varepsilon t+2\nu\varepsilon,t)$$

for some ε.

8.121 Descendants of 6.168

$p^3+2p^2+2p+2+\gcd(p-1,3)$ algebras

$$\langle a,b,c\,|\,ca-baa,cb,pa,pb,pc-baa,\ \text{class 4}\rangle \tag{7.1765}$$

$$\langle a,b,c\,|\,ca-baa,cb,pa-babb,pb,pc-baa,\ \text{class 4}\rangle \tag{7.1766}$$

$$\langle a,b,c\,|\,ca-baa,cb,pa-\omega babb,pb,pc-baa,\ \text{class 4}\rangle\ (p=1\bmod 3) \tag{7.1767}$$

$$\langle a,b,c\,|\,ca-baa,cb,pa-\omega^2 babb,pb,pc-baa,\ \text{class 4}\rangle\ (p=1\bmod 3) \tag{7.1768}$$

$$\langle a,b,c\,|\,ca-baa,cb,pa-xbabb,pb,pc-baa-babb,\ \text{class 4}\rangle\ (\text{all } x,\ x\sim -x) \tag{7.1769}$$

$$\langle a,b,c\,|\,ca-baa,cb,pa-xbabb,pb,pc-baa-\omega babb,\ \text{class 4}\rangle\ (\text{all } x,\ x\sim -x) \tag{7.1770}$$

$$\langle a,b,c\,|\,ca-baa,cb,pa-ybabb,pb-babb,pc-baa-xbabb,\ \text{class 4}\rangle\ (\text{all } x,\ y,\ y\sim -y) \tag{7.1771}$$

$$\langle a,b,c\,|\,ca-baa,cb,pa-ybabb,pb-\omega babb,pc-baa-xbabb,\ \text{class 4}\rangle\ (\text{all } x,\ y,\ y\sim -y) \tag{7.1772}$$

$$\langle a,b,c\,|\,ca-baa-babb,cb,pa-zbabb,pb-xbabb,pc-baa-ybabb,\ \text{class 4}\rangle\ (\text{all } x,\ y,z,\ z\sim -z) \tag{7.1773}$$

$$\langle a,b,c\,|\,ca-baa-\omega babb,cb,pa-zbabb,pb-xbabb,pc-baa-ybabb,\ \text{class 4}\rangle\ (\text{all } x,\ y,z,\ z\sim -z) \tag{7.1774}$$

8.122 Descendants of 6.172

$(p^4+p^2)/2$ algebras

$$\langle a,b,c \,|\, ca-baa, cb-ybaaa, pa-zbaaa, pb-tbaaa, pc-baa-bab-xbaaa, \text{ class } 4\rangle \text{ (all } y,z,t,\ x\neq 0,\ x\sim -x) \quad (7.1775)$$

$$\langle a,b,c \,|\, ca-baa, cb-ybaaa, pa-zbaaa, pb-tbaaa, pc-baa-bab, \text{ class } 4\rangle \text{ (all } y,z,t,\ (z,t)\sim(t,z)) \quad (7.1776)$$

8.123 Descendants of 6.173

Algebra 6.173 has $3p+3+(p^2+2p+3)\gcd(p-1,3)/2$ descendants of order p^7. Algebra 6.173 has presentation

$$\langle a,b,c \,|\, ca-bab,\ cb-\omega baa,\ pa,\ pb,\ pc, \text{ class } 3\rangle.$$

If L is a descendant of 6.173 of order p^7 then the commutator structure of L is given by the classification of nilpotent Lie algebras over $\mathbb{Z}_p$ of order p^7. The commutator relations take the form

$$ca = bab,\ cb = \omega baa,\ baab = \lambda baaa,\ babb = \mu baaa$$

for some parameters λ,μ. Provided $\alpha^2+2\alpha\beta\lambda+\beta^2\mu\neq 0$, the parameters λ,μ give the same algebra (up to isomorphism) as the pair

$$\frac{\pm(\omega\alpha\beta+\alpha^2\lambda+\omega\beta^2\lambda+\alpha\beta\mu)}{\alpha^2+2\alpha\beta\lambda+\beta^2\mu},\ \frac{\omega^2\beta^2+2\omega\alpha\beta\lambda+\alpha^2\mu}{\alpha^2+2\alpha\beta\lambda+\beta^2\mu}.$$

There are $p+2$ orbits of pairs λ,μ under this action.

We pick a set representative pairs λ,μ for these orbits, and get the following presentations for the descendants of 6.173 of order p^7:

$$\langle a,b,c \,|\, ca-bab,\ cb-\omega baa, baab-\lambda baaa,\ babb-\mu baaa,\ pa-ybaaa,\ pb-zbaaa,\ pc-tbaaa, \text{ class } 4\rangle. \quad (7.1777)$$

For each pair λ,μ we compute the set of pairs (α,β) which fixes λ,μ under one of the $\pm$ actions given above. It turns out that we need to treat the pair $\lambda=\mu=0$ separately from the other pairs.

If $\lambda=\mu=0$, then (y,z,t) and (y',z',t') give isomorphic algebras if

$$\begin{pmatrix} y' \\ z' \\ t' \end{pmatrix} = \frac{1}{\alpha^4}\begin{pmatrix} \pm\alpha & 0 & 0 \\ 0 & \alpha & \gamma \\ 0 & 0 & \pm\alpha^2 \end{pmatrix}\begin{pmatrix} y \\ z \\ t \end{pmatrix}$$

for some α,γ with $\alpha\neq 0$.

In all other cases we can assume that if $t\neq 0$ then $y=z=0$. For non-zero t we have

$$t\sim\frac{\pm t}{(\alpha^2+2\alpha\beta\lambda+\beta^2\mu)}$$

for some (α,β) which fixes λ,μ under one of the $\pm$ actions given above. If $t=0$ then (y,z) and (y',z') give isomorphic algebras if

$$\begin{pmatrix} y' \\ z' \end{pmatrix} = \frac{1}{(\alpha^2-\omega\beta^2)(\alpha^2+2\alpha\beta\lambda+\beta^2\mu)}\begin{pmatrix} \pm\alpha & \pm\beta \\ \omega\beta & \alpha \end{pmatrix}\begin{pmatrix} y \\ z \end{pmatrix}$$

for some (α,β) which fixes λ,μ under one of the $\pm$ actions given above.

8.124 Descendants of 6.174

$(p^3+p^2+p+1)/4$ algebras

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-bab-ybaaa, pb, pc-xbaaa, \text{ class } 4\rangle\ (x\neq 0,\ \text{all } y,\ x\sim -x,\ \ y\sim -y) \tag{7.1778}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-bab, pb-ybaaa, pc, \text{ class } 4\rangle\ (\text{all } y,\ y\sim -y) \tag{7.1779}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-zbaa-bab-ybaaa, pb, pc-xbaaa, \text{ class } 4\rangle\ (x,z\neq 0,\ \text{all } y, y\sim -y, z\sim -z) \tag{7.1780}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-zbaa-bab, pb-ybaaa, pc, \text{ class } 4\rangle\ (\text{all } y,\ z\neq 0,\ y\sim -y,\ z\sim -z) \tag{7.1781}$$

8.125 Descendants of 6.175

$(p^3+p^2+p+1)/4$ algebras

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-\omega bab-ybaaa, pb, pc-xbaaa, \text{ class } 4\rangle\ (x\neq 0,\ \text{all } y,\ x\sim -x,\ \ y\sim -y) \tag{7.1782}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-\omega bab, pb-ybaaa, pc, \text{ class } 4\rangle\ (\text{all } y,\ y\sim -y) \tag{7.1783}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-zbaa-\omega bab-ybaaa, pb, pc-xbaaa, \text{ class } 4\rangle\ (x,z\neq 0,\ \text{all } y,\ y\sim -y,\ z\sim -z) \tag{7.1784}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-zbaa-\omega bab, pb-ybaaa, pc, \text{ class } 4\rangle\ (\text{all } y,\ z\neq 0,\ y\sim -y,\ z\sim -z) \tag{7.1785}$$

8.126 Descendants of 6.176

$p(p-1)+p\gcd(p-1,4)/2$ algebras

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-baa, pb, pc-xbabb, \text{ class } 4\rangle\ (\text{all } x) \tag{7.1786}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-baa, pb-ybabb, pc-xbabb, \text{ class } 4\rangle\ (\text{all } x,\ y\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p) \tag{7.1787}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-baa-ybabb, pb, pc-babb, \text{ class } 4\rangle\ (y\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p) \tag{7.1788}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-baa-zbabb, pb-ybabb, pc-babb, \text{ class } 4\rangle\ (y,z\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p) \tag{7.1789}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-\omega baa, pb, pc-xbabb, \text{ class } 4\rangle\ (\text{all } x,\ p=1 \bmod 4) \tag{7.1790}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-\omega baa, pb-ybabb, pc-xbabb, \text{ class } 4\rangle\ (\text{all } x,\ \ y\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p,\ p=1 \bmod 4) \tag{7.1791}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-\omega baa-ybabb, pb, pc-\omega babb, \text{ class } 4\rangle\ (y\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p,\ p=1 \bmod 4) \tag{7.1792}$$

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-\omega baa-zbabb, pb-ybabb, pc-\omega babb, \text{ class } 4\rangle\ (y,z\neq 0,\ y\sim y' \text{ if } y^4=y'^4 \bmod p,\ p=1 \bmod 4) \tag{7.1793}$$

8.127 Descendants of 6.178

Algebra 6.178 has $(3p^2-1)/2$ descendants of order p^7. Algebra 6.178 has presentation

$$\langle a,b,c \,|\, ca-bab,\ cb-\omega baa,\ pa-\lambda baa-\mu bab,\ pb-\nu baa-\xi bab,\ pc,\ \text{class } 3\rangle,$$

where we write $A=\begin{pmatrix} \lambda & \mu \\ \nu & \xi \end{pmatrix}$, and A ranges over a set of representatives for the orbits of non-singular 2×2 matrices under the action

$$A \to \frac{1}{\det P} PAP^{-1}$$

as P ranges over non-singular matrices

$$P=\begin{pmatrix} \alpha & \beta \\ \pm\omega\beta & \pm\alpha \end{pmatrix}.$$

These algebras are terminal unless $\xi=-\lambda$. The number of orbits of non-singular matrices with $\xi=-\lambda$ is $(3p-1)/2$. The matrices split up into one orbit of size $p-1$ (matrices $\begin{pmatrix} 0 & y \\ \omega y & 0 \end{pmatrix}$), $p-1$ orbits of size $(p^2-1)/2$ (including two orbits of elements $\begin{pmatrix} x & y \\ -\omega y & -x \end{pmatrix}$), and $(p-1)/2$ orbits of size p^2-1. All orbits contain matrices where $\lambda=0$ or $\lambda=1$.

It is possible to choose orbit representatives of the following 6 types:

1. $\begin{pmatrix} 0 & 1 \\ \omega & 0 \end{pmatrix}$,
2. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ when $p=1 \bmod 4$,
3. $\begin{pmatrix} 0 & 1 \\ -\omega & 0 \end{pmatrix}$,
4. one representative $\begin{pmatrix} 1 & \mu \\ -\omega\mu & -1 \end{pmatrix}$ $(\mu\neq 0)$ which is not in the same orbit as $\begin{pmatrix} 0 & 1 \\ -\omega & 0 \end{pmatrix}$ when $p=3 \bmod 4$,
5. $p-3$ representatives $\begin{pmatrix} 0 & \mu \\ \nu & 0 \end{pmatrix}$ $(\nu\neq\pm\omega\mu)$, and
6. $(p-1)/2$ representatives of the form $\begin{pmatrix} 1 & \mu \\ \nu & -1 \end{pmatrix}$ $(\nu\neq-\omega\mu)$.

We then obtain the following presentations for the descendants of 6.178. The first four cases of the matrix A are straightforward.

$$\langle a,b,c \,|\, ca-bab, cb-\omega baa, pa-bab, pb-\omega baa, pc-xbaaa,\ \text{class } 4\rangle\ (\text{all } x,\ x\sim -x) \tag{7.1794}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baa, pa-baa, pb+bab, pc-xbaab,\ \text{class } 4\rangle\ (\text{all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.1795}$$

$$\langle a,b,c \,|\, ca-bab, cb-\omega baa, pa-bab, pb+\omega baa, pc-xbaaa,\ \text{class } 4\rangle\ (\text{all } x,\ x\sim -x) \tag{7.1796}$$

For the one matrix $\begin{pmatrix} 1 & \mu \\ -\omega\mu & -1 \end{pmatrix}$ $(\mu\neq 0)$ (when $p=3 \bmod 4$) we have:

$$\langle a,b,c \,|\, ca-bab, cb-\omega baa, pa-baa-\mu bab, pb+\omega\mu baa+bab, pc-xbaaa,\ \text{class } 4\rangle\ (\text{all } x,\ x\sim -x,\ p=3 \bmod 4) \tag{7.1797}$$

For the $p-3$ matrices $\begin{pmatrix} 0 & \mu \\ \nu & 0 \end{pmatrix}$ $(\nu\neq\pm\omega\mu)$ we have:

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-\mu bab,\ pb-\nu baa,\ pc-xbaaa,\ \text{class 4}\rangle\ (\text{all } x,\ x\sim -x).$$

But we have extra descendants if $(\omega\mu+2\nu)(2\omega\nu+\mu^{-1}\nu^2)$ is a square. If $\omega\mu+2\nu=0$ then we have,

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-\mu bab-xbaaa,\ pb-\nu baa,\ pc,\ \text{class 4}\rangle\ (x\neq 0,\ x\sim -x),$$

If $2\omega\nu+\mu^{-1}\nu^2=0$ we have

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-\mu bab,\ pb-\nu baa-xbaaa,\ pc,\ \text{class 4}\rangle\ (x\neq 0,\ x\sim -x),$$

and if $(\omega\mu+2\nu)(2\omega\nu+\mu^{-1}\nu^2)=y^2\neq 0$ then for one such value y we have

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-\mu bab,\ pb-\nu baa-xbaaa,\ pc-ybaaa,\ \text{class 4}\rangle\ (x\neq 0,\ x\sim -x).$$

We combine these four possibilities into a single presentation

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-\mu bab-xbaaa, pb-\nu baa-ybaaa, pc-zbaaa,\ \text{class 4}\rangle. \tag{7.1798}$$

For the $(p-1)/2$ matrices $\begin{pmatrix} 1 & \mu \\ \nu & -1 \end{pmatrix}$ $(\nu\neq -\omega\mu)$ the situation is even more complicated. First we have

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-baa-\mu bab,\ pb-\nu baa+bab,\ pc-xbaab,\ \text{class 4}\rangle\ (\text{all } x).$$

But if $(1+yz)\left(2\left(\omega y+z\right)^2+\omega(1+yz)\right)$ is a square we have an additional $p-1$ descendants. It is not that easy to prove, but $(1+yz)\left(2\left(\omega y+z\right)^2+\omega(1+yz)\right)$ cannot equal zero, under the assumption that A is not in the same orbit as a matrix with $(1,1)$ entry equal to zero. If $(1+yz)\left(2\left(\omega y+z\right)^2+\omega(1+yz)\right)=x^2\neq 0$, then if $x-\omega\mu-\nu=\omega\mu^2+2\mu\nu+1=0$ we have

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-baa-\mu bab-ybaab,\ pb-\nu baa+bab,\ pc-xbaab,\ \text{class 4}\rangle\ (y\neq 0,\ y\sim -y),$$

but if one of $x-\omega\mu-\nu$, $\omega\mu^2+2\mu\nu+1$ is non-zero we have

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-baa-\mu bab,\ pb-\nu baa+bab-ybaab,\ pc-xbaab,\ \text{class 4}\rangle\ (y\neq 0,\ y\sim -y).$$

And similarly for $-x$, $x+\omega\mu+\nu=\omega\mu^2+2\mu\nu+1=0$ we have

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-baa-\mu bab-ybaab,\ pb-\nu baa+bab,\ pc+xbaab,\ \text{class 4}\rangle\ (y\neq 0,\ y\sim -y),$$

but if one of $x+\omega\mu+\nu$, $\omega\mu^2+2\mu\nu+1$ is non-zero we have

$$\langle a,b,c\,|\,ca-bab,\ cb-\omega baa,\ pa-baa-\mu bab,\ pb-\nu baa+bab-ybaab,\ pc+xbaab,\ \text{class 4}\rangle\ (y\neq 0,\ y\sim -y).$$

We combine these five possibilities into a single presentation

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa, pa-baa-\mu bab-xbaab, pb-\nu baa+bab-ybaab, pc-zbaab,\ \text{class 4}\rangle. \tag{7.1799}$$

8.128 Descendants of 6.179

$(p^4+p^2)/2$ algebras

$$\langle a,b,c\,|\,ca-bab, cb-\omega baa-xbaaa, pa-ybaaa, pb-zbaaa, pc-bab-tbaaa,\ \text{class 4}\rangle\ (\text{all } x,\ y,z,t,\ y\sim -y,\ \text{if } y=0 \text{ then } t\sim -t) \tag{7.1800}$$

8.129 Descendants of 6.182

4 algebras

$$\langle a, b, c \,|\, ba, ca, cb, p^2b, pc, \text{ class } 4\rangle \tag{7.1801}$$

$$\langle a, b, c \,|\, ba, ca, cb - p^3a, p^2b, pc, \text{ class } 4\rangle \tag{7.1802}$$

$$\langle a, b, c \,|\, ba, ca - p^3a, cb, p^2b, pc, \text{ class } 4\rangle \tag{7.1803}$$

$$\langle a, b, c \,|\, ba - p^3a, ca, cb, p^2b, pc, \text{ class } 4\rangle \tag{7.1804}$$

8.130 Descendants of 6.183

2 algebras

$$\langle a, b, c \,|\, ba - p^2a, ca, cb, p^2b, pc, \text{ class } 4\rangle \tag{7.1805}$$

$$\langle a, b, c \,|\, ba - p^2a, ca, cb - p^3a, p^2b, pc, \text{ class } 4\rangle \tag{7.1806}$$

8.131 Descendants of 6.184

9 algebras

$$\langle a, b, c \,|\, baa, bab, ca, cb, pb, pc, \text{ class } 4\rangle \tag{7.1807}$$

$$\langle a, b, c \,|\, baa, bab, ca - p^3a, cb, pb, pc, \text{ class } 4\rangle \tag{7.1808}$$

$$\langle a, b, c \,|\, baa, bab, ca, cb - p^3a, pb, pc, \text{ class } 4\rangle \tag{7.1809}$$

$$\langle a, b, c \,|\, baa - p^3a, bab, ca, cb, pb, pc, \text{ class } 4\rangle \tag{7.1810}$$

$$\langle a, b, c \,|\, baa - p^3a, bab, ca, cb - p^3a, pb, pc, \text{ class } 4\rangle \tag{7.1811}$$

$$\langle a, b, c \,|\, baa, bab - p^3a, ca, cb, pb, pc, \text{ class } 4\rangle \tag{7.1812}$$

$$\langle a, b, c \,|\, baa, bab - p^3a, ca - p^3a, cb, pb, pc, \text{ class } 4\rangle \tag{7.1813}$$

$$\langle a, b, c \,|\, baa, bab - \omega p^3a, ca, cb, pb, pc, \text{ class } 4\rangle \tag{7.1814}$$

$$\langle a, b, c \,|\, baa, bab - \omega p^3a, ca - p^3a, cb, pb, pc, \text{ class } 4\rangle \tag{7.1815}$$

8.132 Descendants of 6.187

$11 + 4\gcd(p-1,3) + 2\gcd(p-1,4)$ algebras

$$\langle a,b,c \mid bab, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1816}$$

$$\langle a,b,c \mid bab - baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1817}$$

$$\langle a,b,c \mid bab - \omega baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1818}$$

$$\langle a,b,c \mid bab - \omega^2 baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1819}$$

$$\langle a,b,c \mid bab - \omega^3 baaa, ca, cb, p^2a - baaa, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1820}$$

$$\langle a,b,c \mid bab, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1821}$$

$$\langle a,b,c \mid bab - baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1822}$$

$$\langle a,b,c \mid bab - \omega baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1823}$$

$$\langle a,b,c \mid bab - \omega^2 baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1824}$$

$$\langle a,b,c \mid bab - \omega^3 baaa, ca, cb - baaa, p^2a - baaa, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1825}$$

$$\langle a,b,c \mid bab, ca, cb, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1826}$$

$$\langle a,b,c \mid bab, ca, cb, p^2a, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1827}$$

$$\langle a,b,c \mid bab, ca, cb, p^2a, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1828}$$

$$\langle a,b,c \mid bab, ca, cb, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1829}$$

$$\langle a,b,c \mid bab, ca, cb, p^2a, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1830}$$

$$\langle a,b,c \mid bab, ca, cb - baaa, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1831}$$

$$\langle a,b,c \mid bab, ca, cb - baaa, p^2a, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1832}$$

$$\langle a,b,c \mid bab, ca, cb - baaa, p^2a, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1833}$$

$$\langle a,b,c \mid bab, ca, cb - baaa, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1834}$$

$$\langle a,b,c \mid bab, ca, cb - baaa, p^2a, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1835}$$

$$\langle a,b,c \mid bab - baaa, ca, cb, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1836}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, p^2a, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1837}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, p^2a, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1838}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1839}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, p^2a, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1840}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1841}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, p^2a, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1842}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, p^2a, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1843}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1844}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, p^2a, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1845}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, p^2a, pb, pc - \omega baaa, \text{ class } 4\rangle \tag{7.1846}$$

8.133 Descendants of 6.188

$3 + 2\gcd(p-1, 3)$ algebras

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a - baaa, pb, pc, \text{ class } 4\rangle \tag{7.1847}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1848}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb - baaa, pc, \text{ class } 4\rangle \tag{7.1849}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1850}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1851}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb, pc - baaa, \text{ class } 4\rangle \tag{7.1852}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb - baaa, pc - baaa, \text{ class } 4\rangle \tag{7.1853}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb - \omega baaa, pc - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1854}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, p^2a, pb - \omega^2 baaa, pc - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1855}$$

8.134 Descendants of 6.189

$(5p+7)/2$ algebras

$$\langle a,b,c \,|\, bab, ca, cb, p^2a, pb-baa-xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1856}$$

$$\langle a,b,c \,|\, bab, ca, cb, p^2a-baaa, pb-baa, pc, \text{ class } 4\rangle \tag{7.1857}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb, p^2a-xbaaa, pb-baa, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.1858}$$

$$\langle a,b,c \,|\, bab, ca, cb, p^2a, pb-baa, pc-baaa, \text{ class } 4\rangle \tag{7.1859}$$

$$\langle a,b,c \,|\, bab, ca, cb, p^2a-baaa, pb-baa, pc-baaa, \text{ class } 4\rangle \tag{7.1860}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb, p^2a-xbaaa, pb-baa, pc-baaa, \text{ class } 4\rangle \text{ (all } x) \tag{7.1861}$$

8.135 Descendants of 6.190

2 algebras

$$\langle a,b,c \,|\, bab, ca, cb-baa, p^2a, pb-baa, pc, \text{ class } 4\rangle \tag{7.1862}$$

$$\langle a,b,c \,|\, bab, ca, cb-baa, p^2a-baaa, pb-baa, pc, \text{ class } 4\rangle \tag{7.1863}$$

8.136 Descendants of 6.191

$(5p+7)/2$ algebras

$$\langle a,b,c \,|\, bab, ca, cb, p^2a, pb-\omega baa-xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.1864}$$

$$\langle a,b,c \,|\, bab, ca, cb, p^2a-baaa, pb-\omega baa, pc, \text{ class } 4\rangle \tag{7.1865}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb, p^2a-xbaaa, pb-\omega baa, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.1866}$$

$$\langle a,b,c \,|\, bab, ca, cb, p^2a, pb-\omega baa, pc-baaa, \text{ class } 4\rangle \tag{7.1867}$$

$$\langle a,b,c \,|\, bab, ca, cb, p^2a-baaa, pb-\omega baa, pc-baaa, \text{ class } 4\rangle \tag{7.1868}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb, p^2a-xbaaa, pb-\omega baa, pc-baaa, \text{ class } 4\rangle \text{ (all } x) \tag{7.1869}$$

8.137 Descendants of 6.192

2 algebras

$$\langle a,b,c \,|\, bab, ca, cb-baa, p^2a, pb-\omega baa, pc, \text{ class } 4\rangle \tag{7.1870}$$

$$\langle a,b,c \,|\, bab, ca, cb-baa, p^2a-baaa, pb-\omega baa, pc, \text{ class } 4\rangle \tag{7.1871}$$

8.138 Descendants of 6.197

$11 + 4\gcd(p-1,3) + 2\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, baa, ca, cb, p^2a - babb, pb, pc, \text{ class } 4\rangle \tag{7.1872}$$

$$\langle a,b,c \,|\, baa, ca, cb, p^2a - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1873}$$

$$\langle a,b,c \,|\, baa, ca, cb, p^2a - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1874}$$

$$\langle a,b,c \,|\, baa, ca - babb, cb, p^2a - babb, pb, pc, \text{ class } 4\rangle \tag{7.1875}$$

$$\langle a,b,c \,|\, baa, ca - babb, cb, p^2a - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1876}$$

$$\langle a,b,c \,|\, baa, ca - babb, cb, p^2a - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1877}$$

$$\langle a,b,c \,|\, baa - babb, ca, cb, p^2a - babb, pb, pc, \text{ class } 4\rangle \tag{7.1878}$$

$$\langle a,b,c \,|\, baa - babb, ca, cb, p^2a - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1879}$$

$$\langle a,b,c \,|\, baa - babb, ca, cb, p^2a - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1880}$$

$$\langle a,b,c \,|\, baa - babb, ca - babb, cb, p^2a - babb, pb, pc, \text{ class } 4\rangle \tag{7.1881}$$

$$\langle a,b,c \,|\, baa - babb, ca - babb, cb, p^2a - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1882}$$

$$\langle a,b,c \,|\, baa - babb, ca - babb, cb, p^2a - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1883}$$

$$\langle a,b,c \,|\, baa, ca, cb, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1884}$$

$$\langle a,b,c \,|\, baa, ca, cb, p^2a, pb, pc - babb, \text{ class } 4\rangle \tag{7.1885}$$

$$\langle a,b,c \,|\, baa, ca, cb, p^2a, pb - babb, pc, \text{ class } 4\rangle \tag{7.1886}$$

$$\langle a,b,c \,|\, baa, ca - babb, cb, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1887}$$

$$\langle a,b,c \,|\, baa, ca - babb, cb, p^2a, pb, pc - babb, \text{ class } 4\rangle \tag{7.1888}$$

$$\langle a,b,c \,|\, baa, ca - babb, cb, p^2a, pb - babb, pc, \text{ class } 4\rangle \tag{7.1889}$$

$$\langle a,b,c \,|\, baa - babb, ca, cb, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1890}$$

$$\langle a,b,c \,|\, baa - babb, ca, cb, p^2a, pb, pc - babb, \text{ class } 4\rangle \tag{7.1891}$$

$$\langle a,b,c \,|\, baa - babb, ca, cb, p^2a, pb - babb, pc, \text{ class } 4\rangle \tag{7.1892}$$

$$\langle a, b, c \,|\, baa - babb, ca, cb, p^2a, pb - \omega babb, pc, \text{ class } 4\rangle \tag{7.1893}$$

$$\langle a, b, c \,|\, baa - babb, ca, cb, p^2a, pb - \omega^2 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1894}$$

$$\langle a, b, c \,|\, baa - babb, ca, cb, p^2a, pb - \omega^3 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1895}$$

$$\langle a, b, c \,|\, baa - babb, ca - babb, cb, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1896}$$

$$\langle a, b, c \,|\, baa - babb, ca - babb, cb, p^2a, pb, pc - babb, \text{ class } 4\rangle \tag{7.1897}$$

$$\langle a, b, c \,|\, baa - babb, ca - babb, cb, p^2a, pb, pc - \omega babb, \text{ class } 4\rangle \tag{7.1898}$$

$$\langle a, b, c \,|\, baa - babb, ca - babb, cb, p^2a, pb - babb, pc, \text{ class } 4\rangle \tag{7.1899}$$

$$\langle a, b, c \,|\, baa - babb, ca - babb, cb, p^2a, pb - \omega babb, pc, \text{ class } 4\rangle \tag{7.1900}$$

$$\langle a, b, c \,|\, baa - babb, ca - babb, cb, p^2a, pb - \omega^2 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1901}$$

$$\langle a, b, c \,|\, baa - babb, ca - babb, cb, p^2a, pb - \omega^3 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1902}$$

8.139 Descendants of 6.198

$4 + \gcd(p - 1, 3)$ algebras

$$\langle a, b, c \,|\, baa, ca - bab, cb, p^2a - babb, pb, pc, \text{ class } 4\rangle \tag{7.1903}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, p^2a - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1904}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, p^2a - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1905}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, p^2a, pb, pc, \text{ class } 4\rangle \tag{7.1906}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, p^2a, pb, pc - babb, \text{ class } 4\rangle \tag{7.1907}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, p^2a, pb - babb, pc, \text{ class } 4\rangle \tag{7.1908}$$

$$\langle a, b, c \,|\, baa, ca - bab, cb, p^2a, pb - babb, pc - babb, \text{ class } 4\rangle \tag{7.1909}$$

8.140 Descendants of 6.207

5 algebras

$$\langle a, b, c \,|\, baa, ca, cb, pb - ba, pc, \text{ class } 4\rangle \tag{7.1910}$$

$$\langle a, b, c \,|\, baa, ca, cb - p^3a, pb - ba, pc, \text{ class } 4\rangle \tag{7.1911}$$

$$\langle a, b, c \,|\, baa - p^3a, ca, cb, pb - ba, pc, \text{ class } 4\rangle \tag{7.1912}$$

$$\langle a, b, c \,|\, baa - p^3a, ca, cb - p^3a, pb - ba, pc, \text{ class } 4\rangle \tag{7.1913}$$

$$\langle a, b, c \,|\, baa, ca - p^3a, cb, pb - ba, pc, \text{ class } 4\rangle \tag{7.1914}$$

8.141 Descendants of 6.212

4 algebras

$$\langle a, b, c \,|\, ca, cb, pb, pc - ba, \text{ class } 4\rangle \tag{7.1915}$$

$$\langle a, b, c \,|\, ca - p^3a, cb, pb, pc - ba, \text{ class } 4\rangle \tag{7.1916}$$

$$\langle a, b, c \,|\, ca, cb - p^3a, pb, pc - ba, \text{ class } 4\rangle \tag{7.1917}$$

$$\langle a, b, c \,|\, ca, cb - \omega p^3a, pb, pc - ba, \text{ class } 4\rangle \tag{7.1918}$$

8.142 Descendants of 6.215

3 algebras

$$\langle a, b, c \,|\, ca - p^2a, cb, pb, pc - ba, \text{ class } 4\rangle \tag{7.1919}$$

$$\langle a, b, c \,|\, ca - p^2a, cb - p^3a, pb, pc - ba, \text{ class } 4\rangle \tag{7.1920}$$

$$\langle a, b, c \,|\, ca - p^2a, cb - \omega p^3a, pb, pc - ba, \text{ class } 4\rangle \tag{7.1921}$$

8.143 Descendants of 6.216

4 algebras

$$\langle a, b, c \,|\, baa, bab, ca, cb, pa, pb, \text{ class } 4\rangle \tag{7.1922}$$

$$\langle a, b, c \,|\, baa, bab, ca, cb - p^3c, pa, pb, \text{ class } 4\rangle \tag{7.1923}$$

$$\langle a, b, c \,|\, baa, bab - p^3c, ca, cb, pa, pb, \text{ class } 4\rangle \tag{7.1924}$$

$$\langle a, b, c \,|\, baa, bab - p^3c, ca - p^3c, cb, pa, pb, \text{ class } 4\rangle \tag{7.1925}$$

8.144 Descendants of 6.218

$11 + 4\gcd(p-1,3) + 2\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, bab, ca, cb, pa, pb, p^2c, \text{ class } 4\rangle \tag{7.1926}$$

$$\langle a,b,c \,|\, bab, ca, cb, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1927}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb, pa, pb, p^2c, \text{ class } 4\rangle \tag{7.1928}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1929}$$

$$\langle a,b,c \,|\, bab, ca, cb - baaa, pa, pb, p^2c, \text{ class } 4\rangle \tag{7.1930}$$

$$\langle a,b,c \,|\, bab, ca, cb - baaa, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1931}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb - baaa, pa, pb, p^2c, \text{ class } 4\rangle \tag{7.1932}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb - baaa, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1933}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb - \omega baaa, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1934}$$

$$\langle a,b,c \,|\, bab, ca, cb, pa, pb - baaa, p^2c, \text{ class } 4\rangle \tag{7.1935}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb, pa, pb - baaa, p^2c, \text{ class } 4\rangle \tag{7.1936}$$

$$\langle a,b,c \,|\, bab, ca, cb - baaa, pa, pb - baaa, p^2c, \text{ class } 4\rangle \tag{7.1937}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb - baaa, pa, pb - baaa, p^2c, \text{ class } 4\rangle \tag{7.1938}$$

$$\langle a,b,c \,|\, bab, ca, cb, pa, pb - \omega baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1939}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb, pa, pb - \omega baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1940}$$

$$\langle a,b,c \,|\, bab, ca, cb - baaa, pa, pb - \omega baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1941}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb - baaa, pa, pb - \omega baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1942}$$

$$\langle a,b,c \,|\, bab, ca, cb, pa, pb - \omega^2 baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1943}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb, pa, pb - \omega^2 baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1944}$$

$$\langle a,b,c \,|\, bab, ca, cb - baaa, pa, pb - \omega^2 baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1945}$$

$$\langle a,b,c \,|\, bab - baaa, ca, cb - baaa, pa, pb - \omega^2 baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1946}$$

$$\langle a, b, c \,|\, bab, ca, cb, pa - baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1947}$$

$$\langle a, b, c \,|\, bab, ca, cb - baaa, pa - baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1948}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, pa - baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1949}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, pa - baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1950}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, pa - \omega baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1951}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, pa - \omega baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1952}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, pa - \omega^2 baaa, pb, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1953}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, pa - \omega^2 baaa, pb, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1954}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb, pa - \omega^3 baaa, pb, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1955}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baaa, pa - \omega^3 baaa, pb, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.1956}$$

8.145 Descendants of 6.222

$2p + 5 + 3\gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa, pb, p^2c, \text{ class } 4\rangle \tag{7.1957}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa - baaa, pa, pb, p^2c, \text{ class } 4\rangle \tag{7.1958}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1959}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa - baaa, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1960}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baa, pa, pb, p^2c, \text{ class } 4\rangle \tag{7.1961}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baa, pa, pb, p^2c - baaa, \text{ class } 4\rangle \tag{7.1962}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baa, pa, pb, p^2c - \omega baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1963}$$

$$\langle a, b, c \,|\, bab - baaa, ca, cb - baa, pa, pb, p^2c - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1964}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa, pb - baaa, p^2c, \text{ class } 4\rangle \tag{7.1965}$$

$$\langle a, b, c \,|\, bab, ca, cb - baa, pa, pb - \omega baaa, p^2c, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1966}$$

$$\langle a,b,c \,|\, bab, ca, cb-baa, pa, pb-\omega^2 baaa, p^2c, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1967}$$

$$\langle a,b,c \,|\, bab, ca, cb-baa, pa-baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1968}$$

$$\langle a,b,c \,|\, bab, ca, cb-baa-baaa, pa, pb-xbaaa, p^2c, \text{ class } 4\rangle \ (x \neq 0) \tag{7.1969}$$

$$\langle a,b,c \,|\, bab, ca, cb-baa-baaa, pa-baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1970}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa-baaa, pa-baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1971}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa-\omega baaa, pb, p^2c, \text{ class } 4\rangle \tag{7.1972}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa-\omega^2 baaa, pb, p^2c, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1973}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa-\omega^3 baaa, pb, p^2c, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.1974}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa, pb-baaa, p^2c, \text{ class } 4\rangle \tag{7.1975}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa, pb-\omega baaa, p^2c, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1976}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa, pb-\omega^2 baaa, p^2c, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.1977}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa-xbaaa, pb-baaa, p^2c, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.1978}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa-xbaaa, pb-\omega baaa, p^2c, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1979}$$

$$\langle a,b,c \,|\, bab-baaa, ca, cb-baa, pa-xbaaa, pb-\omega^2 baaa, p^2c, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.1980}$$

8.146 Descendants of 6.228

$p+1$ algebras

$$\langle a,b,c \,|\, ca, cb-xp^3c, pa-ba, pb, \text{ class } 4\rangle \ (\text{all } x) \tag{7.1981}$$

$$\langle a,b,c \,|\, ca-p^3c, cb, pa-ba, pb, \text{ class } 4\rangle \tag{7.1982}$$

8.147 Descendants of 6.231

$p^2 + 5p + 14 + (p + 17)\gcd(p - 1, 3) + 2\gcd(p - 1, 4) + \gcd(p - 1, 7) + \gcd(p - 1, 8)$ algebras

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.1983}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1984}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1985}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1986}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.1987}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa - babb, pb - babb, pc, \text{ class } 4\rangle \tag{7.1988}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa - \omega babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1989}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa - \omega^2 babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1990}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa, cac, pa, pb, pc - babb, \text{ class } 4\rangle \tag{7.1991}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.1992}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa - babb, pb, pc, \text{ class } 4\rangle \tag{7.1993}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1994}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1995}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.1996}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa - babb, pb - babb, pc, \text{ class } 4\rangle \tag{7.1997}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa - \omega babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1998}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa - \omega^2 babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.1999}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa, pb, pc - babb, \text{ class } 4\rangle \tag{7.2000}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa, pb - babb, pc - babb, \text{ class } 4\rangle \tag{7.2001}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa, pb - \omega babb, pc - babb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2002}$$

$$\langle a, b, c \,|\, cb, baa, bac, caa - babb, cac, pa, pb, pc - \omega babb, \text{ class } 4\rangle \tag{7.2003}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa - babb, cac, pa, pb - babb, pc - \omega babb, \text{ class } 4\rangle \tag{7.2004}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa - babb, cac, pa, pb - \omega babb, pc - \omega babb, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2005}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.2006}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - babb, pb, pc, \text{ class } 4\rangle \tag{7.2007}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - \omega babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2008}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - \omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2009}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.2010}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - babb, pb - babb, pc, \text{ class } 4\rangle \tag{7.2011}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - \omega babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2012}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - \omega^2 babb, pb - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2013}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa, pb, pc - babb, \text{ class } 4\rangle \tag{7.2014}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - babb, pb, pc - babb, \text{ class } 4\rangle \tag{7.2015}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - \omega babb, pb, pc - babb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2016}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa, cac, pa - \omega^2 babb, pb, pc - babb, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2017}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa - babb, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.2018}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa - babb, cac, pa - babb, pb - xbabb, pc, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2019}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa - babb, cac, pa - \omega babb, pb - xbabb, pc, \text{ class } 4\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.2020}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa - babb, cac, pa - \omega^2 babb, pb - xbabb, pc, \text{ class } 4\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.2021}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa - babb, cac, pa, pb - babb, pc, \text{ class } 4\rangle \tag{7.2022}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa - babb, cac, pa, pb - \omega babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2023}$$

$$\langle a,b,c \,|\, cb, baa, bac - babb, caa - babb, cac, pa, pb - \omega^2 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2024}$$

$$\langle a,b,c \,|\, cb, baa, bac-babb, caa-babb, cac, pa, pb-xbabb, pc-babb, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.2025}$$

$$\langle a,b,c \,|\, cb, baa, bac-babb, caa-babb, cac, pa, pb-xbabb, pc-\omega babb, \text{ class } 4\rangle \text{ (all } x,\ x\sim -x) \tag{7.2026}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.2027}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa, pb-babb, pc, \text{ class } 4\rangle \tag{7.2028}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa, pb-\omega babb, pc, \text{ class } 4\rangle \tag{7.2029}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa, pb-\omega^2 babb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2030}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa, pb-\omega^3 babb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2031}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa-babb, pb, pc, \text{ class } 4\rangle \tag{7.2032}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa-\omega babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2033}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa-\omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2034}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa-babb, pb-xbabb, pc, \text{ class } 4\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p) \tag{7.2035}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa-\omega babb, pb-xbabb, pc, \text{ class } 4\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.2036}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa-\omega^2 babb, pb-xbabb, pc, \text{ class } 4\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.2037}$$

$$\langle a,b,c \,|\, cb, baa-babb, bac, caa, cac, pa, pb, pc-babb, \text{ class } 4\rangle \tag{7.2038}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa, pb, pc, \text{ class } 4\rangle \tag{7.2039}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa, pb, pc-babb, \text{ class } 4\rangle \tag{7.2040}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa, pb-babb, pc, \text{ class } 4\rangle \tag{7.2041}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa, pb-babb, pc-babb, \text{ class } 4\rangle \tag{7.2042}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-babb, pb, pc, \text{ class } 4\rangle \tag{7.2043}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega babb, pb, pc, \text{ class } 4\rangle \tag{7.2044}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^2 babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2045}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^3 babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2046}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^4 babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2047}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^5 babb, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2048}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-babb, pb, pc-babb, \text{ class } 4\rangle \tag{7.2049}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega babb, pb, pc-babb, \text{ class } 4\rangle \tag{7.2050}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^2 babb, pb, pc-babb, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2051}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^3 babb, pb, pc-babb, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2052}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^4 babb, pb, pc-babb, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2053}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^5 babb, pb, pc-babb, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2054}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-babb, pb-babb, pc, \text{ class } 4\rangle \tag{7.2055}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega babb, pb-babb, pc, \text{ class } 4\rangle \tag{7.2056}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^2 babb, pb-babb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2057}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^3 babb, pb-babb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2058}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^4 babb, pb-babb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2059}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^5 babb, pb-babb, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2060}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-babb, pb-babb, pc-xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^6 = x'^6 \bmod p) \tag{7.2061}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega babb, pb-babb, pc-xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^6 = x'^6 \bmod p) \tag{7.2062}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^2 babb, pb-babb, pc-xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^6 = x'^6 \bmod p, \ p=1 \bmod 3) \tag{7.2063}$$

$$\langle a,b,c \,|\, cb, baa, bac, caa, cac-babb, pa-\omega^3 babb, pb-babb, pc-xbabb, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^6 = x'^6 \bmod p, \ p=1 \bmod 3) \tag{7.2064}$$

$$\langle a,b,c\,|\,cb,baa,bac,caa,cac-babb,pa-\omega^4babb,pb-babb,pc-xbabb,\text{ class }4\rangle\ (x\neq 0, x\sim x' \text{ if } x^6=x'^6 \bmod p,\ p=1 \bmod 3) \tag{7.2065}$$

$$\langle a,b,c\,|\,cb,baa,bac,caa,cac-babb,pa-\omega^5babb,pb-babb,pc-xbabb,\text{ class }4\rangle\ (x\neq 0, x\sim x' \text{ if } x^6=x'^6 \bmod p,\ p=1 \bmod 3) \tag{7.2066}$$

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa,pb,pc,\text{ class }4\rangle \tag{7.2067}$$

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa,pb,pc-babb,\text{ class }4\rangle \tag{7.2068}$$

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa,pb,pc-xbabb,\text{ class }4\rangle\ (x=\omega^i, i=1,2,3,4,5,6,\ p=1 \bmod 7) \tag{7.2069}$$

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa,pb-xbabb,pc,\text{ class }4\rangle\ (x=\omega^i, 0\leq i<\gcd(p-1,8)) \tag{7.2070}$$

In the following presentation we have $x=\omega^i,\ 0\leq i<\gcd(p-1,8),\ y\neq 0,\ y\sim y'$ if $y^8=y'^8 \bmod p$:

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa,pb-xbabb,pc-ybabb,\text{ class }4\rangle \tag{7.2071}$$

In the following presentation we have $x=\omega^i,\ 0\leq i<2\gcd(p-1,3)$:

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa-xbabb,pb,pc,\text{ class }4\rangle \tag{7.2072}$$

In the following presentation we have $x=\omega^i,\ 0\leq i<2\gcd(p-1,3),\ y\neq 0,\ y\sim y'^6=y'^6 \bmod p$:

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa-xbabb,pb,pc-ybabb,\text{ class }4\rangle \tag{7.2073}$$

In the following presentation we have $x=\omega^i,\ 0\leq i<2\gcd(p-1,3),\ y\neq 0,\ y\sim y'^3=y'^3 \bmod p$, all $z,\ z\sim -z$:

$$\langle a,b,c\,|\,cb,baa-babb,bac,caa,cac-babb,pa-xbabb,pb-ybabb,pc-zbabb,\text{ class }4\rangle \tag{7.2074}$$

8.148 Descendants of 6.256

$20+(p+11)\gcd(p-1,3)+4\gcd(p-1,4)$ algebras

$$\langle a,b,c\,|\,cb,bab,bac,caa,cac,pa,pb,pc,\text{ class }4\rangle \tag{7.2075}$$

$$\langle a,b,c\,|\,cb,bab,bac,caa,cac,pa-baaa,pb,pc,\text{ class }4\rangle \tag{7.2076}$$

$$\langle a,b,c\,|\,cb,bab,bac,caa,cac,pa,pb-baaa,pc,\text{ class }4\rangle \tag{7.2077}$$

$$\langle a,b,c\,|\,cb,bab,bac,caa,cac,pa,pb-\omega baaa,pc,\text{ class }4\rangle\ (p=1 \bmod 3) \tag{7.2078}$$

$$\langle a,b,c\,|\,cb,bab,bac,caa,cac,pa,pb-\omega^2baaa,pc,\text{ class }4\rangle\ (p=1 \bmod 3) \tag{7.2079}$$

$$\langle a,b,c\,|\,cb,bab,bac,caa,cac,pa,pb,pc-baaa,\text{ class }4\rangle \tag{7.2080}$$

$$\langle a,b,c\,|\,cb-baaa,bab,bac,caa,cac,pa,pb,pc,\text{ class }4\rangle \tag{7.2081}$$

$$\langle a,b,c \,|\, cb-baaa, bab, bac, caa, cac, pa-baaa, pb, pc, \text{ class } 4\rangle \tag{7.2082}$$

$$\langle a,b,c \,|\, cb-baaa, bab, bac, caa, cac, pa, pb-baaa, pc, \text{ class } 4\rangle \tag{7.2083}$$

$$\langle a,b,c \,|\, cb-baaa, bab, bac, caa, cac, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2084}$$

$$\langle a,b,c \,|\, cb-baaa, bab, bac, caa, cac, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2085}$$

$$\langle a,b,c \,|\, cb-baaa, bab, bac, caa, cac, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.2086}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.2087}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa-baaa, pb, pc, \text{ class } 4\rangle \tag{7.2088}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa-\omega baaa, pb, pc, \text{ class } 4\rangle \tag{7.2089}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa-\omega^2 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2090}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa-\omega^3 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2091}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa, pb-baaa, pc, \text{ class } 4\rangle \tag{7.2092}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2093}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2094}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.2095}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.2096}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa-baaa, pb, pc, \text{ class } 4\rangle \tag{7.2097}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa-\omega baaa, pb, pc, \text{ class } 4\rangle \tag{7.2098}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa-\omega^2 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2099}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa-\omega^3 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2100}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa, pb-baaa, pc, \text{ class } 4\rangle \tag{7.2101}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2102}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2103}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.2104}$$

$$\langle a,b,c \,|\, cb-baaa, bab-baaa, bac, caa, cac, pa, pb, pc-\omega baaa, \text{ class } 4\rangle \tag{7.2105}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.2106}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa-baaa, pb, pc, \text{ class } 4\rangle \tag{7.2107}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb-baaa, pc, \text{ class } 4\rangle \tag{7.2108}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2109}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2110}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb-baaa, pc-baaa, \text{ class } 4\rangle \tag{7.2111}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb-\omega baaa, pc-baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2112}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb-\omega^2 baaa, pc-baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2113}$$

$$\langle a,b,c \,|\, cb, bab, bac-baaa, caa, cac, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.2114}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa-baaa, pb, pc, \text{ class } 4\rangle \tag{7.2115}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa-\omega baaa, pb, pc, \text{ class } 4\rangle \tag{7.2116}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb, pc, \text{ class } 4\rangle \tag{7.2117}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb-baaa, pc, \text{ class } 4\rangle \tag{7.2118}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2119}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2120}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.2121}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb-baaa, pc-baaa, \text{ class } 4\rangle \tag{7.2122}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb-\omega baaa, pc-baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2123}$$

$$\langle a,b,c \,|\, cb, bab, bac, caa, cac-baaa, pa, pb-\omega^2 baaa, pc-baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2124}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa, pb, pc, \text{ class } 4\rangle \tag{7.2125}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa-baaa, pb, pc, \text{ class } 4\rangle \tag{7.2126}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa-\omega baaa, pb, pc, \text{ class } 4\rangle \tag{7.2127}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa-\omega^2 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2128}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa-\omega^3 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2129}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.2130}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa, pb, pc-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2131}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa, pb, pc-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2132}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa, pb-baaa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x) \tag{7.2133}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa, pb-\omega baaa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.2134}$$

$$\langle a,b,c \,|\, cb, bab-baaa, bac, caa, cac-baaa, pa, pb-\omega^2 baaa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.2135}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa, pb, pc, \text{ class } 4\rangle \tag{7.2136}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa-baaa, pb, pc, \text{ class } 4\rangle \tag{7.2137}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa-\omega baaa, pb, pc, \text{ class } 4\rangle \tag{7.2138}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa-\omega^2 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2139}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa-\omega^3 baaa, pb, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2140}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa, pb, pc-baaa, \text{ class } 4\rangle \tag{7.2141}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa, pb, pc-\omega baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2142}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa, pb, pc-\omega^2 baaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2143}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa, pb-baaa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x) \tag{7.2144}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa, pb-\omega baaa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.2145}$$

$$\langle a,b,c \,|\, cb, bab-\omega baaa, bac, caa, cac-baaa, pa, pb-\omega^2 baaa, pc-xbaaa, \text{ class } 4\rangle \ (\text{all } x,\ x\sim -x,\ p=1 \bmod 3) \tag{7.2146}$$

8.149 Descendants of 6.261

$4p + 2 + (p^2 + 3p + 1)\gcd(p-1,3) + (p+1)\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb, pc, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2}) \tag{7.2147}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa - baaa, pb, pc, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2}) \tag{7.2148}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb, pc - baaa, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2}) \tag{7.2149}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - baaa, pc, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2}) \tag{7.2150}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2},\ p = 1 \bmod 3) \tag{7.2151}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2},\ p = 1 \bmod 3) \tag{7.2152}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - baaa, pc - baaa, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2}) \tag{7.2153}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega baaa, pc - baaa, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2},\ p = 1 \bmod 3) \tag{7.2154}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac - xbaaa, caa, cac, pa, pb - \omega^2 baaa, pc - baaa, \text{ class } 4\rangle \ (x \neq -1, -\frac{1}{2},\ p = 1 \bmod 3) \tag{7.2155}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb, pc, \text{ class } 4\rangle \tag{7.2156}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa - baaa, pb, pc, \text{ class } 4\rangle \tag{7.2157}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb, pc - baaa, \text{ class } 4\rangle \tag{7.2158}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa - baaa, pb, pc - baaa, \text{ class } 4\rangle \tag{7.2159}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb - baaa, pc, \text{ class } 4\rangle \tag{7.2160}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2161}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2162}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb - baaa, pc - baaa, \text{ class } 4\rangle \tag{7.2163}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega baaa, pc - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2164}$$

$$\langle a,b,c \,|\, cb - baa, bab, bac + baaa, caa, cac, pa, pb - \omega^2 baaa, pc - baaa, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2165}$$

$$\langle a,b,c\,|\,cb-baa,bab,bac+\frac{1}{2}baaa,caa,cac,pa,pb,pc,\text{ class }4\rangle \tag{7.2166}$$

$$\langle a,b,c\,|\,cb-baa,bab,bac+\frac{1}{2}baaa,caa,cac,pa-baaa,pb,pc,\text{ class }4\rangle \tag{7.2167}$$

$$\langle a,b,c\,|\,cb-baa,bab,bac+\frac{1}{2}baaa,caa,cac,pa,pb-baaa,pc,\text{ class }4\rangle \tag{7.2168}$$

$$\langle a,b,c\,|\,cb-baa,bab,bac+\frac{1}{2}baaa,caa,cac,pa,pb-\omega baaa,pc,\text{ class }4\rangle\ (p=1\bmod 3) \tag{7.2169}$$

$$\langle a,b,c\,|\,cb-baa,bab,bac+\frac{1}{2}baaa,caa,cac,pa,pb-\omega^2 baaa,pc,\text{ class }4\rangle\ (p=1\bmod 3) \tag{7.2170}$$

$$\langle a,b,c\,|\,cb-baa,bab,bac+\frac{1}{2}baaa,caa,cac,pa,pb,pc-baaa,\text{ class }4\rangle \tag{7.2171}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+zbaaa,caa,cac,pa,pb,pc,\text{ class }4\rangle \tag{7.2172}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa-baaa,pb,pc,\text{ class }4\rangle \tag{7.2173}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa-\omega baaa,pb,pc,\text{ class }4\rangle \tag{7.2174}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa-\omega^2 baaa,pb,pc,\text{ class }4\rangle\ (p=1\bmod 4) \tag{7.2175}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa-\omega^3 baaa,pb,pc,\text{ class }4\rangle\ (p=1\bmod 4) \tag{7.2176}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa,pb-baaa,pc,\text{ class }4\rangle \tag{7.2177}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+zbaaa,caa,cac,pa,pb-\omega baaa,pc,\text{ class }4\rangle\ (p=1\bmod 3) \tag{7.2178}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa,pb-\omega^2 baaa,pc,\text{ class }4\rangle\ (p=1\bmod 3) \tag{7.2179}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa,pb,pc-baaa,\text{ class }4\rangle \tag{7.2180}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa,pb,pc-\omega baaa,\text{ class }4\rangle\ (p=1\bmod 3) \tag{7.2181}$$

$$\langle a,b,c\,|\,cb-baa,bab-baaa,bac+\frac{1}{2}baaa,caa,cac,pa,pb,pc-\omega^2 baaa,\text{ class }4\rangle\ (p=1\bmod 3) \tag{7.2182}$$

$$\langle a,b,c\,|\,cb-baa,bab-xbaaa,bac,caa,cac,pa,pb,pc,\text{ class }4\rangle\ (\text{all }x) \tag{7.2183}$$

$$\langle a,b,c\,|\,cb-baa,bab-xbaaa,bac,caa,cac,pa-baaa,pb,pc,\text{ class }4\rangle\ (\text{all }x) \tag{7.2184}$$

$$\langle a,b,c\,|\,cb-baa,bab-xbaaa,bac,caa,cac,pa-\omega baaa,pb,pc,\text{ class }4\rangle\ (\text{all }x) \tag{7.2185}$$

$$\langle a,b,c\,|\,cb-baa,bab-xbaaa,bac,caa,cac,pa-\omega^2 baaa,pb,pc,\text{ class }4\rangle\ (\text{all }x,\ p=1\bmod 4) \tag{7.2186}$$

$$\langle a,b,c \,|\, cb-baa, bab-xbaaa, bac, caa, cac, pa-\omega^3 baaa, pb, pc, \text{ class } 4\rangle \text{ (all } x,\ p=1 \bmod 4) \tag{7.2187}$$

$$\langle a,b,c \,|\, cb-baa, bab-xbaaa, bac, caa, cac, pa, pb-baaa, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.2188}$$

$$\langle a,b,c \,|\, cb-baa, bab-xbaaa, bac, caa, cac, pa, pb-\omega baaa, pc, \text{ class } 4\rangle \text{ (all } x,\ p=1 \bmod 3) \tag{7.2189}$$

$$\langle a,b,c \,|\, cb-baa, bab-xbaaa, bac, caa, cac, pa, pb-\omega^2 baaa, pc, \text{ class } 4\rangle \text{ (all } x,\ p=1 \bmod 3) \tag{7.2190}$$

$$\langle a,b,c \,|\, cb-baa, bab-xbaaa, bac, caa, cac, pa, pb-ybaaa, pc-baaa, \text{ class } 4\rangle \text{ (all } x,\ y) \tag{7.2191}$$

$$\langle a,b,c \,|\, cb-baa, bab-xbaaa, bac, caa, cac, pa, pb-ybaaa, pc-\omega baaa, \text{ class } 4\rangle \text{ (all } x,\ y,\ p=1 \bmod 3) \tag{7.2192}$$

$$\langle a,b,c \,|\, cb-baa, bab-xbaaa, bac, caa, cac, pa, pb-ybaaa, pc-\omega^2 baaa, \text{ class } 4\rangle \text{ (all } x,\ y,\ p=1 \bmod 3) \tag{7.2193}$$

8.150 Descendants of 6.267

$p+3+3\gcd(p-1,3)+\gcd(p-1,4)+\gcd(p-1,5)$ algebras

$$\langle a,b,c \,|\, cb, caa, pa-ba, pb, pc, \text{ class } 4\rangle \tag{7.2194}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba-cacc, pb, pc, \text{ class } 4\rangle \tag{7.2195}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba-\omega cacc, pb, pc, \text{ class } 4\rangle \text{ } (p=1 \bmod 3) \tag{7.2196}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba-\omega^2 cacc, pb, pc, \text{ class } 4\rangle \text{ } (p=1 \bmod 3) \tag{7.2197}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba, pb, pc-cacc, \text{ class } 4\rangle \tag{7.2198}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba-cacc, pb, pc-cacc, \text{ class } 4\rangle \tag{7.2199}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba-\omega cacc, pb, pc-cacc, \text{ class } 4\rangle \text{ } (p=1 \bmod 3) \tag{7.2200}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba-\omega^2 cacc, pb, pc-cacc, \text{ class } 4\rangle \text{ } (p=1 \bmod 3) \tag{7.2201}$$

$$\langle a,b,c \,|\, cb, caa, pa-ba, pb-cacc, pc, \text{ class } 4\rangle \tag{7.2202}$$

$$\langle a,b,c \,|\, cb, caa-cacc, pa-ba, pb-cacc, pc, \text{ class } 4\rangle \tag{7.2203}$$

$$\langle a,b,c \,|\, cb, caa-cacc, pa-ba, pb-xcacc, pc, \text{ class } 4\rangle \text{ } (x=\omega^i,\ i=1,2,3,4,\ p=1 \bmod 5) \tag{7.2204}$$

$$\langle a,b,c \,|\, cb, caa-cacc, pa-ba, pb, pc, \text{ class } 4\rangle \tag{7.2205}$$

$$\langle a,b,c \,|\, cb, caa-cacc, pa-ba, pb, pc-cacc, \text{ class } 4\rangle \tag{7.2206}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba, pb, pc - \omega cacc, \text{ class } 4\rangle \tag{7.2207}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba, pb, pc - \omega^2 cacc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2208}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba, pb, pc - \omega^3 cacc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2209}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba - cacc, pb, pc, \text{ class } 4\rangle \tag{7.2210}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba - \omega cacc, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2211}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba - \omega cacc, pb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2212}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba - cacc, pb, pc - xcacc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.2213}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba - \omega cacc, pb, pc - xcacc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.2214}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - ba - \omega cacc, pb, pc - xcacc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.2215}$$

8.151 Descendants of 6.269

$(p + 5)/2$ algebras

$$\langle a, b, c \,|\, cb, caa, pa - ba - cac, pb, pc, \text{ class } 4\rangle \tag{7.2216}$$

$$\langle a, b, c \,|\, cb, caa, pa - ba - cac, pb - cacc, pc, \text{ class } 4\rangle \tag{7.2217}$$

$$\langle a, b, c \,|\, cb, caa, pa - ba - cac, pb - xcacc, pc - cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2218}$$

8.152 Descendants of 6.271

$(p + 5)/2$ algebras

$$\langle a, b, c \,|\, cb, caa, pa - ba - \omega cac, pb, pc, \text{ class } 4\rangle \tag{7.2219}$$

$$\langle a, b, c \,|\, cb, caa, pa - ba - \omega cac, pb - cacc, pc, \text{ class } 4\rangle \tag{7.2220}$$

$$\langle a, b, c \,|\, cb, caa, pa - ba - \omega cac, pb - xcacc, pc - cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2221}$$

8.153 Descendants of 6.273

$1 + \gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, cb, cac, pa-ba, pb-caa, pc, \text{ class } 4\rangle \tag{7.2222}$$

$$\langle a,b,c \,|\, cb, cac-caaa, pa-ba, pb-caa, pc, \text{ class } 4\rangle \tag{7.2223}$$

$$\langle a,b,c \,|\, cb, cac-\omega caaa, pa-ba, pb-caa, pc, \text{ class } 4\rangle \tag{7.2224}$$

$$\langle a,b,c \,|\, cb, cac-\omega^2 caaa, pa-ba, pb-caa, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2225}$$

$$\langle a,b,c \,|\, cb, cac-\omega^3 caaa, pa-ba, pb-caa, pc, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.2226}$$

$$\langle a,b,c \,|\, cb-caaa, cac, pa-ba, pb-caa, pc, \text{ class } 4\rangle \tag{7.2227}$$

$$\langle a,b,c \,|\, cb-\omega caaa, cac, pa-ba, pb-caa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2228}$$

$$\langle a,b,c \,|\, cb-\omega^2 caaa, cac, pa-ba, pb-caa, pc, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2229}$$

8.154 Descendants of 6.274

$p + 2 + (2p+3)\gcd(p-1,3) + \gcd(p-1,5)$ algebras

$$\langle a,b,c \,|\, cb, cac, pa-ba, pb, pc, \text{ class } 4\rangle \tag{7.2230}$$

$$\langle a,b,c \,|\, cb, cac, pa-ba, pb, pc-caaa, \text{ class } 4\rangle \tag{7.2231}$$

$$\langle a,b,c \,|\, cb, cac, pa-ba, pb, pc-\omega caaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2232}$$

$$\langle a,b,c \,|\, cb, cac, pa-ba, pb, pc-\omega^2 caaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2233}$$

$$\langle a,b,c \,|\, cb, cac-caaa, pa-ba, pb, pc, \text{ class } 4\rangle \tag{7.2234}$$

$$\langle a,b,c \,|\, cb, cac-caaa, pa-ba, pb, pc-caaa, \text{ class } 4\rangle \tag{7.2235}$$

$$\langle a,b,c \,|\, cb, cac-caaa, pa-ba, pb, pc-\omega caaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2236}$$

$$\langle a,b,c \,|\, cb, cac-caaa, pa-ba, pb, pc-\omega^2 caaa, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.2237}$$

$$\langle a,b,c \,|\, cb-caaa, cac, pa-ba, pb, pc-xcaaa, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2238}$$

$$\langle a,b,c \,|\, cb-\omega caaa, cac, pa-ba, pb, pc-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.2239}$$

$$\langle a,b,c \,|\, cb-\omega^2 caaa, cac, pa-ba, pb, pc-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.2240}$$

$$\langle a,b,c \,|\, cb - caaa, cac - caaa, pa - ba, pb, pc - xcaaa, \text{ class } 4\rangle \text{ (all } x) \tag{7.2241}$$

$$\langle a,b,c \,|\, cb - \omega caaa, cac - caaa, pa - ba, pb, pc - xcaaa, \text{ class } 4\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.2242}$$

$$\langle a,b,c \,|\, cb - \omega^2 caaa, cac - caaa, pa - ba, pb, pc - xcaaa, \text{ class } 4\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.2243}$$

$$\langle a,b,c \,|\, cb, cac, pa - ba, pb - caaa, pc, \text{ class } 4\rangle \tag{7.2244}$$

$$\langle a,b,c \,|\, cb - caaa, cac, pa - ba, pb - caaa, pc, \text{ class } 4\rangle \tag{7.2245}$$

$$\langle a,b,c \,|\, cb - \omega caaa, cac, pa - ba, pb - caaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2246}$$

$$\langle a,b,c \,|\, cb - \omega^2 caaa, cac, pa - ba, pb - caaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2247}$$

$$\langle a,b,c \,|\, cb, cac - caaa, pa - ba, pb - caaa, pc, \text{ class } 4\rangle \tag{7.2248}$$

$$\langle a,b,c \,|\, cb, cac - caaa, pa - ba, pb - xcaaa, pc, \text{ class } 4\rangle \ (x = \omega^i, i = 1,2,3,4,\ p = 1 \bmod 5) \tag{7.2249}$$

$$\langle a,b,c \,|\, cb - caaa, cac - caaa, pa - ba, pb - xcaaa, pc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.2250}$$

$$\langle a,b,c \,|\, cb - \omega caaa, cac - caaa, pa - ba, pb - xcaaa, pc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.2251}$$

$$\langle a,b,c \,|\, cb - \omega^2 caaa, cac - caaa, pa - ba, pb - xcaaa, pc, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.2252}$$

8.155 Descendants of 6.275

1 algebra

$$\langle a,b,c \,|\, cb, cac, pa - ba, pb, pc - caa, \text{ class } 4\rangle \tag{7.2253}$$

8.156 Descendants of 6.276

1 algebra

$$\langle a,b,c \,|\, cb, cac, pa - ba, pb, pc - \omega caa, \text{ class } 4\rangle \tag{7.2254}$$

8.157 Descendants of 6.277

p algebras

$$\langle a,b,c \,|\, cb - caa, cac - xcaaa, pa - ba, pb - caa, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.2255}$$

8.158 Descendants of 6.278

$(5p+3)/2$ algebras

$$\langle a,b,c \,|\, cb-caa, cac, pa-ba, pb, pc-xcaa, \text{ class } 4\rangle \ (x \neq 0,-1,2,\frac{1}{2}) \tag{7.2256}$$

$$\langle a,b,c \,|\, cb-caa, cac, pa-ba, pb, pc-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2257}$$

$$\langle a,b,c \,|\, cb-caa, cac, pa-ba, pb-caaa, pc-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2258}$$

$$\langle a,b,c \,|\, cb-caa, cac, pa-ba, pb, pc+caa, \text{ class } 4\rangle \tag{7.2259}$$

$$\langle a,b,c \,|\, cb-caa, cac, pa-ba, pb-caaa, pc+caa, \text{ class } 4\rangle \tag{7.2260}$$

$$\langle a,b,c \,|\, cb-caa, cac, pa-ba, pb, pc-2caa-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2261}$$

$$\langle a,b,c \,|\, cb-caa, cac, pa-ba, pb, pc-\frac{1}{2}caa, \text{ class } 4\rangle \tag{7.2262}$$

$$\langle a,b,c \,|\, cb-caa, cac-caaa, pa-ba, pb, pc-\frac{1}{2}caa, \text{ class } 4\rangle \tag{7.2263}$$

8.159 Descendants of 6.279

p algebras

$$\langle a,b,c \,|\, cb-\omega caa, cac-xcaaa, pa-ba, pb-caa, pc, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2264}$$

8.160 Descendants of 6.280

$(5p+3)/2$ algebras

$$\langle a,b,c \,|\, cb-\omega caa, cac, pa-ba, pb, pc-xcaa, \text{ class } 4\rangle \ (x \neq 0,-\omega,2\omega,\frac{\omega}{2}) \tag{7.2265}$$

$$\langle a,b,c \,|\, cb-\omega caa, cac, pa-ba, pb, pc-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2266}$$

$$\langle a,b,c \,|\, cb-\omega caa, cac, pa-ba, pb-caaa, pc-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2267}$$

$$\langle a,b,c \,|\, cb-\omega caa, cac, pa-ba, pb, pc+\omega caa, \text{ class } 4\rangle \tag{7.2268}$$

$$\langle a,b,c \,|\, cb-\omega caa, cac, pa-ba, pb-caaa, pc+\omega caa, \text{ class } 4\rangle \tag{7.2269}$$

$$\langle a,b,c \,|\, cb-\omega caa, cac, pa-ba, pb, pc-2\omega caa-xcaaa, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2270}$$

$$\langle a,b,c \,|\, cb-\omega caa, cac, pa-ba, pb, pc-\frac{\omega}{2}caa, \text{ class } 4\rangle \tag{7.2271}$$

$$\langle a,b,c \,|\, cb-\omega caa, cac-caaa, pa-ba, pb, pc-\frac{\omega}{2}caa, \text{ class } 4\rangle \tag{7.2272}$$

8.161 Descendants of 6.281

$2p^2 + 4p + 4 + 2\gcd(p-1, 3)$ algebras

$$\langle a, b, c \,|\, cb, caa, pa, pb - ba, pc, \text{ class } 4\rangle \tag{7.2273}$$

$$\langle a, b, c \,|\, cb, caa, pa - cacc, pb - ba, pc, \text{ class } 4\rangle \tag{7.2274}$$

$$\langle a, b, c \,|\, cb, caa, pa - \omega cacc, pb - ba, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2275}$$

$$\langle a, b, c \,|\, cb, caa, pa - \omega^2 cacc, pb - ba, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2276}$$

$$\langle a, b, c \,|\, cb, caa, pa - xcacc, pb - ba, pc - cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2277}$$

$$\langle a, b, c \,|\, cb, caa, pa - xcacc, pb - ba, pc - \omega cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2278}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - xcacc, pb - ba, pc - ycacc, \text{ class } 4\rangle \ (\text{all } x,\ y, x \sim -x) \tag{7.2279}$$

$$\langle a, b, c \,|\, cb, caa - \omega cacc, pa - xcacc, pb - ba, pc - ycacc, \text{ class } 4\rangle \ (\text{all } x,\ y,\ x \sim -x) \tag{7.2280}$$

$$\langle a, b, c \,|\, cb, caa, pa, pb - ba - cacc, pc, \text{ class } 4\rangle \tag{7.2281}$$

$$\langle a, b, c \,|\, cb, caa, pa - cacc, pb - ba - cacc, pc, \text{ class } 4\rangle \tag{7.2282}$$

$$\langle a, b, c \,|\, cb, caa, pa - \omega cacc, pb - ba - cacc, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2283}$$

$$\langle a, b, c \,|\, cb, caa, pa - \omega^2 cacc, pb - ba - cacc, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2284}$$

$$\langle a, b, c \,|\, cb, caa, pa - xcacc, pb - ba - cacc, pc - cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2285}$$

$$\langle a, b, c \,|\, cb, caa, pa - xcacc, pb - ba - cacc, pc - \omega cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2286}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - xcacc, pb - ba - cacc, pc - ycacc, \text{ class } 4\rangle \ (\text{all } x,\ y, x \sim -x) \tag{7.2287}$$

$$\langle a, b, c \,|\, cb, caa - \omega cacc, pa - xcacc, pb - ba - cacc, pc - ycacc, \text{ class } 4\rangle \ (\text{all } x,\ y,\ x \sim -x) \tag{7.2288}$$

8.162 Descendants of 6.282

$p^3 + p^2 + 2p + 2 + \gcd(p-1, 3)$ algebras

$$\langle a, b, c \,|\, cb, caa, pa, pb - ba - cac, pc, \text{ class } 4\rangle \tag{7.2289}$$

$$\langle a, b, c \,|\, cb, caa, pa - cacc, pb - ba - cac, pc, \text{ class } 4\rangle \tag{7.2290}$$

$$\langle a, b, c \,|\, cb, caa, pa - \omega cacc, pb - ba - cac, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2291}$$

$$\langle a, b, c \,|\, cb, caa, pa - \omega^2 cacc, pb - ba - cac, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2292}$$

$$\langle a, b, c \,|\, cb, caa, pa - xcacc, pb - ba - cac, pc - cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2293}$$

$$\langle a, b, c \,|\, cb, caa, pa - xcacc, pb - ba - cac, pc - \omega cacc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2294}$$

$$\langle a, b, c \,|\, cb, caa - cacc, pa - xcacc, pb - ba - cac, pc - ycacc, \text{ class } 4\rangle \ (\text{all } x,\ y,\ x \sim -x) \tag{7.2295}$$

$$\langle a, b, c \,|\, cb, caa - \omega cacc, pa - xcacc, pb - ba - cac, pc - ycacc, \text{ class } 4\rangle \ (\text{all } x,\ y,\ x \sim -x) \tag{7.2296}$$

$$\langle a, b, c \,|\, cb, caa - xcacc, pa - ycacc, pb - ba - cac - cacc, pc - zcacc, \text{ class } 4\rangle \ (\text{all } x,\ y, z) \tag{7.2297}$$

8.163 Descendants of 6.289

$6p$ algebras

$$\langle a, b, c \,|\, cb, cac, pa, pb - ba, pc - xcaaa, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2298}$$

$$\langle a, b, c \,|\, cb, cac - caaa, pa, pb - ba, pc - xcaaa, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2299}$$

$$\langle a, b, c \,|\, cb - caaa, cac, pa, pb - ba, pc - xcaaa, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2300}$$

$$\langle a, b, c \,|\, cb - caaa, cac - caaa, pa, pb - ba, pc - xcaaa, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2301}$$

$$\langle a, b, c \,|\, cb, cac, pa - caaa, pb - ba, pc, \text{ class } 4\rangle \tag{7.2302}$$

$$\langle a, b, c \,|\, cb - caaa, cac, pa - caaa, pb - ba, pc, \text{ class } 4\rangle \tag{7.2303}$$

$$\langle a, b, c \,|\, cb, cac - caaa, pa - xcaaa, pb - ba, pc, \text{ class } 4\rangle \ (x \neq 0) \tag{7.2304}$$

$$\langle a, b, c \,|\, cb - caaa, cac - caaa, pa - xcaaa, pb - ba, pc, \text{ class } 4\rangle \ (x \neq 0) \tag{7.2305}$$

8.164 Descendants of 6.290

$2p^2 + p$ algebras

$$\langle a, b, c \,|\, cb - caa - xcaaa, cac, pa, pb - ba, pc - ycaaa, \text{ class } 4\rangle \text{ (all } x,\ y) \tag{7.2306}$$

$$\langle a, b, c \,|\, cb - caa - xcaaa, cac, pa - caaa, pb - ba, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.2307}$$

$$\langle a, b, c \,|\, cb - caa, cac - caaa, pa - xcaaa, pb - ba, pc - ycaaa, \text{ class } 4\rangle \text{ (all } x,\ y) \tag{7.2308}$$

8.165 Descendants of 6.294

$p + 4 + 5\gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b, c \,|\, cb, baa, pa, pb - ca, pc - babb, \text{ class } 4\rangle \tag{7.2309}$$

$$\langle a, b, c \,|\, cb, baa, pa, pb - ca, pc - \omega babb, \text{ class } 4\rangle \tag{7.2310}$$

$$\langle a, b, c \,|\, cb, baa, pa, pb - ca, pc, \text{ class } 4\rangle \tag{7.2311}$$

$$\langle a, b, c \,|\, cb, baa, pa - babb, pb - ca, pc, \text{ class } 4\rangle \tag{7.2312}$$

$$\langle a, b, c \,|\, cb, baa, pa - \omega babb, pb - ca, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2313}$$

$$\langle a, b, c \,|\, cb, baa, pa - \omega^2 babb, pb - ca, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2314}$$

$$\langle a, b, c \,|\, cb, baa, pa, pb - ca - babb, pc, \text{ class } 4\rangle \tag{7.2315}$$

$$\langle a, b, c \,|\, cb, baa, pa - babb, pb - ca - babb, pc, \text{ class } 4\rangle \tag{7.2316}$$

$$\langle a, b, c \,|\, cb, baa, pa - \omega babb, pb - ca - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2317}$$

$$\langle a, b, c \,|\, cb, baa, pa - \omega^2 babb, pb - ca - babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2318}$$

$$\langle a, b, c \,|\, cb, baa - babb, pa, pb - ca, pc - babb, \text{ class } 4\rangle \tag{7.2319}$$

$$\langle a, b, c \,|\, cb, baa - babb, pa, pb - ca, pc - \omega babb, \text{ class } 4\rangle \tag{7.2320}$$

$$\langle a, b, c \,|\, cb, baa - babb, pa, pb - ca, pc - xbabb, \text{ class } 4\rangle \ (x = \omega^i,\ i = 2, 3, 4, 5,\ p = 1 \bmod 3) \tag{7.2321}$$

$$\langle a, b, c \,|\, cb, baa - babb, pa, pb - ca, pc, \text{ class } 4\rangle \tag{7.2322}$$

$$\langle a, b, c \,|\, cb, baa - babb, pa, pb - ca - babb, pc, \text{ class } 4\rangle \tag{7.2323}$$

$$\langle a, b, c \,|\, cb, baa - babb, pa, pb - ca - \omega babb, pc, \text{ class } 4\rangle \tag{7.2324}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa, pb - ca - \omega^2 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2325}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa, pb - ca - \omega^3 babb, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2326}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa - babb, pb - ca, pc, \text{ class } 4\rangle \tag{7.2327}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa - \omega babb, pb - ca, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2328}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa - \omega^2 babb, pb - ca, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2329}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa - babb, pb - ca - xbabb, pc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.2330}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa - \omega babb, pb - ca - xbabb, pc, \text{ class } 4\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.2331}$$

$$\langle a,b,c \,|\, cb, baa - babb, pa - \omega^2 babb, pb - ca - xbabb, pc, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.2332}$$

8.166 Descendants of 6.295

$(p+5)/2$ algebras

$$\langle a,b,c \,|\, cb, baa, pa - bab - xbabb, pb - ca, pc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2333}$$

$$\langle a,b,c \,|\, cb, baa, pa - bab, pb - ca, pc - babb, \text{ class } 4\rangle \tag{7.2334}$$

$$\langle a,b,c \,|\, cb, baa, pa - bab, pb - ca, pc - \omega babb, \text{ class } 4\rangle \tag{7.2335}$$

8.167 Descendants of 6.296

$(p+5)/2$ algebras

$$\langle a,b,c \,|\, cb, baa, pa - \omega bab - xbabb, pb - ca, pc, \text{ class } 4\rangle \ (\text{all } x,\ x \sim -x) \tag{7.2336}$$

$$\langle a,b,c \,|\, cb, baa, pa - \omega bab, pb - ca, pc - babb, \text{ class } 4\rangle \tag{7.2337}$$

$$\langle a,b,c \,|\, cb, baa, pa - \omega bab, pb - ca, pc - \omega babb, \text{ class } 4\rangle \tag{7.2338}$$

8.168 Descendants of 6.297

$2p + p\gcd(p-1,3)$ algebras

$$\langle a,b,c \,|\, cb, baa, pa, pb - ca - bab, pc - xbabb, \text{ class } 4\rangle \text{ (all } x) \tag{7.2339}$$

$$\langle a,b,c \,|\, cb, baa, pa - babb, pb - ca - bab, pc - xbabb, \text{ class } 4\rangle \text{ (all } x) \tag{7.2340}$$

$$\langle a,b,c \,|\, cb, baa, pa - \omega babb, pb - ca - bab, pc - xbabb, \text{ class } 4\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.2341}$$

$$\langle a,b,c \,|\, cb, baa, pa - \omega^2 babb, pb - ca - bab, pc - xbabb, \text{ class } 4\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.2342}$$

$$\langle a,b,c \,|\, cb, baa, pa - xbabb, pb - ca - bab - babb, pc + 2babb, \text{ class } 4\rangle \text{ (all } x) \tag{7.2343}$$

8.169 Descendants of 6.298

$(3p-1)/2$ algebras

$$\langle a,b,c \,|\, cb, baa, pa - bab, pb - ca - bab, pc - xbabb, \text{ class } 4\rangle \text{ (all } x) \tag{7.2344}$$

$$\langle a,b,c \,|\, cb, baa, pa - bab - xbabb, pb - ca - bab, pc - 2babb, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.2345}$$

8.170 Descendants of 6.299

$(3p-1)/2$ algebras

$$\langle a,b,c \,|\, cb, baa, pa - \omega bab, pb - ca - bab, pc - xbabb, \text{ class } 4\rangle \text{ (all } x) \tag{7.2346}$$

$$\langle a,b,c \,|\, cb, baa, pa - \omega bab - xbabb, pb - ca - bab, pc - 2babb, \text{ class } 4\rangle \ (x \neq 0, x \sim -x) \tag{7.2347}$$

8.171 Descendants of 6.303

$p^2 + p + (p+1)\gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, cb, bab - baa, pa, pb - ca, pc, \text{ class } 4\rangle \tag{7.2348}$$

$$\langle a,b,c \,|\, cb, bab - baa, pa, pb - ca - baaa, pc, \text{ class } 4\rangle \tag{7.2349}$$

$$\langle a,b,c \,|\, cb, bab - baa, pa, pb - ca - \omega baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2350}$$

$$\langle a,b,c \,|\, cb, bab - baa, pa, pb - ca - \omega^2 baaa, pc, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.2351}$$

$$\langle a,b,c \,|\, cb, bab - baa, pa - baaa, pb - ca - xbaaa, pc, \text{ class } 4\rangle \text{ (all } x) \tag{7.2352}$$

$$\langle a,b,c \,|\, cb, bab - baa, pa - \omega baaa, pb - ca - xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.2353}$$

$$\langle a,b,c \,|\, cb, bab - baa, pa - \omega^2 baaa, pb - ca - xbaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ p = 1 \bmod 3) \tag{7.2354}$$

$$\langle a,b,c \,|\, cb, bab - baa - baaa, pa - xbaaa, pb - ca - ybaaa, pc, \text{ class } 4\rangle \text{ (all } x,\ y) \tag{7.2355}$$

$$\langle a,b,c\,|\,cb, bab-baa, pa, pb-ca, pc-baaa, \text{ class } 4\rangle \tag{7.2356}$$

$$\langle a,b,c\,|\,cb, bab-baa, pa, pb-ca, pc-\omega baaa, \text{ class } 4\rangle \tag{7.2357}$$

$$\langle a,b,c\,|\,cb, bab-baa, pa, pb-ca, pc-\omega^2 baaa, \text{ class } 4\rangle\ (p=1 \bmod 4) \tag{7.2358}$$

$$\langle a,b,c\,|\,cb, bab-baa, pa, pb-ca, pc-\omega^3 baaa, \text{ class } 4\rangle\ (p=1 \bmod 4) \tag{7.2359}$$

$$\langle a,b,c\,|\,cb, bab-baa-baaa, pa, pb-ca, pc-xbaaa, \text{ class } 4\rangle\ (x\neq 0) \tag{7.2360}$$

8.172 Descendants of 6.304

$p(p+1)/2$ algebras

$$\langle a,b,c\,|\,cb, bab-baa, pa, pb-ca-baa-ybaaa, pc-xbaaa, \text{ class } 4\rangle\ (x\neq 0,\ \text{all } y,\ y\sim -y) \tag{7.2361}$$

$$\langle a,b,c\,|\,cb, bab-baa, pa-ybaaa, pb-ca-baa, pc, \text{ class } 4\rangle\ (\text{all } y,\ y\sim -y) \tag{7.2362}$$

8.173 Descendants of 6.305

$p(p+1)/2$ algebras

$$\langle a,b,c\,|\,cb, bab-baa, pa, pb-ca-\omega baa-ybaaa, pc-xbaaa, \text{ class } 4\rangle\ (x\neq 0,\ \text{all } y, y\sim -y) \tag{7.2363}$$

$$\langle a,b,c\,|\,cb, bab-baa, pa-ybaaa, pb-ca-\omega baa, pc, \text{ class } 4\rangle\ (\text{all } y,\ y\sim -y) \tag{7.2364}$$

8.174 Descendants of 6.312

$4p+2+2\gcd(p-1,3)+4\gcd(p-1,4)$ algebras

$$\langle a,b,c\,|\,cb, bab, pa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2365}$$

$$\langle a,b,c\,|\,cb, bab, pa-baaa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2366}$$

$$\langle a,b,c\,|\,cb-baaa, bab, pa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2367}$$

$$\langle a,b,c\,|\,cb-baaa, bab, pa-baaa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2368}$$

$$\langle a,b,c\,|\,cb-baaa, bab, pa-\omega baaa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2369}$$

$$\langle a,b,c\,|\,cb-baaa, bab, pa-xbaaa, pb-ca, pc, \text{ class } 4\rangle\ (x=\omega^i,\ i=2,3,4,5,\ p=1 \bmod 3) \tag{7.2370}$$

$$\langle a,b,c\,|\,cb, bab-baaa, pa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2371}$$

$$\langle a,b,c\,|\,cb, bab-baaa, pa-baaa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2372}$$

$$\langle a,b,c\,|\,cb, bab-baaa, pa-\omega baaa, pb-ca, pc, \text{ class } 4\rangle \tag{7.2373}$$

$$\langle a, b, c \,|\, cb, bab - baaa, pa - \omega^2 baaa, pb - ca, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2374}$$

$$\langle a, b, c \,|\, cb, bab - baaa, pa - \omega^3 baaa, pb - ca, pc, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2375}$$

$$\langle a, b, c \,|\, cb - baaa, bab - baaa, pa - xbaaa, pb - ca, pc, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2376}$$

$$\langle a, b, c \,|\, cb - \omega baaa, bab - baaa, pa - xbaaa, pb - ca, pc, \text{ class } 4\rangle \ (\text{all } x) \tag{7.2377}$$

$$\langle a, b, c \,|\, cb, bab, pa, pb - ca, pc - baaa, \text{ class } 4\rangle \tag{7.2378}$$

$$\langle a, b, c \,|\, cb, bab, pa, pb - ca, pc - \omega baaa, \text{ class } 4\rangle \tag{7.2379}$$

$$\langle a, b, c \,|\, cb, bab, pa, pb - ca, pc - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2380}$$

$$\langle a, b, c \,|\, cb, bab, pa, pb - ca, pc - \omega^3 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2381}$$

$$\langle a, b, c \,|\, cb - baaa, bab, pa, pb - ca, pc - baaa, \text{ class } 4\rangle \tag{7.2382}$$

$$\langle a, b, c \,|\, cb - baaa, bab, pa, pb - ca, pc - \omega baaa, \text{ class } 4\rangle \tag{7.2383}$$

$$\langle a, b, c \,|\, cb - baaa, bab, pa, pb - ca, pc - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2384}$$

$$\langle a, b, c \,|\, cb - baaa, bab, pa, pb - ca, pc - \omega^3 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2385}$$

$$\langle a, b, c \,|\, cb, bab - baaa, pa, pb - ca, pc - baaa, \text{ class } 4\rangle \tag{7.2386}$$

$$\langle a, b, c \,|\, cb, bab - baaa, pa, pb - ca, pc - \omega baaa, \text{ class } 4\rangle \tag{7.2387}$$

$$\langle a, b, c \,|\, cb, bab - baaa, pa, pb - ca, pc - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2388}$$

$$\langle a, b, c \,|\, cb, bab - baaa, pa, pb - ca, pc - \omega^3 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2389}$$

$$\langle a, b, c \,|\, cb - baaa, bab - baaa, pa, pb - ca, pc - xbaaa, \text{ class } 4\rangle \ (x \neq 0) \tag{7.2390}$$

$$\langle a, b, c \,|\, cb - \omega baaa, bab - baaa, pa, pb - ca, pc - xbaaa, \text{ class } 4\rangle \ (x \neq 0) \tag{7.2391}$$

8.175 Descendants of 6.313

$p + \gcd(p-1,4) + \gcd(p-1,5)$ algebras

$$\langle a,b,c \,|\, cb - baa, bab, pa, pb - ca, pc, \text{ class } 4\rangle \tag{7.2392}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa, pb - ca, pc - baaa, \text{ class } 4\rangle \tag{7.2393}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa, pb - ca, pc - \omega baaa, \text{ class } 4\rangle \tag{7.2394}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa, pb - ca, pc - \omega^2 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2395}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa, pb - ca, pc - \omega^3 baaa, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.2396}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa - baaa, pb - ca, pc, \text{ class } 4\rangle \tag{7.2397}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa - ybaaa, pb - ca, pc, \text{ class } 4\rangle \ (y = \omega^i,\ i = 1,2,3,4,\ p = 1 \bmod 5) \tag{7.2398}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa - baaa, pb - ca, pc - xbaaa, \text{ class } 4\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^5 = x'^5 \bmod p) \tag{7.2399}$$

$$\langle a,b,c \,|\, cb - baa, bab, pa - ybaaa, pb - ca, pc - xbaaa, \text{ class } 4\rangle \ (y = \omega^i, i = 1,2,3,4,\ x \neq 0,\ x \sim x' \text{ if } x^5 = x'^5 \bmod p,\ p = 1 \bmod 5) \tag{7.2400}$$

8.176 Descendants of 6.322

3 algebras

$$\langle a,b,c \,|\, ba, ca, cb, pb, pc, \text{ class } 5\rangle \tag{7.2401}$$

$$\langle a,b,c \,|\, ba - p^4 a, ca, cb, pb, pc, \text{ class } 5\rangle \tag{7.2402}$$

$$\langle a,b,c \,|\, ba, ca, cb - p^4 a, pb, pc, \text{ class } 5\rangle \tag{7.2403}$$

8.177 Descendants of 6.325

$19 + 5\gcd(p-1,3) + 6\gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, baaaa, bab, ca, cb, pa, pb, pc, \text{ class } 5\rangle \tag{7.2404}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb, pa - baaab, pb, pc, \text{ class } 5\rangle \tag{7.2405}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb, pa - \omega baaab, pb, pc, \text{ class } 5\rangle \tag{7.2406}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb, pa, pb - baaab, pc, \text{ class } 5\rangle \tag{7.2407}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb, pa - baaab, pb - baaab, pc, \text{ class } 5\rangle \tag{7.2408}$$

$$\langle a, b, c \,|\, baaaa, bab, ca, cb, pa - \omega baaab, pb - baaab, pc, \text{ class } 5\rangle \tag{7.2409}$$

$$\langle a, b, c \,|\, baaaa, bab, ca, cb, pa - \omega^2 baaab, pb - baaab, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2410}$$

$$\langle a, b, c \,|\, baaaa, bab, ca, cb, pa - \omega^3 baaab, pb - baaab, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2411}$$

$$\langle a, b, c \,|\, baaaa, bab, ca, cb, pa, pb, pc - baaab, \text{ class } 5\rangle \tag{7.2412}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa, pb, pc, \text{ class } 5\rangle \tag{7.2413}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa - baaab, pb, pc, \text{ class } 5\rangle \tag{7.2414}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa - \omega baaab, pb, pc, \text{ class } 5\rangle \tag{7.2415}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa, pb - baaab, pc, \text{ class } 5\rangle \tag{7.2416}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa - baaab, pb - baaab, pc, \text{ class } 5\rangle \tag{7.2417}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa - \omega baaab, pb - baaab, pc, \text{ class } 5\rangle \tag{7.2418}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa - \omega^2 baaab, pb - baaab, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2419}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa - \omega^3 baaab, pb - baaab, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2420}$$

$$\langle a, b, c \,|\, baaaa, bab, ca - baaab, cb, pa, pb, pc - baaab, \text{ class } 5\rangle \tag{7.2421}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb, pa, pb, pc, \text{ class } 5\rangle \tag{7.2422}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb, pa - baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2423}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb, pa, pb - baaaa, pc, \text{ class } 5\rangle \tag{7.2424}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb, pa, pb - \omega baaaa, pc, \text{ class } 5\rangle \tag{7.2425}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb, pa, pb - \omega^2 baaaa, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2426}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb, pa, pb - \omega^3 baaaa, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2427}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb, pa, pb, pc - baaaa, \text{ class } 5\rangle \tag{7.2428}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb - baaaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2429}$$

$$\langle a, b, c \,|\, baaab, bab, ca, cb - baaaa, pa - baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2430}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb - baaaa, pa, pb - baaaa, pc, \text{ class } 5\rangle \tag{7.2431}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb - baaaa, pa, pb - \omega baaaa, pc, \text{ class } 5\rangle \tag{7.2432}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb - baaaa, pa, pb - \omega^2 baaaa, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2433}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb - baaaa, pa, pb - \omega^3 baaaa, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2434}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb - baaaa, pa, pb, pc - baaaa, \text{ class } 5\rangle \tag{7.2435}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa, pb, pc, \text{ class } 5\rangle \tag{7.2436}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa - baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2437}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa - \omega baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2438}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa - xbaaaa, pb, pc, \text{ class } 5\rangle \ (x = \omega^i,\ i = 2,3,4,5,\ p = 1 \bmod 3) \tag{7.2439}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa, pb - baaaa, pc, \text{ class } 5\rangle \tag{7.2440}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa, pb - \omega baaaa, pc, \text{ class } 5\rangle \tag{7.2441}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa, pb - \omega^2 baaaa, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2442}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa, pb - \omega^3 baaaa, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2443}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb, pa, pb, pc - baaaa, \text{ class } 5\rangle \tag{7.2444}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb - baaaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2445}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb - baaaa, pa - baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2446}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb - baaaa, pa - \omega baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2447}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb - baaaa, pa - xbaaaa, pb, pc, \text{ class } 5\rangle \ (x = \omega^i,\ i = 2,3,4,5,\ p = 1 \bmod 3) \tag{7.2448}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb - baaaa, pa, pb - baaaa, pc, \text{ class } 5\rangle \tag{7.2449}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb - baaaa, pa, pb - \omega baaaa, pc, \text{ class } 5\rangle \tag{7.2450}$$

$$\langle a,b,c \,|\, baaab, bab - baaaa, ca, cb - baaaa, pa, pb - \omega^2 baaaa, pc, \text{ class } 5\rangle \ (p = 1 \bmod 4) \tag{7.2451}$$

$$\langle a,b,c \,|\, baaab, bab-baaaa, ca, cb-baaaa, pa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2452}$$

$$\langle a,b,c \,|\, baaab, bab-baaaa, ca, cb-baaaa, pa, pb, pc-baaaa, \text{ class } 5\rangle \tag{7.2453}$$

$$\langle a,b,c \,|\, baaab, bab-baaaa, ca, cb-baaaa, pa, pb, pc-\omega baaaa, \text{ class } 5\rangle \ (p=1 \bmod 3) \tag{7.2454}$$

$$\langle a,b,c \,|\, baaab, bab-baaaa, ca, cb-baaaa, pa, pb, pc-\omega^2 baaaa, \text{ class } 5\rangle \ (p=1 \bmod 3) \tag{7.2455}$$

8.178 Descendants of 6.326

$4p+5+4\gcd(p-1,3)+2\gcd(p-1,4)+4\gcd(p-1,5)$ algebras

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa, pb, pc, \text{ class } 5\rangle \tag{7.2456}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa-baaab, pb, pc, \text{ class } 5\rangle \tag{7.2457}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa-\omega baaab, pb, pc, \text{ class } 5\rangle \tag{7.2458}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa-xbaaab, pb, pc, \text{ class } 5\rangle \ (x=\omega^i,\ i=2,3,4,5,\ p=1 \bmod 3) \tag{7.2459}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa, pb-baaab, pc, \text{ class } 5\rangle \tag{7.2460}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa, pb-ybaaab, pc, \text{ class } 5\rangle \ (y=\omega^i,\ i=1,2,3,4,\ p=1 \bmod 5) \tag{7.2461}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa-xbaaab, pb-baaab, pc, \text{ class } 5\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p) \tag{7.2462}$$

In the following presentation we have $y=\omega^i,\ i=1,2,3,4,\ x\neq 0,\ x\sim x'$ if $x^5=x'^5 \bmod p,\ p=1 \bmod 5$:

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa-xbaaab, pb-ybaaab, pc, \text{ class } 5\rangle \tag{7.2463}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca, cb, pa, pb, pc-baaab, \text{ class } 5\rangle \tag{7.2464}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb, pa, pb, pc, \text{ class } 5\rangle \tag{7.2465}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb, pa-baaab, pb, pc, \text{ class } 5\rangle \tag{7.2466}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb, pa-\omega baaab, pb, pc, \text{ class } 5\rangle \tag{7.2467}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb, pa-xbaaab, pb, pc, \text{ class } 5\rangle \ (x=\omega^i,\ i=2,3,4,5,\ p=1 \bmod 3) \tag{7.2468}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb, pa, pb-baaab, pc, \text{ class } 5\rangle \tag{7.2469}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb, pa, pb-xbaaab, pc, \text{ class } 5\rangle \ (x=\omega^i,\ i=1,2,3,4,\ p=1 \bmod 5) \tag{7.2470}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca-baaab, cb, pa-xbaaab, pb-baaab, pc, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p) \tag{7.2471}$$

In the following presentation we have $y=\omega^i,\ i=1,2,3,4,\ x\neq 0,\ x\sim x'$ if $x^5=x'^5 \bmod p,\ p=1 \bmod 5$:

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca-baaab, cb, pa-xbaaab, pb-ybaaab, pc, \text{ class } 5\rangle \tag{7.2472}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca-baaab, cb, pa, pb, pc-baaab, \text{ class } 5\rangle \tag{7.2473}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa, pb, pc, \text{ class } 5\rangle \tag{7.2474}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa-baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2475}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa-xbaaaa, pb, pc, \text{ class } 5\rangle\ (x=\omega^i,\ i=1,2,3,4,\ p=1 \bmod 5) \tag{7.2476}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa, pb-baaaa, pc, \text{ class } 5\rangle \tag{7.2477}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa, pb-\omega baaaa, pc, \text{ class } 5\rangle \tag{7.2478}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa, pb-\omega^2 baaaa, pc, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2479}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2480}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa-xbaaaa, pb-baaaa, pc, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.2481}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa-xbaaaa, pb-\omega baaaa, pc, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.2482}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa-xbaaaa, pb-\omega^2 baaaa, pc, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.2483}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa-xbaaaa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.2484}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb, pa, pb, pc-baaaa, \text{ class } 5\rangle \tag{7.2485}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2486}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaaa, pa-baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2487}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaaa, pa-xbaaaa, pb, pc, \text{ class } 5\rangle\ (x=\omega^i,\ i=1,2,3,4,\ p=1 \bmod 5) \tag{7.2488}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa, pb-baaaa, pc, \text{ class } 5\rangle \tag{7.2489}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa, pb-\omega baaaa, pc, \text{ class } 5\rangle \tag{7.2490}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa, pb-\omega^2 baaaa, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2491}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2492}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa-xbaaaa, pb-baaaa, pc, \text{ class } 5\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.2493}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa-xbaaaa, pb-\omega baaaa, pc, \text{ class } 5\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p) \tag{7.2494}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa-xbaaaa, pb-\omega^2 baaaa, pc, \text{ class } 5\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.2495}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa-xbaaaa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^4=x'^4 \bmod p,\ p=1 \bmod 4) \tag{7.2496}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa, pb, pc-baaaa, \text{ class } 5\rangle \tag{7.2497}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaaa, pa, pb, pc-\omega baaaa, \text{ class } 5\rangle \tag{7.2498}$$

8.179 Descendants of 6.327

$3p+12+2\gcd(p-1,3)+3\gcd(p-1,4)+\gcd(p-1,7)$ algebras

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2499}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa-baaab, pb, pc, \text{ class } 5\rangle \tag{7.2500}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa-\omega baaab, pb, pc, \text{ class } 5\rangle \tag{7.2501}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa, pb-baaab, pc, \text{ class } 5\rangle \tag{7.2502}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa-baaab, pb-baaab, pc, \text{ class } 5\rangle \tag{7.2503}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa-\omega baaab, pb-baaab, pc, \text{ class } 5\rangle \tag{7.2504}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa-\omega^2 baaab, pb-baaab, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2505}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa-\omega^3 baaab, pb-baaab, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2506}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa, pb, pc-baaab, \text{ class } 5\rangle \tag{7.2507}$$

$$\langle a,b,c \,|\, baaaa, bab, ca, cb-baaa, pa, pb, pc-\omega baaab, \text{ class } 5\rangle \tag{7.2508}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2509}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa-baaab, pb, pc, \text{ class } 5\rangle \tag{7.2510}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa-\omega baaab, pb, pc, \text{ class } 5\rangle \tag{7.2511}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa-xbaaab, pb, pc, \text{ class } 5\rangle \ (x=\omega^i,\ i=2,3,4,5,\ p=1 \bmod 3) \tag{7.2512}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa, pb-baaab, pc, \text{ class } 5\rangle \tag{7.2513}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa, pb-xbaaab, pc, \text{ class } 5\rangle \ (x=\omega^i,\ i=1,2,3,4,5,6,\ p=1 \bmod 7) \tag{7.2514}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa-xbaaab, pb-baaab, pc, \text{ class } 5\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^7=x'^7 \bmod p) \tag{7.2515}$$

In the following presentation we have $y=\omega^i,\ i=1,2,3,4,5,6,\ x\neq 0,\ x\sim x'$ if $x^7=x'^7 \bmod p,\ p=1 \bmod 7$:

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa-xbaaab, pb-ybaaab, pc, \text{ class } 5\rangle \tag{7.2516}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa, pb, pc-baaab, \text{ class } 5\rangle \tag{7.2517}$$

$$\langle a,b,c \,|\, baaaa, bab, ca-baaab, cb-baaa, pa, pb, pc-\omega baaab, \text{ class } 5\rangle \tag{7.2518}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2519}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb-baaaa, pc, \text{ class } 5\rangle \tag{7.2520}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb-\omega baaaa, pc, \text{ class } 5\rangle \tag{7.2521}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb-\omega^2 baaaa, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2522}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2523}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb, pc-baaaa, \text{ class } 5\rangle \tag{7.2524}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb-baaaa, pc-baaaa, \text{ class } 5\rangle \tag{7.2525}$$

$$\langle a,b,c \,|\, baaab, bab, ca, cb-baaa, pa, pb-\omega baaaa, pc-baaaa, \text{ class } 5\rangle \tag{7.2526}$$

$$\langle a,b,c\,|\,baaab, bab, ca, cb-baaa, pa, pb-\omega^2 baaaa, pc-baaaa, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2527}$$

$$\langle a,b,c\,|\,baaab, bab, ca, cb-baaa, pa, pb-\omega^3 baaaa, pc-baaaa, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2528}$$

$$\langle a,b,c\,|\,baaab, bab, ca, cb-baaa, pa-baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2529}$$

$$\langle a,b,c\,|\,baaab, bab, ca, cb-baaa-baaaa, pa, pb-xbaaaa, pc, \text{ class } 5\rangle\ (\text{all } x) \tag{7.2530}$$

$$\langle a,b,c\,|\,baaab, bab, ca, cb-baaa-baaaa, pa, pb-xbaaaa, pc-baaaa, \text{ class } 5\rangle\ (\text{all } x) \tag{7.2531}$$

$$\langle a,b,c\,|\,baaab, bab, ca, cb-baaa-baaaa, pa-baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2532}$$

8.180 Descendants of 6.328

$p^2+3p-2+(p+3)\gcd(p-1,3)+2\gcd(p-1,4)+2\gcd(p-1,5)$ algebras

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2533}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa-baaab, pb, pc, \text{ class } 5\rangle \tag{7.2534}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa-\omega baaab, pb, pc, \text{ class } 5\rangle \tag{7.2535}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa-xbaaab, pb, pc, \text{ class } 5\rangle\ (x=\omega^i,\ i=2,3,4,5,\ p=1 \bmod 3) \tag{7.2536}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa, pb-baaab, pc, \text{ class } 5\rangle \tag{7.2537}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa, pb-xbaaab, pc, \text{ class } 5\rangle\ (x=\omega^i,\ i=1,2,3,4,\ p=1 \bmod 5) \tag{7.2538}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa-xbaaab, pb-baaab, pc, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p) \tag{7.2539}$$

In the following presentation we have $y=\omega^i,\ i=1,2,3,4,\ x\neq 0,\ x\sim x'$ if $x^5=x'^5 \bmod p,\ p=1 \bmod 5$:

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa-xbaaab, pb-ybaaab, pc, \text{ class } 5\rangle \tag{7.2540}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa, pb, pc-baaab, \text{ class } 5\rangle \tag{7.2541}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa, pb, pc-\omega baaab, \text{ class } 5\rangle \tag{7.2542}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa, pb, pc-\omega^2 baaab, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2543}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca, cb-baaa, pa, pb, pc-\omega^3 baaab, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2544}$$

$$\langle a,b,c\,|\,baaaa, bab-baaa, ca-baaab, cb-baaa, pa-xbaaab, pb, pc, \text{ class } 5\rangle\ (\text{all } x) \tag{7.2545}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb-baaa, pa-xbaaab, pb-ybaaab, pc, \text{ class } 5\rangle \ (\text{all } x,\ y\neq 0,\ y\sim y' \text{ if } y^3=y'^3 \bmod p) \tag{7.2546}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-baaab, cb-baaa, pa, pb, pc-ybaaab, \text{ class } 5\rangle \ (y\neq 0,\ y\sim y' \text{ if } y^3=y'^3 \bmod p) \tag{7.2547}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-\omega baaab, cb-baaa, pa-xbaaab, pb, pc, \text{ class } 5\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.2548}$$

In the next presentation we have all $x,\ y\neq 0,\ y\sim y'$ if $y^3=y'^3 \bmod p,\ p=1 \bmod 3$:

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-\omega baaab, cb-baaa, pa-xbaaab, pb-ybaaab, pc, \text{ class } 5\rangle \ (\text{all } x,\ y\neq 0,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod \tag{7.2549}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-\omega baaab, cb-baaa, pa, pb, pc-ybaaab, \text{ class } 5\rangle \ (y\neq 0,\ y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.2550}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-\omega^2 baaab, cb-baaa, pa-xbaaab, pb, pc, \text{ class } 5\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.2551}$$

In the next presentation we have all $x,\ y\neq 0,\ y\sim y'$ if $y^3=y'^3 \bmod p,\ p=1 \bmod 3$:

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-\omega^2 baaab, cb-baaa, pa-xbaaab, pb-ybaaab, pc, \text{ class } 5\rangle \tag{7.2552}$$

$$\langle a,b,c \,|\, baaaa, bab-baaa, ca-\omega^2 baaab, cb-baaa, pa, pb, pc-ybaaab, \text{ class } 5\rangle \ (y\neq 0, y\sim y' \text{ if } y^3=y'^3 \bmod p,\ p=1 \bmod 3) \tag{7.2553}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2554}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa, pb-baaaa, pc, \text{ class } 5\rangle \tag{7.2555}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa, pb-\omega baaaa, pc, \text{ class } 5\rangle \tag{7.2556}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa, pb-\omega^2 baaaa, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2557}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle \ (p=1 \bmod 4) \tag{7.2558}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa-baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2559}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa-ybaaaa, pb, pc, \text{ class } 5\rangle \ (y=\omega^i,\ i=1,2,3,4,\ p=1 \bmod 5) \tag{7.2560}$$

$$\langle a,b,c \,|\, baaab, bab-baaa, ca, cb-baaa, pa-baaaa, pb-xbaaaa, pc, \text{ class } 5\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^5=x'^5 \bmod p) \tag{7.2561}$$

In the next presentation we have $y=\omega^i,\ i=1,2,3,4,\ x\neq 0,\ x\sim x'$ if $x^5=x'^5 \bmod p,\ p=1 \bmod 5$:

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaa, pa-ybaaaa, pb-xbaaaa, pc, \text{ class } 5\rangle \tag{7.2562}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaa, pa, pb, pc-baaaa, \text{ class } 5\rangle \tag{7.2563}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaa, pa, pb, pc-\omega baaaa, \text{ class } 5\rangle\ (p=1 \bmod 3) \tag{7.2564}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaa, pa, pb, pc-\omega^2 baaaa, \text{ class } 5\rangle\ (p=1 \bmod 3) \tag{7.2565}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaa, pa, pb-xbaaaa, pc-baaaa, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^3=x'^3 \bmod p) \tag{7.2566}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaa, pa, pb-xbaaaa, pc-\omega baaaa, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.2567}$$

$$\langle a,b,c\,|\,baaab, bab-baaa, ca, cb-baaa, pa, pb-xbaaaa, pc-\omega^2 baaaa, \text{ class } 5\rangle\ (x\neq 0,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.2568}$$

8.181 Descendants of 6.362

$p^2+7p+3+2\gcd(p-1,3)+3\gcd(p-1,4)+\gcd(p-1,5)$ algebras

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb, pc, \text{ class } 5\rangle \tag{7.2569}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-baaaa, pc, \text{ class } 5\rangle \tag{7.2570}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-\omega baaaa, pc, \text{ class } 5\rangle \tag{7.2571}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-\omega^2 baaaa, pc, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2572}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-\omega^3 baaaa, pc, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2573}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb, pc-baaaa, \text{ class } 5\rangle \tag{7.2574}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-baaaa, pc-baaaa, \text{ class } 5\rangle \tag{7.2575}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-\omega baaaa, pc-baaaa, \text{ class } 5\rangle \tag{7.2576}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-\omega^2 baaaa, pc-baaaa, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2577}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa, pb-\omega^3 baaaa, pc-baaaa, \text{ class } 5\rangle\ (p=1 \bmod 4) \tag{7.2578}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa-baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2579}$$

$$\langle a,b,c\,|\,bab, ca, cb-baa, pa-baaaa, pb, pc-baaaa, \text{ class } 5\rangle \tag{7.2580}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-baaaa,pa,pb-xbaaaa,pc,\text{ class }5\rangle\ (\text{all }x) \tag{7.2581}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-baaaa,pa,pb-xbaaaa,pc-baaaa,\text{ class }5\rangle\ (\text{all }x) \tag{7.2582}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-baaaa,pa-baaaa,pb,pc,\text{ class }5\rangle \tag{7.2583}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-baaaa,pa-xbaaaa,pb,pc-baaaa,\text{ class }5\rangle\ (x\neq 0,\ x\sim -x) \tag{7.2584}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-\omega baaaa,pa,pb-xbaaaa,pc,\text{ class }5\rangle\ (\text{all }x) \tag{7.2585}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-\omega baaaa,pa,pb-xbaaaa,pc-baaaa,\text{ class }5\rangle\ (\text{all }x) \tag{7.2586}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-\omega baaaa,pa-baaaa,pb,pc,\text{ class }5\rangle \tag{7.2587}$$

$$\langle a,b,c\,|\,bab,ca,cb-baa-\omega baaaa,pa-xbaaaa,pb,pc-baaaa,\text{ class }5\rangle\ (x\neq 0,\ x\sim -x) \tag{7.2588}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb,pc,\text{ class }5\rangle \tag{7.2589}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb,pc-baaaa,\text{ class }5\rangle \tag{7.2590}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb,pc-ybaaaa,\text{ class }5\rangle\ (y=\omega^i,\ i=1,2,3,4,\ p=1\bmod 5) \tag{7.2591}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-baaaa,pc,\text{ class }5\rangle \tag{7.2592}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-\omega baaaa,pc,\text{ class }5\rangle \tag{7.2593}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-\omega^2 baaaa,pc,\text{ class }5\rangle\ (p=1\bmod 4) \tag{7.2594}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-\omega^3 baaaa,pc,\text{ class }5\rangle\ (p=1\bmod 4) \tag{7.2595}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-baaaa,pc-xbaaaa,\text{ class }5\rangle\ (x\neq 0,\ x\sim x'\text{ if }x^4=x'^4\bmod p) \tag{7.2596}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-\omega baaaa,pc-xbaaaa,\text{ class }5\rangle\ (x\neq 0, x\sim x'\text{ if }x^4=x'^4\bmod p) \tag{7.2597}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-\omega^2 baaaa,pc-xbaaaa,\text{ class }5\rangle\ (x\neq 0,\ x\sim x'\text{ if }x^4=x'^4\bmod p,\ p=1\bmod 4) \tag{7.2598}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa,pb-\omega^3 baaaa,pc-xbaaaa,\text{ class }5\rangle\ (x\neq 0, x\sim x'\text{ if }x^4=x'^4\bmod p,\ p=1\bmod 4) \tag{7.2599}$$

$$\langle a,b,c\,|\,bab-baaaa,ca,cb-baa,pa-baaaa,pb,pc,\text{ class }5\rangle \tag{7.2600}$$

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - \omega baaaa, pb, pc, \text{ class } 5\rangle \tag{7.2601}$$

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - ybaaaa, pb, pc, \text{ class } 5\rangle \ (y = \omega^i,\ i = 2,3,4,5,\ p = 1 \bmod 3) \tag{7.2602}$$

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - baaaa, pb - xbaaaa, pc, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.2603}$$

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - \omega baaaa, pb - xbaaaa, pc, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.2604}$$

In the next presentation we have $y = \omega^i,\ i = 2,3,4,5,\ x \neq 0,\ x \sim x'$ if $x^3 = x'^3 \bmod p,\ p = 1 \bmod 3$:

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - ybaaaa, pb - xbaaaa, pc, \text{ class } 5\rangle \tag{7.2605}$$

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - baaaa, pb - zbaaaa, pc - xbaaaa, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^6 = x'^6 \bmod p,\ \text{all } z) \tag{7.2606}$$

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - \omega baaaa, pb - zbaaaa, pc - xbaaaa, \text{ class } 5\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^6 = x'^6 \bmod p,\ \text{all } z) \tag{7.2607}$$

In the next presentation we have $y = \omega^i,\ i = 2,3,4,5,\ x \neq 0,\ x \sim x'$ if $x^6 = x'^6 \bmod p$, all z, $p = 1 \bmod 3$:

$$\langle a,b,c \,|\, bab - baaaa, ca, cb - baa, pa - ybaaaa, pb - zbaaaa, pc - xbaaaa, \text{ class } 5\rangle \tag{7.2608}$$

8.182 Descendants of 6.85

3 algebras

$$\langle a,b,c \,|\, ba, ca, cb, p^2b, p^2c, \text{ class } 3\rangle \tag{7.2609}$$

$$\langle a,b,c \,|\, ba, ca - p^2a, cb, p^2b, p^2c, \text{ class } 3\rangle \tag{7.2610}$$

$$\langle a,b,c \,|\, ba, ca, cb - p^2a, p^2b, p^2c, \text{ class } 3\rangle \tag{7.2611}$$

8.183 Descendants of 6.86

$p + 17$ algebras

$$\langle a,b,c \,|\, baa, bab, p^2a, p^2b, ca, cb, pc, \text{ class } 3\rangle \tag{7.2612}$$

$$\langle a,b,c \,|\, baa, bab, pba, p^2a, ca, cb, pc, \text{ class } 3\rangle \tag{7.2613}$$

$$\langle a,b,c \,|\, baa, bab, pba, p^2a, ca, cb - p^2b, pc, \text{ class } 3\rangle \tag{7.2614}$$

$$\langle a,b,c \,|\, baa, bab, pba, p^2a, ca - p^2b, cb, pc, \text{ class } 3\rangle \tag{7.2615}$$

$$\langle a,b,c \,|\, baa, bab, p^2a, p^2b - pba, ca, cb, pc, \text{ class } 3\rangle \tag{7.2616}$$

$$\langle a, b, c \,|\, baa, pba, p^2a, p^2b, ca, cb, pc, \text{ class } 3\rangle \tag{7.2617}$$

$$\langle a, b, c \,|\, baa, pba, p^2a, p^2b, ca, cb, pc - bab, \text{ class } 3\rangle \tag{7.2618}$$

$$\langle a, b, c \,|\, baa, pba, p^2a, p^2b, ca - bab, cb, pc, \text{ class } 3\rangle \tag{7.2619}$$

$$\langle a, b, c \,|\, baa, pba, p^2a, p^2b, ca - bab, cb, pc - bab, \text{ class } 3\rangle \tag{7.2620}$$

$$\langle a, b, c \,|\, baa, pba, p^2a, p^2b - bab, ca, cb, pc, \text{ class } 3\rangle \tag{7.2621}$$

$$\langle a, b, c \,|\, baa, pba, p^2a, p^2b - bab, ca - bab, cb, pc, \text{ class } 3\rangle \tag{7.2622}$$

$$\langle a, b, c \,|\, baa, pba, p^2a - bab, p^2b, ca, cb, pc, \text{ class } 3\rangle \tag{7.2623}$$

$$\langle a, b, c \,|\, baa, pba, p^2a - bab, p^2b, ca - bab, cb, pc, \text{ class } 3\rangle \tag{7.2624}$$

$$\langle a, b, c \,|\, baa, pba, p^2a - \omega bab, p^2b, ca, cb, pc, \text{ class } 3\rangle \tag{7.2625}$$

$$\langle a, b, c \,|\, baa, pba, p^2a - \omega bab, p^2b, ca - bab, cb, pc, \text{ class } 3\rangle \tag{7.2626}$$

$$\langle a, b, c \,|\, baa, pba - bab, p^2a - xbab, p^2b, ca, cb, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.2627}$$

$$\langle a, b, c \,|\, baa, pba - bab, p^2a + bab, p^2b, ca, cb, pc - bab, \text{ class } 3\rangle \tag{7.2628}$$

$$\langle a, b, c \,|\, baa, pba - bab, p^2a, p^2b - bab, ca, cb, pc, \text{ class } 3\rangle \tag{7.2629}$$

8.184 Descendants of 6.87

5 algebras

$$\langle a, b, c \,|\, p^2a, p^2b, ca, cb, pc - ba, \text{ class } 3\rangle \tag{7.2630}$$

$$\langle a, b, c \,|\, p^2a, pba, ca, cb, pc - ba, \text{ class } 3\rangle \tag{7.2631}$$

$$\langle a, b, c \,|\, p^2a, pba, ca, cb - p^2b, pc - ba, \text{ class } 3\rangle \tag{7.2632}$$

$$\langle a, b, c \,|\, p^2a, pba, ca - p^2b, cb, pc - ba, \text{ class } 3\rangle \tag{7.2633}$$

$$\langle a, b, c \,|\, p^2a, pba, ca - \omega * p^2b, cb, pc - ba, \text{ class } 3\rangle \tag{7.2634}$$

8.185 Descendants of 6.88

26 algebras

$$\langle a, b, c \,|\, baa, bab, p^2b, ca, cb, pa, \text{ class } 3\rangle \tag{7.2635}$$

$$\langle a, b, c \,|\, baa, bab, p^2b, ca, cb - p^2c, pa, \text{ class } 3\rangle \tag{7.2636}$$

$$\langle a, b, c \,|\, baa, bab, p^2b, ca - p^2c, cb, pa, \text{ class } 3\rangle \tag{7.2637}$$

$$\langle a, b, c \,|\, baa, bab, p^2c, ca, cb, pa, \text{ class } 3\rangle \tag{7.2638}$$

$$\langle a, b, c \,|\, baa, bab, p^2c, ca, cb - p^2b, pa, \text{ class } 3\rangle \tag{7.2639}$$

$$\langle a, b, c \,|\, baa, bab, p^2c, ca - p^2b, cb, pa, \text{ class } 3\rangle \tag{7.2640}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c, ca, cb, pa, \text{ class } 3\rangle \tag{7.2641}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c, ca - bab, cb, pa, \text{ class } 3\rangle \tag{7.2642}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c, ca, cb, pa - bab, \text{ class } 3\rangle \tag{7.2643}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c, ca - bab, cb, pa - bab, \text{ class } 3\rangle \tag{7.2644}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c, ca, cb, pa - \omega bab, \text{ class } 3\rangle \tag{7.2645}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c, ca - bab, cb, pa - \omega bab, \text{ class } 3\rangle \tag{7.2646}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c - bab, ca, cb, pa, \text{ class } 3\rangle \tag{7.2647}$$

$$\langle a, b, c \,|\, baa, p^2b, p^2c - bab, ca - bab, cb, pa, \text{ class } 3\rangle \tag{7.2648}$$

$$\langle a, b, c \,|\, baa, p^2b - bab, p^2c, ca, cb, pa, \text{ class } 3\rangle \tag{7.2649}$$

$$\langle a, b, c \,|\, baa, p^2b - bab, p^2c, ca - bab, cb, pa, \text{ class } 3\rangle \tag{7.2650}$$

$$\langle a, b, c \,|\, bab, p^2b, p^2c, ca, cb, pa, \text{ class } 3\rangle \tag{7.2651}$$

$$\langle a, b, c \,|\, bab, p^2b, p^2c, ca, cb, pa - baa, \text{ class } 3\rangle \tag{7.2652}$$

$$\langle a, b, c \,|\, bab, p^2b, p^2c, ca, cb - baa, pa, \text{ class } 3\rangle \tag{7.2653}$$

$$\langle a, b, c \,|\, bab, p^2b, p^2c, ca, cb - baa, pa - baa, \text{ class } 3\rangle \tag{7.2654}$$

$$\langle a, b, c \,|\, bab, p^2b, p^2c - baa, ca, cb, pa, \text{ class } 3\rangle \tag{7.2655}$$

$$\langle a, b, c \,|\, bab, p^2b, p^2c - baa, ca, cb - baa, pa, \text{ class } 3\rangle \tag{7.2656}$$

$$\langle a, b, c \,|\, bab, p^2b - baa, p^2c, ca, cb, pa, \text{ class } 3\rangle \tag{7.2657}$$

$$\langle a, b, c \,|\, bab, p^2b - baa, p^2c, ca, cb - baa, pa, \text{ class } 3\rangle \tag{7.2658}$$

$$\langle a, b, c \,|\, bab, p^2b - \omega baa, p^2c, ca, cb, pa, \text{ class } 3\rangle \tag{7.2659}$$

$$\langle a, b, c \,|\, bab, p^2b - \omega baa, p^2c, ca, cb - baa, pa, \text{ class } 3\rangle \tag{7.2660}$$

8.186 Descendants of 6.89

$p + 6$ algebras

$$\langle a, b, c \,|\, p^2b, p^2c, ca, cb, pa - ba, \text{ class } 3\rangle \tag{7.2661}$$

$$\langle a, b, c \,|\, bab, p^2c, ca, cb, pa - ba, \text{ class } 3\rangle \tag{7.2662}$$

$$\langle a, b, c \,|\, bab, p^2c, ca, cb - p^2b, pa - ba, \text{ class } 3\rangle \tag{7.2663}$$

$$\langle a, b, c \,|\, bab, p^2c, ca - p^2b, cb, pa - ba, \text{ class } 3\rangle \tag{7.2664}$$

$$\langle a, b, c \,|\, bab, p^2b, ca, cb - xp^2c, pa - ba, \text{ class } 3\rangle \text{ (all } x) \tag{7.2665}$$

$$\langle a, b, c \,|\, bab, p^2b, ca - p^2c, cb, pa - ba, \text{ class } 3\rangle \tag{7.2666}$$

$$\langle a, b, c \,|\, p^2b, p^2c - bab, ca, cb, pa - ba, \text{ class } 3\rangle \tag{7.2667}$$

8.187 Descendants of 6.90

30 algebras

$$\langle a, b, c \,|\, baa, bab, bac, caa, cac, cb, pb, pc, \text{ class } 3\rangle \tag{7.2668}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cac, cb - p^2a, pb, pc, \text{ class } 3\rangle \tag{7.2669}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb, pb, pc, \text{ class } 3\rangle \tag{7.2670}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb - caa, pb, pc, \text{ class } 3\rangle \tag{7.2671}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb, pb, pc - caa, \text{ class } 3\rangle \tag{7.2672}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb - caa, pb, pc - caa, \text{ class } 3\rangle \tag{7.2673}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.2674}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb - caa, pb, pc - \omega caa, \text{ class } 3\rangle \tag{7.2675}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb, pb - caa, pc, \text{ class } 3\rangle \tag{7.2676}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a, cb - caa, pb - caa, pc, \text{ class } 3\rangle \tag{7.2677}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a - caa, cb, pb, pc, \text{ class } 3\rangle \tag{7.2678}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, p^2a - caa, cb - caa, pb, pc, \text{ class } 3\rangle \tag{7.2679}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2a, pb, pc, \text{ class } 3\rangle \tag{7.2680}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2a, pb, pc - cac, \text{ class } 3\rangle \tag{7.2681}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2a, pb - cac, pc, \text{ class } 3\rangle \tag{7.2682}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2a - cac, pb, pc, \text{ class } 3\rangle \tag{7.2683}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2a - \omega cac, pb, pc, \text{ class } 3\rangle \tag{7.2684}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac - baa, cb, p^2a, pb, pc, \text{ class } 3\rangle \tag{7.2685}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac - baa, cb, p^2a, pb, pc - baa, \text{ class } 3\rangle \tag{7.2686}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac - baa, cb, p^2a, pb - baa, pc, \text{ class } 3\rangle \tag{7.2687}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac - baa, cb, p^2a, pb - \omega baa, pc, \text{ class } 3\rangle \tag{7.2688}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac - baa, cb, p^2a - baa, pb, pc, \text{ class } 3\rangle \tag{7.2689}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac - baa, cb, p^2a - \omega baa, pb, pc, \text{ class } 3\rangle \tag{7.2690}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2a, pb, pc, \text{ class } 3\rangle \tag{7.2691}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2a, pb, pc - bab, \text{ class } 3\rangle \tag{7.2692}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2a, pb - bab, pc - bab, \text{ class } 3\rangle \tag{7.2693}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2a - bab, pb, pc, \text{ class } 3\rangle \tag{7.2694}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2a, pb, pc, \text{ class } 3\rangle \tag{7.2695}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2a, pb, pc - bab, \text{ class } 3\rangle \tag{7.2696}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2a - bab, pb, pc, \text{ class } 3\rangle \tag{7.2697}$$

8.188 Descendants of 6.91

$2p + 88 + \gcd(p-1, 4)$ algebras

$$\langle a, b, c \,|\, baa, bab, bac, caa, cac, cb, pa, pc, \text{ class } 3\rangle \tag{7.2698}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cac, cb - p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2699}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2700}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2b, pa - cac, pc, \text{ class } 3\rangle \tag{7.2701}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2b, pa - \omega cac, pc, \text{ class } 3\rangle \tag{7.2702}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2b, pa, pc - cac, \text{ class } 3\rangle \tag{7.2703}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2b, pa - cac, pc - cac, \text{ class } 3\rangle \tag{7.2704}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2b, pa - \omega cac, pc - cac, \text{ class } 3\rangle \tag{7.2705}$$

$$\langle a, b, c \,|\, baa, bab, bac, caa, cb, p^2b - cac, pa, pc, \text{ class } 3\rangle \tag{7.2706}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2707}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2b, pa - bab, pc, \text{ class } 3\rangle \tag{7.2708}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2b, pa - \omega bab, pc, \text{ class } 3\rangle \tag{7.2709}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2b, pa, pc - bab, \text{ class } 3\rangle \tag{7.2710}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2b, pa - bab, pc - bab, \text{ class } 3\rangle \tag{7.2711}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2b, pa - \omega bab, pc - bab, \text{ class } 3\rangle \tag{7.2712}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + bab, cb, p^2b - bab, pa, pc, \text{ class } 3\rangle \tag{7.2713}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2714}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2b, pa - bab, pc, \text{ class } 3\rangle \tag{7.2715}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2b, pa - \omega bab, pc, \text{ class } 3\rangle \tag{7.2716}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2b, pa, pc - bab, \text{ class } 3\rangle \tag{7.2717}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac + \omega bab, cb, p^2b, pa - bab, pc - bab, \text{ class } 3\rangle \tag{7.2718}$$

$$\langle a,b,c\,|\,baa,bac,caa,cac+\omega bab,cb,p^2b,pa-\omega bab,pc-bab,\text{ class 3}\rangle \tag{7.2719}$$

$$\langle a,b,c\,|\,baa,bac,caa,cac+\omega bab,cb,p^2b-bab,pa,pc,\text{ class 3}\rangle \tag{7.2720}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b,pa,pc,\text{ class 3}\rangle \tag{7.2721}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b,pa-baa,pc,\text{ class 3}\rangle \tag{7.2722}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b,pa-\omega baa,pc,\text{ class 3}\rangle \tag{7.2723}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b,pa,pc-baa,\text{ class 3}\rangle \tag{7.2724}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b,pa-baa,pc-baa,\text{ class 3}\rangle \tag{7.2725}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b,pa-\omega baa,pc-baa,\text{ class 3}\rangle \tag{7.2726}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b-baa,pa,pc,\text{ class 3}\rangle \tag{7.2727}$$

$$\langle a,b,c\,|\,bab,bac,caa,cac-baa,cb,p^2b-\omega baa,pa,pc,\text{ class 3}\rangle \tag{7.2728}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+baa,cb,p^2b,pa,pc,\text{ class 3}\rangle \tag{7.2729}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+baa,cb,p^2b,pa,pc-baa,\text{ class 3}\rangle \tag{7.2730}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+baa,cb,p^2b,pa,pc-\omega baa,\text{ class 3}\rangle\ (p=1\bmod 4) \tag{7.2731}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+baa,cb,p^2b,pa-baa,pc-xbaa,\text{ class 3}\rangle\ (\text{all } x,\ x\sim -x) \tag{7.2732}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+baa,cb,p^2b,pa-\omega baa,pc-xbaa,\text{ class 3}\rangle\ (\text{all } x,\ x\sim -x) \tag{7.2733}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+baa,cb,p^2b-baa,pa,pc,\text{ class 3}\rangle \tag{7.2734}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+baa,cb,p^2b-\omega baa,pa,pc,\text{ class 3}\rangle \tag{7.2735}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+\omega baa,cb,p^2b,pa,pc,\text{ class 3}\rangle \tag{7.2736}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+\omega baa,cb,p^2b,pa,pc-baa,\text{ class 3}\rangle \tag{7.2737}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+\omega baa,cb,p^2b,pa,pc-\omega baa,\text{ class 3}\rangle\ (p=1\bmod 4) \tag{7.2738}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+\omega baa,cb,p^2b,pa-baa,pc-xbaa,\text{ class 3}\rangle\ (\text{all } x,\ x\sim -x) \tag{7.2739}$$

$$\langle a,b,c\,|\,bab-baa,bac,caa,cac+\omega baa,cb,p^2b,pa-\omega baa,pc-xbaa,\text{ class 3}\rangle\ (\text{all } x,\ x\sim -x) \tag{7.2740}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac + \omega baa, cb, p^2b - baa, pa, pc, \text{ class } 3\rangle \tag{7.2741}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac + \omega baa, cb, p^2b - \omega baa, pa, pc, \text{ class } 3\rangle \tag{7.2742}$$

$$\langle a, b, c \,|\, baa, bab, caa, cac, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2743}$$

$$\langle a, b, c \,|\, baa, bab, caa, cac, cb, p^2b, pa, pc - bac, \text{ class } 3\rangle \tag{7.2744}$$

$$\langle a, b, c \,|\, baa, bab, caa, cac, cb, p^2b, pa - bac, pc, \text{ class } 3\rangle \tag{7.2745}$$

$$\langle a, b, c \,|\, baa, bab, caa, cac, cb, p^2b, pa - bac, pc - bac, \text{ class } 3\rangle \tag{7.2746}$$

$$\langle a, b, c \,|\, baa, bab, caa, cac, cb, p^2b - bac, pa, pc, \text{ class } 3\rangle \tag{7.2747}$$

$$\langle a, b, c \,|\, baa, bab, caa - bac, cac, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2748}$$

$$\langle a, b, c \,|\, baa, bab, caa - bac, cac, cb, p^2b, pa, pc - bac, \text{ class } 3\rangle \tag{7.2749}$$

$$\langle a, b, c \,|\, baa, bab, caa - bac, cac, cb, p^2b, pa, pc - \omega bac, \text{ class } 3\rangle \tag{7.2750}$$

$$\langle a, b, c \,|\, baa, bab, caa - bac, cac, cb, p^2b, pa - bac, pc, \text{ class } 3\rangle \tag{7.2751}$$

$$\langle a, b, c \,|\, baa, bab, caa - bac, cac, cb, p^2b, pa - bac, pc - bac, \text{ class } 3\rangle \tag{7.2752}$$

$$\langle a, b, c \,|\, baa, bab, caa - bac, cac, cb, p^2b, pa - bac, pc - \omega bac, \text{ class } 3\rangle \tag{7.2753}$$

$$\langle a, b, c \,|\, baa, bab, caa - bac, cac, cb, p^2b - bac, pa, pc, \text{ class } 3\rangle \tag{7.2754}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2755}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb, p^2b, pa, pc - caa, \text{ class } 3\rangle \tag{7.2756}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb, p^2b, pa, pc - \omega caa, \text{ class } 3\rangle \tag{7.2757}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb - caa, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2758}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb - caa, p^2b, pa, pc - caa, \text{ class } 3\rangle \tag{7.2759}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb - caa, p^2b, pa, pc - \omega caa, \text{ class } 3\rangle \tag{7.2760}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb, p^2b, pa - caa, pc, \text{ class } 3\rangle \tag{7.2761}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb - caa, p^2b, pa - caa, pc, \text{ class } 3\rangle \tag{7.2762}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb, p^2b - caa, pa, pc, \text{ class } 3\rangle \tag{7.2763}$$

$$\langle a, b, c \,|\, baa, bab, bac, cac, cb - caa, p^2b - caa, pa, pc, \text{ class } 3\rangle \tag{7.2764}$$

$$\langle a, b, c \,|\, baa, bac, caa - bab, cac, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2765}$$

$$\langle a, b, c \,|\, baa, bac, caa - bab, cac, cb, p^2b, pa, pc - bab, \text{ class } 3\rangle \tag{7.2766}$$

$$\langle a, b, c \,|\, baa, bac, caa - bab, cac, cb, p^2b, pa, pc - \omega bab, \text{ class } 3\rangle \tag{7.2767}$$

$$\langle a, b, c \,|\, baa, bac, caa - bab, cac, cb, p^2b, pa - bab, pc, \text{ class } 3\rangle \tag{7.2768}$$

$$\langle a, b, c \,|\, baa, bac, caa - bab, cac, cb, p^2b, pa - \omega bab, pc, \text{ class } 3\rangle \tag{7.2769}$$

$$\langle a, b, c \,|\, baa, bac, caa - bab, cac, cb, p^2b - bab, pa, pc, \text{ class } 3\rangle \tag{7.2770}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2771}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac, cb, p^2b, pa, pc - bab, \text{ class } 3\rangle \tag{7.2772}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac, cb, p^2b, pa - bab, pc, \text{ class } 3\rangle \tag{7.2773}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac, cb, p^2b, pa - \omega bab, pc, \text{ class } 3\rangle \tag{7.2774}$$

$$\langle a, b, c \,|\, baa, bac, caa, cac, cb, p^2b - bab, pa, pc, \text{ class } 3\rangle \tag{7.2775}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2776}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb, p^2b, pa, pc - baa, \text{ class } 3\rangle \tag{7.2777}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb, p^2b, pa - baa, pc, \text{ class } 3\rangle \tag{7.2778}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb, p^2b - baa, pa, pc, \text{ class } 3\rangle \tag{7.2779}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb, p^2b - \omega baa, pa, pc, \text{ class } 3\rangle \tag{7.2780}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb - baa, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2781}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb - baa, p^2b, pa, pc - baa, \text{ class } 3\rangle \tag{7.2782}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb - baa, p^2b, pa - baa, pc, \text{ class } 3\rangle \tag{7.2783}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb - baa, p^2b - baa, pa, pc, \text{ class } 3\rangle \tag{7.2784}$$

$$\langle a, b, c \,|\, bab, bac, caa, cac, cb - baa, p^2b - \omega baa, pa, pc, \text{ class } 3\rangle \tag{7.2785}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac, cb, p^2b, pa, pc, \text{ class } 3\rangle \tag{7.2786}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac, cb, p^2b, pa - baa, pc, \text{ class } 3\rangle \tag{7.2787}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac, cb, p^2b, pa - \omega baa, pc, \text{ class } 3\rangle \tag{7.2788}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac, cb, p^2b, pa, pc - baa, \text{ class } 3\rangle \tag{7.2789}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac, cb, p^2b - baa, pa, pc, \text{ class } 3\rangle \tag{7.2790}$$

$$\langle a, b, c \,|\, bab - baa, bac, caa, cac, cb, p^2b - \omega baa, pa, pc, \text{ class } 3\rangle \tag{7.2791}$$

8.189 Descendants of 6.92

$2p + 13$ algebras

$$\langle a, b, c \,|\, baa, caa, cac, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2792}$$

$$\langle a, b, c \,|\, baa, caa, cac, cb - p^2a, pb - ba, pc, \text{ class } 3\rangle \tag{7.2793}$$

$$\langle a, b, c \,|\, baa, caa, p^2a, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2794}$$

$$\langle a, b, c \,|\, baa, caa, p^2a, cb, pb - ba, pc - cac, \text{ class } 3\rangle \tag{7.2795}$$

$$\langle a, b, c \,|\, baa, caa, p^2a, cb, pb - ba - cac, pc, \text{ class } 3\rangle \tag{7.2796}$$

$$\langle a, b, c \,|\, baa, caa, p^2a, cb, pb - ba - cac, pc - cac, \text{ class } 3\rangle \tag{7.2797}$$

$$\langle a, b, c \,|\, baa, caa, p^2a - cac, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2798}$$

$$\langle a, b, c \,|\, baa, caa, p^2a - \omega cac, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2799}$$

$$\langle a, b, c \,|\, baa, cac, p^2a, cb, pb - ba, pc - xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.2800}$$

$$\langle a, b, c \,|\, baa, cac, p^2a, cb - caa, pb - ba, pc - xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.2801}$$

$$\langle a, b, c \,|\, baa, cac, p^2a - caa, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2802}$$

$$\langle a, b, c \,|\, baa, cac, p^2a - caa, cb - caa, pb - ba, pc, \text{ class } 3\rangle \tag{7.2803}$$

$$\langle a, b, c \,|\, caa, cac, p^2a, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2804}$$

$$\langle a, b, c \,|\, caa, cac - baa, p^2a, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2805}$$

$$\langle a, b, c \,|\, caa - baa, cac, p^2a, cb, pb - ba, pc, \text{ class } 3\rangle \tag{7.2806}$$

8.190 Descendants of 6.93

$p + 15 + 2\gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a,b,c \,|\, baa, bab, caa, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2807}$$

$$\langle a,b,c \,|\, baa, bab, caa, cb - p^2a, pb - ca, pc, \text{ class } 3\rangle \tag{7.2808}$$

$$\langle a,b,c \,|\, baa, bab, caa, cb - \omega p^2a, pb - ca, pc, \text{ class } 3\rangle \tag{7.2809}$$

$$\langle a,b,c \,|\, baa, bab, p^2a, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2810}$$

$$\langle a,b,c \,|\, baa, bab, p^2a - caa, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2811}$$

$$\langle a,b,c \,|\, baa, caa, p^2a, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2812}$$

$$\langle a,b,c \,|\, baa, caa, p^2a, cb, pb - ca, pc - bab, \text{ class } 3\rangle \tag{7.2813}$$

$$\langle a,b,c \,|\, baa, caa, p^2a, cb, pb - ca - bab, pc, \text{ class } 3\rangle \tag{7.2814}$$

$$\langle a,b,c \,|\, baa, caa, p^2a - bab, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2815}$$

$$\langle a,b,c \,|\, baa, caa, p^2a - \omega bab, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2816}$$

$$\langle a,b,c \,|\, baa, caa - bab, p^2a - xbab, cb, pb - ca, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.2817}$$

$$\langle a,b,c \,|\, baa, caa - bab, p^2a + bab, cb, pb - ca, pc - bab, \text{ class } 3\rangle \tag{7.2818}$$

$$\langle a,b,c \,|\, baa, caa - bab, p^2a + bab, cb, pb - ca, pc - \omega bab, \text{ class } 3\rangle \tag{7.2819}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2820}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb, pb - ca, pc - baa, \text{ class } 3\rangle \tag{7.2821}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb, pb - ca, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.2822}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb, pb - ca, pc - \omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.2823}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb - baa, pb - ca, pc, \text{ class } 3\rangle \tag{7.2824}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb - baa, pb - ca, pc - baa, \text{ class } 3\rangle \tag{7.2825}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb - baa, pb - ca, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.2826}$$

$$\langle a,b,c \,|\, bab, caa, p^2a, cb - baa, pb - ca, pc - \omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.2827}$$

$$\langle a, b, c \,|\, bab, caa, p^2a - baa, cb, pb - ca, pc, \text{ class } 3\rangle \tag{7.2828}$$

$$\langle a, b, c \,|\, bab, caa, p^2a - baa, cb - baa, pb - ca, pc, \text{ class } 3\rangle \tag{7.2829}$$

$$\langle a, b, c \,|\, bab, caa, p^2a - baa, cb - \omega baa, pb - ca, pc, \text{ class } 3\rangle \tag{7.2830}$$

$$\langle a, b, c \,|\, bab, caa, p^2a - baa, cb - \omega^2 baa, pb - ca, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.2831}$$

$$\langle a, b, c \,|\, bab, caa, p^2a - baa, cb - \omega^3 baa, pb - ca, pc, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.2832}$$

8.191 Descendants of 6.94

$2p + 15 + 3\gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b, c \,|\, caa, cac, cb, pa, pc - ba, \text{ class } 3\rangle \tag{7.2833}$$

$$\langle a, b, c \,|\, caa, cac, cb - p^2b, pa, pc - ba, \text{ class } 3\rangle \tag{7.2834}$$

$$\langle a, b, c \,|\, caa, p^2b, cb, pa, pc - ba, \text{ class } 3\rangle \tag{7.2835}$$

$$\langle a, b, c \,|\, caa, p^2b, cb, pa - cac, pc - ba, \text{ class } 3\rangle \tag{7.2836}$$

$$\langle a, b, c \,|\, caa, p^2b, cb, pa - \omega cac, pc - ba, \text{ class } 3\rangle \tag{7.2837}$$

$$\langle a, b, c \,|\, caa, p^2b, cb, pa, pc - ba - cac, \text{ class } 3\rangle \tag{7.2838}$$

$$\langle a, b, c \,|\, caa, p^2b, cb, pa - cac, pc - ba - cac, \text{ class } 3\rangle \tag{7.2839}$$

$$\langle a, b, c \,|\, caa, p^2b, cb, pa - \omega cac, pc - ba - cac, \text{ class } 3\rangle \tag{7.2840}$$

$$\langle a, b, c \,|\, caa, p^2b - cac, cb, pa, pc - ba, \text{ class } 3\rangle \tag{7.2841}$$

$$\langle a, b, c \,|\, cac, p^2b, cb, pa, pc - ba, \text{ class } 3\rangle \tag{7.2842}$$

$$\langle a, b, c \,|\, cac, p^2b, cb, pa - caa, pc - ba, \text{ class } 3\rangle \tag{7.2843}$$

$$\langle a, b, c \,|\, cac, p^2b, cb - caa, pa, pc - ba, \text{ class } 3\rangle \tag{7.2844}$$

$$\langle a, b, c \,|\, cac, p^2b, cb - caa, pa - caa, pc - ba, \text{ class } 3\rangle \tag{7.2845}$$

$$\langle a, b, c \,|\, cac, p^2b, cb - caa, pa - \omega caa, pc - ba, \text{ class } 3\rangle \tag{7.2846}$$

$$\langle a, b, c \,|\, cac, p^2b, cb - caa, pa - \omega^2 caa, pc - ba, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.2847}$$

$$\langle a,b,c \,|\, cac, p^2b, cb-caa, pa-\omega^3caa, pc-ba, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.2848}$$

$$\langle a,b,c \,|\, cac, p^2b-caa, cb, pa, pc-ba, \text{ class } 3\rangle \tag{7.2849}$$

$$\langle a,b,c \,|\, cac, p^2b-\omega caa, cb, pa, pc-ba, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.2850}$$

$$\langle a,b,c \,|\, cac, p^2b-\omega^2caa, cb, pa, pc-ba, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.2851}$$

$$\langle a,b,c \,|\, cac, p^2b-caa, cb-caa, pa, pc-ba, \text{ class } 3\rangle \tag{7.2852}$$

$$\langle a,b,c \,|\, cac, p^2b-\omega caa, cb-caa, pa, pc-ba, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.2853}$$

$$\langle a,b,c \,|\, cac, p^2b-\omega^2caa, cb-caa, pa, pc-ba, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.2854}$$

$$\langle a,b,c \,|\, cac-caa, p^2b, cb, pa, pc-ba, \text{ class } 3\rangle \tag{7.2855}$$

$$\langle a,b,c \,|\, cac-caa, p^2b, cb, pa, pc-ba-caa, \text{ class } 3\rangle \tag{7.2856}$$

$$\langle a,b,c \,|\, cac-caa, p^2b, cb, pa, pc-ba-\omega caa, \text{ class } 3\rangle \tag{7.2857}$$

$$\langle a,b,c \,|\, cac-caa, p^2b, cb, pa-caa, pc-ba-xcaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.2858}$$

$$\langle a,b,c \,|\, cac-caa, p^2b, cb, pa-\omega caa, pc-ba-xcaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.2859}$$

$$\langle a,b,c \,|\, cac-caa, p^2b-caa, cb, pa, pc-ba, \text{ class } 3\rangle \tag{7.2860}$$

$$\langle a,b,c \,|\, cac-caa, p^2b-\omega caa, cb, pa, pc-ba, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.2861}$$

$$\langle a,b,c \,|\, cac-caa, p^2b-\omega^2caa, cb, pa, pc-ba, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.2862}$$

8.192 Descendants of 6.95

$5p+10$ algebras

$$\langle a,b,c \,|\, baa, bab, cb, pa, pc-ca, \text{ class } 3\rangle \tag{7.2863}$$

$$\langle a,b,c \,|\, baa, bab, cb-p^2b, pa, pc-ca, \text{ class } 3\rangle \tag{7.2864}$$

$$\langle a,b,c \,|\, baa, p^2b, cb, pa, pc-ca, \text{ class } 3\rangle \tag{7.2865}$$

$$\langle a,b,c \,|\, baa, p^2b, cb, pa-bab, pc-ca, \text{ class } 3\rangle \tag{7.2866}$$

$$\langle a,b,c \,|\, baa, p^2b, cb, pa-\omega bab, pc-ca, \text{ class } 3\rangle \tag{7.2867}$$

$$\langle a, b, c \,|\, baa, p^2b, cb, pa, pc - ca - bab, \text{ class } 3\rangle \tag{7.2868}$$

$$\langle a, b, c \,|\, baa, p^2b, cb, pa - bab, pc - ca - bab, \text{ class } 3\rangle \tag{7.2869}$$

$$\langle a, b, c \,|\, baa, p^2b, cb, pa - \omega bab, pc - ca - bab, \text{ class } 3\rangle \tag{7.2870}$$

$$\langle a, b, c \,|\, baa, p^2b - bab, cb, pa, pc - ca, \text{ class } 3\rangle \tag{7.2871}$$

$$\langle a, b, c \,|\, bab, p^2b - xbaa, cb, pa, pc - ca, \text{ class } 3\rangle \text{ (all } x) \tag{7.2872}$$

$$\langle a, b, c \,|\, bab, p^2b - xbaa, cb - baa, pa, pc - ca, \text{ class } 3\rangle \text{ (all } x) \tag{7.2873}$$

$$\langle a, b, c \,|\, bab, p^2b, cb, pa - baa, pc - ca, \text{ class } 3\rangle \tag{7.2874}$$

$$\langle a, b, c \,|\, bab, p^2b, cb - baa, pa - baa, pc - ca, \text{ class } 3\rangle \tag{7.2875}$$

$$\langle a, b, c \,|\, bab - baa, p^2b - xbaa, cb, pa, pc - ca, \text{ class } 3\rangle \ (x \neq 0) \tag{7.2876}$$

$$\langle a, b, c \,|\, bab - baa, p^2b, cb, pa - xbaa, pc - ca, \text{ class } 3\rangle \text{ (all } x) \tag{7.2877}$$

$$\langle a, b, c \,|\, bab - baa, p^2b, cb, pa - xbaa, pc - ca - baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.2878}$$

8.193 Descendants of 6.96

$2p + 26$ algebras

$$\langle a, b, c \,|\, bab, caa, cac, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2879}$$

$$\langle a, b, c \,|\, bab, caa, cac, cb - p^2b, pa - ba, pc, \text{ class } 3\rangle \tag{7.2880}$$

$$\langle a, b, c \,|\, bab, caa, p^2b, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2881}$$

$$\langle a, b, c \,|\, bab, caa, p^2b, cb, pa - ba - cac, pc, \text{ class } 3\rangle \tag{7.2882}$$

$$\langle a, b, c \,|\, bab, caa, p^2b, cb, pa - ba - \omega cac, pc, \text{ class } 3\rangle \tag{7.2883}$$

$$\langle a, b, c \,|\, bab, caa, p^2b, cb, pa - ba, pc - cac, \text{ class } 3\rangle \tag{7.2884}$$

$$\langle a, b, c \,|\, bab, caa, p^2b, cb, pa - ba - cac, pc - cac, \text{ class } 3\rangle \tag{7.2885}$$

$$\langle a, b, c \,|\, bab, caa, p^2b, cb, pa - ba - \omega cac, pc - cac, \text{ class } 3\rangle \tag{7.2886}$$

$$\langle a, b, c \,|\, bab, caa, p^2b - cac, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2887}$$

$$\langle a, b, c \,|\, bab, cac, p^2b, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2888}$$

$$\langle a, b, c \,|\, bab, cac, p^2b, cb, pa - ba, pc - caa, \text{ class } 3\rangle \tag{7.2889}$$

$$\langle a, b, c \,|\, bab, cac, p^2b, cb, pa - ba, pc - \omega caa, \text{ class } 3\rangle \tag{7.2890}$$

$$\langle a, b, c \,|\, bab, cac, p^2b, cb - caa, pa - ba, pc - xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.2891}$$

$$\langle a, b, c \,|\, bab, cac, p^2b, cb - \omega caa, pa - ba, pc - xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.2892}$$

$$\langle a, b, c \,|\, bab, cac, p^2b - caa, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2893}$$

$$\langle a, b, c \,|\, bab, cac, p^2b - caa, cb - caa, pa - ba, pc, \text{ class } 3\rangle \tag{7.2894}$$

$$\langle a, b, c \,|\, bab, cac, p^2b - caa, cb - \omega caa, pa - ba, pc, \text{ class } 3\rangle \tag{7.2895}$$

$$\langle a, b, c \,|\, caa, cac, p^2b, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2896}$$

$$\langle a, b, c \,|\, caa, cac, p^2b, cb, pa - ba, pc - bab, \text{ class } 3\rangle \tag{7.2897}$$

$$\langle a, b, c \,|\, caa, cac, p^2b - bab, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2898}$$

$$\langle a, b, c \,|\, caa, cac + bab, p^2b, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2899}$$

$$\langle a, b, c \,|\, caa, cac + bab, p^2b - bab, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2900}$$

$$\langle a, b, c \,|\, caa, cac + \omega bab, p^2b, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2901}$$

$$\langle a, b, c \,|\, caa, cac + \omega bab, p^2b - bab, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2902}$$

$$\langle a, b, c \,|\, cac, caa - bab, p^2b, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2903}$$

$$\langle a, b, c \,|\, cac, caa - bab, p^2b, cb, pa - ba, pc - bab, \text{ class } 3\rangle \tag{7.2904}$$

$$\langle a, b, c \,|\, cac, caa - bab, p^2b, cb, pa - ba, pc - \omega bab, \text{ class } 3\rangle \tag{7.2905}$$

$$\langle a, b, c \,|\, cac, caa - bab, p^2b - bab, cb, pa - ba, pc, \text{ class } 3\rangle \tag{7.2906}$$

8.194 Descendants of 6.97

$3p + 18 + \gcd(p-1, 3)$ algebras

$$\langle a, b, c \,|\, baa, bab, bac, cb - xp^2b, pa - ca, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.2907}$$

$$\langle a, b, c \,|\, baa, bab, p^2b, cb, pa - ca, pc, \text{ class } 3\rangle \tag{7.2908}$$

$$\langle a, b, c \,|\, baa, bab, p^2b - bac, cb, pa - ca, pc, \text{ class } 3\rangle \tag{7.2909}$$

$$\langle a, b, c \,|\, baa, bac, p^2b, cb, pa - ca, pc, \text{ class } 3\rangle \tag{7.2910}$$

$$\langle a, b, c \,|\, baa, bac, p^2b, cb, pa - ca - bab, pc, \text{ class } 3\rangle \tag{7.2911}$$

$$\langle a, b, c \,|\, baa, bac, p^2b, cb, pa - ca - \omega bab, pc, \text{ class } 3\rangle \tag{7.2912}$$

$$\langle a, b, c \,|\, baa, bac, p^2b, cb, pa - ca, pc - bab, \text{ class } 3\rangle \tag{7.2913}$$

$$\langle a, b, c \,|\, baa, bac, p^2b - bab, cb, pa - ca, pc, \text{ class } 3\rangle \tag{7.2914}$$

$$\langle a, b, c \,|\, bab, bac, p^2b, cb, pa - ca, pc, \text{ class } 3\rangle \tag{7.2915}$$

$$\langle a, b, c \,|\, bab, bac, p^2b, cb - baa, pa - ca, pc, \text{ class } 3\rangle \tag{7.2916}$$

$$\langle a, b, c \,|\, bab, bac, p^2b, cb - \omega baa, pa - ca, pc, \text{ class } 3\rangle \tag{7.2917}$$

$$\langle a, b, c \,|\, bab, bac, p^2b, cb, pa - ca, pc - baa, \text{ class } 3\rangle \tag{7.2918}$$

$$\langle a, b, c \,|\, bab, bac, p^2b, cb - baa, pa - ca, pc - baa, \text{ class } 3\rangle \tag{7.2919}$$

$$\langle a, b, c \,|\, bab, bac, p^2b, cb - \omega baa, pa - ca, pc - baa, \text{ class } 3\rangle \tag{7.2920}$$

$$\langle a, b, c \,|\, bab, bac, p^2b - baa, cb - xbaa, pa - ca, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.2921}$$

$$\langle a, b, c \,|\, bab, bac, p^2b - \omega baa, cb - xbaa, pa - ca, pc, \text{ class } 3\rangle \text{ (all } x) \tag{7.2922}$$

$$\langle a, b, c \,|\, bab - baa, bac, p^2b, cb, pa - ca, pc, \text{ class } 3\rangle \tag{7.2923}$$

$$\langle a, b, c \,|\, bab - baa, bac, p^2b, cb, pa - ca - baa, pc, \text{ class } 3\rangle \tag{7.2924}$$

$$\langle a, b, c \,|\, bab - baa, bac, p^2b, cb, pa - ca - \omega baa, pc, \text{ class } 3\rangle \tag{7.2925}$$

$$\langle a, b, c \,|\, bab - baa, bac, p^2b, cb, pa - ca, pc - baa, \text{ class } 3\rangle \tag{7.2926}$$

$$\langle a, b, c \,|\, bab - baa, bac, p^2b, cb, pa - ca, pc - \omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.2927}$$

$$\langle a,b,c \,|\, bab-baa, bac, p^2b, cb, pa-ca, pc-\omega^2 baa, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.2928}$$

$$\langle a,b,c \,|\, bab-baa, bac, p^2b-baa, cb, pa-ca, pc, \text{ class } 3\rangle \tag{7.2929}$$

$$\langle a,b,c \,|\, bab-baa, bac, p^2b-\omega baa, cb, pa-ca, pc, \text{ class } 3\rangle \tag{7.2930}$$

8.195 Descendants of 6.98

$(5p+1)/2$ algebras

$$\langle a,b,c \,|\, baa, caa, cb, pb-ba, pc-ca, \text{ class } 3\rangle \tag{7.2931}$$

$$\langle a,b,c \,|\, baa, caa, cb-p^2a, pb-ba, pc-ca, \text{ class } 3\rangle \tag{7.2932}$$

$$\langle a,b,c \,|\, baa, p^2a, cb, pb-ba, pc-ca, \text{ class } 3\rangle \tag{7.2933}$$

$$\langle a,b,c \,|\, baa, p^2a, cb, pb-ba-caa, pc-ca, \text{ class } 3\rangle \tag{7.2934}$$

$$\langle a,b,c \,|\, baa, caa, cb, pb-ba, pc-xca, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.2935}$$

$$\langle a,b,c \,|\, baa, caa, cb-p^2a, pb-ba, pc-xca, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.2936}$$

$$\langle a,b,c \,|\, baa, p^2a, cb, pb-ba, pc-xca, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.2937}$$

$$\langle a,b,c \,|\, caa, p^2a, cb, pb-ba, pc-xca, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.2938}$$

$$\langle a,b,c \,|\, caa-baa, p^2a, cb, pb-ba, pc-xca, \text{ class } 3\rangle \ (x \neq 0,1,-1,\ x \sim x^{-1}) \tag{7.2939}$$

$$\langle a,b,c \,|\, baa, caa, cb, pb-ba, pc+ca, \text{ class } 3\rangle \tag{7.2940}$$

$$\langle a,b,c \,|\, baa, caa, cb-p^2a, pb-ba, pc+ca, \text{ class } 3\rangle \tag{7.2941}$$

$$\langle a,b,c \,|\, baa, p^2a, cb, pb-ba, pc+ca, \text{ class } 3\rangle \tag{7.2942}$$

$$\langle a,b,c \,|\, caa-baa, p^2a, cb, pb-ba, pc+ca, \text{ class } 3\rangle \tag{7.2943}$$

8.196 Descendants of 6.99

$p+4$ algebras

$$\langle a,b,c \,|\, baa, caa, cb, pb-ba-ca, pc-ca, \text{ class } 3\rangle \tag{7.2944}$$

$$\langle a,b,c \,|\, baa, caa, cb-p^2a, pb-ba-ca, pc-ca, \text{ class } 3\rangle \tag{7.2945}$$

$$\langle a,b,c \,|\, baa, caa, cb-\omega p^2a, pb-ba-ca, pc-ca, \text{ class } 3\rangle \tag{7.2946}$$

$$\langle a,b,c \,|\, baa, p^2a, cb, pb-ba-ca, pc-ca, \text{ class } 3\rangle \tag{7.2947}$$

$$\langle a,b,c \,|\, caa, p^2a, cb, pb-ba-ca, pc-ca-xbaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.2948}$$

8.197 Descendants of 4.1

1361 algebras when $p=3$, $p^5+2p^4+7p^3+25p^2+88p+270+(p+4)\gcd(p-1,3)+\gcd(p-1,4)$ algebras when $p>3$

$$\langle a,b,c,d \,|\, ba, ca, da, cb, db, dc, pd, \text{ class } 2\rangle \tag{7.2949}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc, pa, pb, \text{ class } 2\rangle \tag{7.2950}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc, pa-ba, pb, \text{ class } 2\rangle \tag{7.2951}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc, pb, pc-ba, \text{ class } 2\rangle \tag{7.2952}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc, pb, pc, \text{ class } 2\rangle \tag{7.2953}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc, pb-ba, pc, \text{ class } 2\rangle \tag{7.2954}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc, pc, pd, \text{ class } 2\rangle \tag{7.2955}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc, pc-ba, pd, \text{ class } 2\rangle \tag{7.2956}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc-ba, pa, pc, \text{ class } 2\rangle \tag{7.2957}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc-ba, pa-ba, pc, \text{ class } 2\rangle \tag{7.2958}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc-ba, pa, pb, \text{ class } 2\rangle \tag{7.2959}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, dc-ba, pa-ba, pb, \text{ class } 2\rangle \tag{7.2960}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pb, pc, pd, \text{ class } 2\rangle \tag{7.2961}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pc, pd, \text{ class } 2\rangle \tag{7.2962}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pb-ba, pc, pd, \text{ class } 2\rangle \tag{7.2963}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pb-ca, pc, pd, \text{ class } 2\rangle \tag{7.2964}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pc-ba, pd, \text{ class } 2\rangle \tag{7.2965}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pc-ca, pd, \text{ class } 2\rangle \tag{7.2966}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa-ba, pc, pd, \text{ class } 2\rangle \tag{7.2967}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa-ca, pc, pd, \text{ class } 2\rangle \tag{7.2968}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pb-ba, pc-xca, pd, \text{ class } 2\rangle \ (x \neq 0, x \sim x^{-1}) \tag{7.2969}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pb-ba-ca, pc-ca, pd, \text{ class } 2\rangle \tag{7.2970}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pb-\omega ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.2971}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pb-xca, pc-ba-ca, pd, \text{ class } 2\rangle \ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.2972}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa-ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.2973}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa-ba, pc-ca, pd, \text{ class } 2\rangle \tag{7.2974}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pb, pc, \text{ class } 2\rangle \tag{7.2975}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa-ba, pb, pc, \text{ class } 2\rangle \tag{7.2976}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pb-ba, pc, \text{ class } 2\rangle \tag{7.2977}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa-ca, pb-ba, pc, \text{ class } 2\rangle \tag{7.2978}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pb-ca, pc, \text{ class } 2\rangle \tag{7.2979}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa-ba, pb-ca, pc, \text{ class } 2\rangle \tag{7.2980}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pb-ba, pc-xca, \text{ class } 2\rangle \ (x \neq 0,\ x \sim x^{-1}) \tag{7.2981}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pb-ba-ca, pc-ca, \text{ class } 2\rangle \tag{7.2982}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pb-\omega ca, pc-ba, \text{ class } 2\rangle \tag{7.2983}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc, pa, pb-xca, pc-ba-ca, \text{ class } 2\rangle \ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.2984}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa, pb, pd - ca, \text{ class } 2\rangle \tag{7.2985}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa, pb - ba, pd - ca, \text{ class } 2\rangle \tag{7.2986}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa - ba, pb, pd - ca, \text{ class } 2\rangle \tag{7.2987}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa, pc, pd - ca, \text{ class } 2\rangle \tag{7.2988}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa, pc - ba, pd - ca, \text{ class } 2\rangle \tag{7.2989}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa - ba, pc, pd - ca, \text{ class } 2\rangle \tag{7.2990}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pb, pc, pd - ca, \text{ class } 2\rangle \tag{7.2991}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pb, pc - ba, pd - ca, \text{ class } 2\rangle \tag{7.2992}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pb - ba, pc, pd - ca, \text{ class } 2\rangle \tag{7.2993}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb, pc, \text{ class } 2\rangle \tag{7.2994}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb, pc - ba, \text{ class } 2\rangle \tag{7.2995}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb, pc - dc, \text{ class } 2\rangle \tag{7.2996}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb, pc - ba - dc, \text{ class } 2\rangle \tag{7.2997}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - ba, pc, \text{ class } 2\rangle \tag{7.2998}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - ba, pc - ba, \text{ class } 2\rangle \tag{7.2999}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - ba, pc - dc, \text{ class } 2\rangle \tag{7.3000}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - ba, pc - ba - dc, \text{ class } 2\rangle \tag{7.3001}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - dc, pc, \text{ class } 2\rangle \tag{7.3002}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - dc, pc - ba, \text{ class } 2\rangle \tag{7.3003}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - dc, pc - dc, \text{ class } 2\rangle \tag{7.3004}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - dc, pc - ba - dc, \text{ class } 2\rangle \tag{7.3005}$$

$$\langle a, b, c, d \,|\, ca, da, cb, db, pa, pb - ba - dc, pc, \text{ class } 2\rangle \tag{7.3006}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pb - ba - dc, pc - ba, \text{ class } 2\rangle \tag{7.3007}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pb - ba - dc, pc - xba - dc, \text{ class } 2\rangle \text{ (all } x) \tag{7.3008}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pb - dc, pc, \text{ class } 2\rangle \tag{7.3009}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pb - dc, pc - ba, \text{ class } 2\rangle \tag{7.3010}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pb - dc, pc - dc, \text{ class } 2\rangle \tag{7.3011}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pb - dc, pc - ba - dc, \text{ class } 2\rangle \tag{7.3012}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc, pd - pb, \text{ class } 2\rangle \tag{7.3013}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc, pd + ba - pb, \text{ class } 2\rangle \tag{7.3014}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc, pd + ba + dc - pb, \text{ class } 2\rangle \tag{7.3015}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc - ba, pd - pb, \text{ class } 2\rangle \tag{7.3016}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc - ba, pd + dc - pb, \text{ class } 2\rangle \tag{7.3017}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc - dc, pd - pb, \text{ class } 2\rangle \tag{7.3018}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc - dc, pd + ba - pb, \text{ class } 2\rangle \tag{7.3019}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc - ba - dc, pd - pb, \text{ class } 2\rangle \tag{7.3020}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa, pc - ba - dc, pd + ba - pb, \text{ class } 2\rangle \tag{7.3021}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pc - ba, pd - pb, \text{ class } 2\rangle \tag{7.3022}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pc - ba, pd + dc - pb, \text{ class } 2\rangle \tag{7.3023}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pc - ba - xdc, pd - pb, \text{ class } 2\rangle \ (x \neq 0) \tag{7.3024}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba, pc - xdc, pd - pb, \text{ class } 2\rangle \ (x \neq 0,\ x \sim x^{-1}) \tag{7.3025}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - dc, pc - ba, pd - pb, \text{ class } 2\rangle \tag{7.3026}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - dc, pc - \omega ba, pd - pb, \text{ class } 2\rangle \tag{7.3027}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - dc, pc - xba - dc, pd - pb, \text{ class } 2\rangle \ (x \neq 0) \tag{7.3028}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba - dc, pc - xba - ydc, pd - pb, \text{ class } 2\rangle \ (x \neq 0,\ y \neq 0,\ y \sim y^{-1}) \tag{7.3029}$$

$$\langle a,b,c,d \,|\, ca, da, cb, db, pa - ba - dc, pc - xba - xdc, pd + ba - pb, \text{ class } 2\rangle \ (x \neq 0,\ x \sim x^{-1}) \tag{7.3030}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb, pc, \text{ class } 2\rangle \tag{7.3031}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa - ba, pb, pc, \text{ class } 2\rangle \tag{7.3032}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa - ca, pb, pc, \text{ class } 2\rangle \tag{7.3033}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - ba, pc, \text{ class } 2\rangle \tag{7.3034}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - ca, pc, \text{ class } 2\rangle \tag{7.3035}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa - ba, pb - ca, pc, \text{ class } 2\rangle \tag{7.3036}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa - ca, pb - ba, pc, \text{ class } 2\rangle \tag{7.3037}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb, pc - ba, \text{ class } 2\rangle \tag{7.3038}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa - ca, pb, pc - ba, \text{ class } 2\rangle \tag{7.3039}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - ba, pc - ba, \text{ class } 2\rangle \tag{7.3040}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa - ca, pb - ba, pc - ba, \text{ class } 2\rangle \tag{7.3041}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - ba - xca, pc - ba, \text{ class } 2\rangle \ (x \neq 0) \tag{7.3042}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - ca, pc - ba, \text{ class } 2\rangle \tag{7.3043}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - \omega ca, pc - ba, \text{ class } 2\rangle \tag{7.3044}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb, pc - ca, \text{ class } 2\rangle \tag{7.3045}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa - ba, pb, pc - ca, \text{ class } 2\rangle \tag{7.3046}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - xba, pc - ca, \text{ class } 2\rangle \ (x \neq 0) \tag{7.3047}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pa, pb - ba - ca, pc - ca, \text{ class } 2\rangle \tag{7.3048}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pb, pc, pd, \text{ class } 2\rangle \tag{7.3049}$$

$$\langle a,b,c,d \,|\, da, cb, db - ca, dc, pb - ba, pc, pd, \text{ class } 2\rangle \tag{7.3050}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb-ca, pc, pd, \text{ class } 2\rangle \tag{7.3051}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb, pc-ba, pd, \text{ class } 2\rangle \tag{7.3052}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb-ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.3053}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb-\omega ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.3054}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb-xba, pc-ca, pd, \text{ class } 2\rangle \text{ (all } x) \tag{7.3055}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb, pc, pd-ba, \text{ class } 2\rangle \tag{7.3056}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb-ca, pc, pd-ba, \text{ class } 2\rangle \tag{7.3057}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb, pc, pd-ca, \text{ class } 2\rangle \tag{7.3058}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb-ba, pc, pd-ca, \text{ class } 2\rangle \tag{7.3059}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb, pc-ba, pd-ca, \text{ class } 2\rangle \tag{7.3060}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc, pb, pc-ca, pd-ba, \text{ class } 2\rangle \tag{7.3061}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc-\omega ba, pa, pb, pc, \text{ class } 2\rangle \tag{7.3062}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc-\omega ba, pa-ba, pb, pc, \text{ class } 2\rangle \tag{7.3063}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc-\omega ba, pa, pb-ba-xca, pc, \text{ class } 2\rangle \text{ (all } x,\ x\sim -x) \tag{7.3064}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc-\omega ba, pa-ca, pb-ba-xca, pc, \text{ class } 2\rangle \text{ (all } x,\ x\sim -x) \tag{7.3065}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc-\omega ba, pa, pb-ca, pc, \text{ class } 2\rangle \tag{7.3066}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc-\omega ba, pa-ba, pb-ca, pc, \text{ class } 2\rangle \tag{7.3067}$$

$$\langle a,b,c,d \,|\, da, cb, db-ca, dc-\omega ba, pa, pb-xba-yca, pc-zba-tca, \text{ class } 2\rangle \tag{7.3068}$$

Here, in presentation 7.3068, we put the parameters x, y, z, t in a matrix $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$. (We require A to be non-singular.) We have an action on the matrices A defined by

$$A \to \alpha^{-1} \begin{pmatrix} \mu & \nu \\ \pm\omega\nu & \pm\mu \end{pmatrix} A \begin{pmatrix} \mu & \nu \\ \pm\omega\nu & \pm\mu \end{pmatrix}^{-1}.$$

Two matrices give isomorphic algebras if they lie in the same orbit under this action. There are $(p+1)^2/2$ orbits.

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb, pc, pd, \text{ class } 2\rangle \tag{7.3069}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - cb, pb, pc, pd, \text{ class } 2\rangle \tag{7.3070}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb, pc, pd, \text{ class } 2\rangle \tag{7.3071}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ca, pb - xcb, pc, pd, \text{ class } 2\rangle \ (x \neq 0, x \sim x^{-1}) \tag{7.3072}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ca - cb, pb - cb, pc, pd, \text{ class } 2\rangle \tag{7.3073}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - \omega cb, pb - ca, pc, pd, \text{ class } 2\rangle \tag{7.3074}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - xcb, pb - ca - cb, pc, pd, \text{ class } 2\rangle \ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.3075}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb - ca, pc, pd, \text{ class } 2\rangle \tag{7.3076}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb - cb, pc, pd, \text{ class } 2\rangle \tag{7.3077}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb - cb, pc - xba - ca, pd, \text{ class } 2\rangle \ (\text{all } x) \tag{7.3078}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb - ca, pc - cb, pd, \text{ class } 2\rangle \tag{7.3079}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb - ca, pc + cb, pd, \text{ class } 2\rangle \tag{7.3080}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb - ca, pc - \omega ba + cb, pd, \text{ class } 2\rangle \tag{7.3081}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb, pc, pd - cb, \text{ class } 2\rangle \tag{7.3082}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb, pc, pd - cb, \text{ class } 2\rangle \tag{7.3083}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb - ba, pc, pd - cb, \text{ class } 2\rangle \tag{7.3084}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ca, pb - ba, pc, pd - cb, \text{ class } 2\rangle \tag{7.3085}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb - ca, pc, pd - cb, \text{ class } 2\rangle \tag{7.3086}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa - ba, pb - ca, pc, pd - cb, \text{ class } 2\rangle \tag{7.3087}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb - ba, pc - xca, pd - cb, \text{ class } 2\rangle \ (x \neq 0, x \sim x^{-1}) \tag{7.3088}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb - ba - ca, pc - ca, pd - cb, \text{ class } 2\rangle \tag{7.3089}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb - \omega ca, pc - ba, pd - cb, \text{ class } 2\rangle \tag{7.3090}$$

$$\langle a,b,c,d \,|\, da, db, dc, pa, pb - xca, pc - ba - ca, pd - cb, \text{ class } 2\rangle \text{ (all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.3091}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc - cb, pd, \text{ class } 2\rangle \tag{7.3092}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - dc, pc - cb, pd, \text{ class } 2\rangle \tag{7.3093}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - ba, pc - cb, pd, \text{ class } 2\rangle \tag{7.3094}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - ba - dc, pc - cb, pd, \text{ class } 2\rangle \tag{7.3095}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - cb, pc - cb, pd, \text{ class } 2\rangle \tag{7.3096}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - cb, pc - cb - dc, pd, \text{ class } 2\rangle \tag{7.3097}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - cb, pc - ba - cb - dc, pd, \text{ class } 2\rangle \tag{7.3098}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc, pd, \text{ class } 2\rangle \tag{7.3099}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc - dc, pd, \text{ class } 2\rangle \tag{7.3100}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc - ba, pd, \text{ class } 2\rangle \tag{7.3101}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc - ba - dc, pd, \text{ class } 2\rangle \tag{7.3102}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - dc, pc - dc, pd, \text{ class } 2\rangle \tag{7.3103}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - dc, pc - ba, pd, \text{ class } 2\rangle \tag{7.3104}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - dc, pc - ba - dc, pd, \text{ class } 2\rangle \tag{7.3105}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - ba, pc - dc, pd, \text{ class } 2\rangle \tag{7.3106}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - ba, pc - ba - dc, pd, \text{ class } 2\rangle \tag{7.3107}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - ba - dc, pc - ba - xdc, pd, \text{ class } 2\rangle \ (x \neq 0) \tag{7.3108}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb - zcb, pc - xba - ycb, pd - dc, \text{ class } 2\rangle \ (x = 0,1,\ y = 0,1,\ z \neq 0,-1) \tag{7.3109}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc, pd - dc, \text{ class } 2\rangle \tag{7.3110}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc - ba, pd - dc, \text{ class } 2\rangle \tag{7.3111}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba, pc, pd-dc, \text{ class } 2\rangle \tag{7.3112}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba, pc-ba, pd-dc, \text{ class } 2\rangle \tag{7.3113}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb, pc-cb, pd-dc, \text{ class } 2\rangle \tag{7.3114}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba, pc-cb, pd-dc, \text{ class } 2\rangle \tag{7.3115}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb, pc, pd-dc, \text{ class } 2\rangle \tag{7.3116}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb, pc-cb, pd-dc, \text{ class } 2\rangle \tag{7.3117}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb, pc-ba, pd-dc, \text{ class } 2\rangle \tag{7.3118}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb, pc-ba-cb, pd-dc, \text{ class } 2\rangle \tag{7.3119}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb-dc, pc, pd-dc, \text{ class } 2\rangle \tag{7.3120}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb-dc, pc-cb, pd-dc, \text{ class } 2\rangle \tag{7.3121}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb-dc, pc-ba, pd-dc, \text{ class } 2\rangle \tag{7.3122}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb-dc, pc-ba-cb, pd-dc, \text{ class } 2\rangle \tag{7.3123}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba-cb-tdc, pc-ba-ydc, pd-cb, \text{ class } 2\rangle \text{ (all } t, y) \tag{7.3124}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-cb-tdc, pc-xba-ydc, pd-cb, \text{ class } 2\rangle \text{ (all } t,\ x=0,1,\ y=0,1) \tag{7.3125}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-tdc, pc-xba-ydc, pd-cb, \text{ class } 2\rangle\ (t=0,1,\omega,\ x=0,1,\ y=0,1) \tag{7.3126}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba-tdc, pc-ydc, pd-cb, \text{ class } 2\rangle\ (t=0,1,\omega,\ y=0,1) \tag{7.3127}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba-tdc, pc-ba-ydc, pd-cb, \text{ class } 2\rangle\ (t=1,\omega,\ \text{all } y) \tag{7.3128}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba, pc-ba-ydc, pd-cb, \text{ class } 2\rangle\ (y=0,1,\omega) \tag{7.3129}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ba-cb-tdc, pc-ydc, pd-cb, \text{ class } 2\rangle \text{ (all } t,\ y=0,1) \tag{7.3130}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-cb, pc, pd-ba, \text{ class } 2\rangle \tag{7.3131}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-cb, pc-dc, pd-ba, \text{ class } 2\rangle \tag{7.3132}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-cb, pc-cb, pd-ba, \text{ class } 2\rangle \tag{7.3133}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-cb, pc-cb-dc, pd-ba, \text{ class } 2\rangle \tag{7.3134}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-ydc, pc-zcb-tdc, pd-ba, \text{ class } 2\rangle \ (y=0,1,\ z=0,1,\ t=0,1) \tag{7.3135}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-xcb, pc-tdc, pd-ba-dc, \text{ class } 2\rangle \ (\text{all } x,\ t=0,1) \tag{7.3136}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-xcb-dc, pc-ydc, pd-ba-dc, \text{ class } 2\rangle \ (x=0,-1, y=0,1) \tag{7.3137}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb-xcb-ydc, pc-cb, pd-ba-dc, \text{ class } 2\rangle \ (\text{all } x,\ \ y=0,1) \tag{7.3138}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa, pb+cb-xdc, pc-cb-dc, pd-ba-dc, \text{ class } 2\rangle \ (\text{all } x) \tag{7.3139}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-xba-ycb, pc-cb, pd-dc, \text{ class } 2\rangle \ (x=0,1,\ \text{all } y) \tag{7.3140}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-ycb, pc-xba, pd-dc, \text{ class } 2\rangle \ (x=0,1,\ \text{all } y) \tag{7.3141}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-ba, pc-zba, pd-dc, \text{ class } 2\rangle \ (z=0,1) \tag{7.3142}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-xba-cb, pc-yba-dc, pd-cb, \text{ class } 2\rangle \ (\text{all } x,\ y) \tag{7.3143}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-ba-cb, pc-yba, pd-cb, \text{ class } 2\rangle \ (\text{all } y) \tag{7.3144}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-cb, pc-yba, pd-cb, \text{ class } 2\rangle \ (y=0,1,\omega) \tag{7.3145}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-ba, pc-yba-dc, pd-cb, \text{ class } 2\rangle \ (\text{all } y) \tag{7.3146}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb, pc-yba-dc, pd-cb, \text{ class } 2\rangle \ (y=0,1) \tag{7.3147}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-xba, pc-yba, pd-cb, \text{ class } 2\rangle \ (x=0,1,\ y=0,1) \tag{7.3148}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-ba, pc-\omega ba, pd-cb, \text{ class } 2\rangle \ (p=1 \bmod 3) \tag{7.3149}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-ba, pc-\omega^2 ba, pd-cb, \text{ class } 2\rangle \ (p=1 \bmod 3) \tag{7.3150}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb, pc, pd-ba, \text{ class } 2\rangle \tag{7.3151}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-cb, pc-cb-dc, pd-ba, \text{ class } 2\rangle \tag{7.3152}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-cb, pc-ycb, pd-ba, \text{ class } 2\rangle \ (\text{all } y) \tag{7.3153}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-xcb, pc-ycb, pd-ba-dc, \text{ class } 2\rangle \ (\text{all } x,\ y) \tag{7.3154}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-dc, pb-xcb, pc-(1+x)^{-1}cb-dc, pd-ba-dc, \text{ class } 2\rangle \ (x \neq -1) \tag{7.3155}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba-ydc, pc-zba-tcb, pd-dc, \text{ class } 2\rangle \ (y=0,1, \text{ all } z,t) \tag{7.3156}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ydc, pc-zba-cb, pd-dc, \text{ class } 2\rangle \ (y=0,1, \text{ all } z) \tag{7.3157}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ydc, pc-zba, pd-dc, \text{ class } 2\rangle \ (y=0,1, \ z=0,1,\omega) \tag{7.3158}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb, pc, pd-cb, \text{ class } 2\rangle \tag{7.3159}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb, pc-ba, pd-cb, \text{ class } 2\rangle \tag{7.3160}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb, pc-\omega ba, pd-cb, \text{ class } 2\rangle \tag{7.3161}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-dc, pc-ba, pd-cb, \text{ class } 2\rangle \tag{7.3162}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-dc, pc-\omega ba, pd-cb, \text{ class } 2\rangle \tag{7.3163}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-\omega dc, pc-\omega ba, pd-cb, \text{ class } 2\rangle \tag{7.3164}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ydc, pc-zba-dc, pd-cb, \text{ class } 2\rangle \ (y=1,\omega, \text{ all } z) \tag{7.3165}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb, pc-xba-dc, pd-cb, \text{ class } 2\rangle \ (x=0,1,\omega) \tag{7.3166}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba-ydc, pc-zba-tdc, pd-cb, \text{ class } 2\rangle \ (y=1,\omega, \text{ all } z, \ t\neq 0,1,-1, \ t\sim t^{-1}) \tag{7.3167}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba, pc-zba-tdc, pd-cb, \text{ class } 2\rangle \ (z=0,1,\omega, \ t\neq 0,1,-1, \ t\sim t^{-1}) \tag{7.3168}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba-dc, pc-xba-tdc, pd-cb, \text{ class } 2\rangle \ (t=1,-1, x\neq 0) \tag{7.3169}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba-\omega dc, pc-(\omega * x^2)ba-tdc, pd-cb, \text{ class } 2\rangle \ (t=1,-1, \ x\neq 0, \ x\sim -x) \tag{7.3170}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba, pc-zba-tdc, pd-cb, \text{ class } 2\rangle \ (t=1,-1, z=0,1,\omega) \tag{7.3171}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba-ydc, pc-zba-tdc, pd-cb-dc, \text{ class } 2\rangle \ (\text{all } y,z,t) \tag{7.3172}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ydc, pc-zba-dc, pd-cb-dc, \text{ class } 2\rangle \ (\text{all } y,z) \tag{7.3173}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ydc, pc-zba, pd-cb-dc, \text{ class } 2\rangle \ (\text{all } y, \ z=0,1,\omega) \tag{7.3174}$$

$$\langle a,b,c,d \,|\, ca, da, db, pa-cb, pb-ba-ydc, pc-zcb-tdc, pd-ba-dc, \text{ class } 2\rangle \ (\text{all } y,z,t) \tag{7.3175}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-cb,pb-dc,pc-zcb-tdc,pd-ba-dc,\text{ class 2}\rangle\ (\text{all } z,t) \tag{7.3176}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-cb,pb,pc-cb-tdc,pd-ba-dc,\text{ class 2}\rangle\ (\text{all } t) \tag{7.3177}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-cb,pb,pc-tdc,pd-ba-dc,\text{ class 2}\rangle\ (t=0,1,\omega) \tag{7.3178}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-cb,pb-xba-ydc,pc-zba-tdc,pd-ba-cb-dc,\text{ class 2}\rangle\ (\text{all } x,\ y,z,t,\ (x,\ y,z,t)\sim(t+1,z+1,y-1,x-1)) \tag{7.3179}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba,pb-xcb,pc-tcb,pd-dc,\text{ class 2}\rangle\ (\text{all } x,t,\ (x,t)\sim(t,x)) \tag{7.3180}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba,pb-xcb,pc-ba+cb,pd-dc,\text{ class 2}\rangle\ (\text{all } x) \tag{7.3181}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba,pb+cb-dc,pc-ba+cb,pd-dc,\text{ class 2}\rangle \tag{7.3182}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba,pb-xcb,pc-zcb,pd-ba-dc,\text{ class 2}\rangle\ (\text{all } x,z) \tag{7.3183}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba,pb+cb-dc,pc-zcb,pd-ba-dc,\text{ class 2}\rangle\ (\text{all } z) \tag{7.3184}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba,pb-xcb,pc+cb-dc,pd-ba-dc,\text{ class 2}\rangle\ (x\neq-1) \tag{7.3185}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba-dc,pb-xcb,pc-zcb,pd-ba-tdc,\text{ class 2}\rangle\ (t\neq0,\ \text{all } x,z,\ (x,z)\sim(tz,\frac{x}{t})) \tag{7.3186}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba-dc,pb-xcb,pc-zcb-dc,pd-ba-dc,\text{ class 2}\rangle\ (x,\ z\neq0,\ (x,z)\sim(z,x)) \tag{7.3187}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba-dc,pb-dc,pc-zcb,pd-ba-dc,\text{ class 2}\rangle\ (z\neq0) \tag{7.3188}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba-dc,pb,pc-dc,pd-ba-dc,\text{ class 2}\rangle \tag{7.3189}$$

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba-dc,pb-dc,pc-xdc,pd-ba-dc,\text{ class 2}\rangle\ (x\neq0,\ x\sim x^{-1}) \tag{7.3190}$$

In the presentation below we have $t\neq0,1$, all x,z such that $(x+t)(1+z)=1 \bmod p$, $(x,z)\sim(tz,\frac{x}{t})$:

$$\langle a,b,c,d\,|\,ca,da,db,pa-ba-dc,pb-xcb,pc-zcb-dc,pd-ba-tdc,\text{ class 2}\rangle \tag{7.3191}$$

$$\langle a,b,c,d\,|\,cb,db,dc,pa,pb-ba,pc-ca,pd-xda,\text{ class 2}\rangle\ (x\neq0) \tag{7.3192}$$

$$\langle a,b,c,d\,|\,cb,db,dc,pa,pb-ba-ca,pc-ca,pd-da,\text{ class 2}\rangle \tag{7.3193}$$

$$\langle a,b,c,d\,|\,cb,db,dc,pa,pb-xda,pc-ba-yda,pd-ca-da,\text{ class 2}\rangle\ (x\neq0,\ \text{all } y) \tag{7.3194}$$

$$\langle a,b,c,d\,|\,cb,db,dc,pa,pb-xda,pc-ba-yda,pd-ca,\text{ class 2}\rangle\ (x\neq0,\ x\sim-x,\ y=1,\omega) \tag{7.3195}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-da, pc-ba, pd-ca, \text{ class } 2\rangle \tag{7.3196}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-\omega da, pc-ba, pd-ca, \text{ class } 2\rangle \ (p=1 \bmod 3) \tag{7.3197}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-\omega^2 da, pc-ba, pd-ca, \text{ class } 2\rangle \ (p=1 \bmod 3) \tag{7.3198}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-ca, pc-da, pd, \text{ class } 2\rangle \tag{7.3199}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-ba, pb-ca, pc-da, pd, \text{ class } 2\rangle \tag{7.3200}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-ba, pc-da, pd, \text{ class } 2\rangle \tag{7.3201}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-ca, pb-ba, pc-da, pd, \text{ class } 2\rangle \tag{7.3202}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-ba, pc-ca, pd, \text{ class } 2\rangle \tag{7.3203}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-da, pb-ba, pc-ca, pd, \text{ class } 2\rangle \tag{7.3204}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-xca, pc-ba-ca, pd, \text{ class } 2\rangle \ (x \neq 0) \tag{7.3205}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-da, pb-xca, pc-ba-ca, pd, \text{ class } 2\rangle \ (x \neq 0) \tag{7.3206}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.3207}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-da, pb-ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.3208}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-\omega ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.3209}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-da, pb-\omega ca, pc-ba, pd, \text{ class } 2\rangle \tag{7.3210}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-ba, pc, pd, \text{ class } 2\rangle \tag{7.3211}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-ca, pb-ba, pc, pd, \text{ class } 2\rangle \tag{7.3212}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb-ca, pc, pd, \text{ class } 2\rangle \tag{7.3213}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-ba, pb-ca, pc, pd, \text{ class } 2\rangle \tag{7.3214}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-da, pb-ca, pc, pd, \text{ class } 2\rangle \tag{7.3215}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa, pb, pc, pd, \text{ class } 2\rangle \tag{7.3216}$$

$$\langle a,b,c,d \,|\, cb, db, dc, pa-ba, pb, pc, pd, \text{ class } 2\rangle \tag{7.3217}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb, pc, pd, \text{ class } 2\rangle \tag{7.3218}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-ba, pc, pd, \text{ class } 2\rangle \tag{7.3219}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-cb, pc, pd, \text{ class } 2\rangle \tag{7.3220}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-cb, pc-ba, pd, \text{ class } 2\rangle \tag{7.3221}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-xdb, pc-ba, pd, \text{ class } 2\rangle \text{ (all } x) \tag{7.3222}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb, pc-cb, pd, \text{ class } 2\rangle \tag{7.3223}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-ba, pc-cb, pd, \text{ class } 2\rangle \tag{7.3224}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-db, pc-cb, pd, \text{ class } 2\rangle \tag{7.3225}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb, pc-db, pd, \text{ class } 2\rangle \tag{7.3226}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-ba, pc-db, pd, \text{ class } 2\rangle \tag{7.3227}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-\omega ba, pc-db, pd, \text{ class } 2\rangle \tag{7.3228}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-cb, pc-db, pd, \text{ class } 2\rangle \tag{7.3229}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-xdb, pc-ba, pd-cb, \text{ class } 2\rangle \text{ (all } x) \tag{7.3230}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb, pc-ba, pd-db, \text{ class } 2\rangle \tag{7.3231}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-cb, pc-ba, pd-db, \text{ class } 2\rangle \tag{7.3232}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb, pc-cb, pd-db, \text{ class } 2\rangle \tag{7.3233}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-ba, pc-cb, pd-db, \text{ class } 2\rangle \tag{7.3234}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb, pc-cb-db, pd-db, \text{ class } 2\rangle \tag{7.3235}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-ba, pc-cb-db, pd-db, \text{ class } 2\rangle \tag{7.3236}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb-\omega ba, pc-cb-db, pd-db, \text{ class } 2\rangle \tag{7.3237}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa, pb, pc-xdb, pd-cb-db, \text{ class } 2\rangle \ (x \neq 0, -\frac{1}{4}) \tag{7.3238}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa,pb-ba,pc-xdb,pd-cb-db,\text{ class 2}\rangle\ (x\neq 0,-\frac{1}{4}) \tag{7.3239}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa,pb,pc-db,pd-cb,\text{ class 2}\rangle \tag{7.3240}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa,pb-ba,pc-db,pd-cb,\text{ class 2}\rangle \tag{7.3241}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa,pb,pc-\omega db,pd-cb,\text{ class 2}\rangle \tag{7.3242}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa,pb-ba,pc-\omega db,pd-cb,\text{ class 2}\rangle \tag{7.3243}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc,pd,\text{ class 2}\rangle \tag{7.3244}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb-cb,pc,pd,\text{ class 2}\rangle \tag{7.3245}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-xcb,pd,\text{ class 2}\rangle\ (x\neq 0) \tag{7.3246}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb-db,pc-xcb,pd,\text{ class 2}\rangle\ (x\neq 0) \tag{7.3247}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc,pd-cb,\text{ class 2}\rangle \tag{7.3248}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb-db,pc,pd-cb,\text{ class 2}\rangle \tag{7.3249}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-xcb,pd-xdb,\text{ class 2}\rangle\ (x\neq 0) \tag{7.3250}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-ba+cb,pd+db,\text{ class 2}\rangle \tag{7.3251}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-xcb,pd-ydb,\text{ class 2}\rangle\ (x,\ y\neq 0,\ x\neq y,\ (x,\ y)\sim(y,x)) \tag{7.3252}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-xcb,pd-ba+db,\text{ class 2}\rangle\ (x\neq 0,-1) \tag{7.3253}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-xcb,pd-cb-xdb,\text{ class 2}\rangle\ (x\neq 0) \tag{7.3254}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-ba+cb,pd-cb+db,\text{ class 2}\rangle \tag{7.3255}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-ba,pb,pc-xdb,pd-cb-ydb,\text{ class 2}\rangle\ (x\neq 0,\ \text{all } y \text{ with } 4x+y^2 \text{ not a square } \bmod p) \tag{7.3256}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-cb,pb,pc,pd,\text{ class 2}\rangle \tag{7.3257}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-cb,pb-ba,pc,pd,\text{ class 2}\rangle \tag{7.3258}$$

$$\langle a,b,c,d\,|\,ca,da,dc-ba,pa-cb,pb-db,pc,pd,\text{ class 2}\rangle \tag{7.3259}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-\omega db, pc, pd, \text{ class } 2\rangle \tag{7.3260}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb, pc, pd-ba, \text{ class } 2\rangle \tag{7.3261}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-ba, pc, pd-ba, \text{ class } 2\rangle \tag{7.3262}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-db, pc, pd-ba, \text{ class } 2\rangle \tag{7.3263}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-\omega db, pc, pd-ba, \text{ class } 2\rangle \tag{7.3264}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xba-ydb, pc, pd-db, \text{ class } 2\rangle \ (x=0,1, \text{ all } y) \tag{7.3265}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-ydb, pc-ba, pd-xdb, \text{ class } 2\rangle \ (\text{all } x,\ y,\ x\sim -x) \tag{7.3266}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-ydb, pc-\omega ba, pd-xdb, \text{ class } 2\rangle \ (\text{all } x,\ y,\ x\sim -x) \tag{7.3267}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-ba-(x^2+1)db, pc-ba, pd-xdb, \text{ class } 2\rangle \ (\text{all } x,\ x\sim -x) \tag{7.3268}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-ba-(x^2+\omega)db, pc-\omega ba, pd-xdb, \text{ class } 2\rangle \ (\text{all } x,\ x\sim -x) \tag{7.3269}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xba-zdb, pc-db, pd, \text{ class } 2\rangle \ (\text{all } x,\ z=1,\omega) \tag{7.3270}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb, pc-db, pd, \text{ class } 2\rangle \tag{7.3271}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-ba, pc-db, pd, \text{ class } 2\rangle \tag{7.3272}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-\omega ba, pc-db, pd, \text{ class } 2\rangle \tag{7.3273}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-\omega^2 ba, pc-db, pd, \text{ class } 2\rangle \ (p=1 \bmod 4) \tag{7.3274}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-\omega^3 ba, pc-db, pd, \text{ class } 2\rangle \ (p=1 \bmod 4) \tag{7.3275}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb, pc-db, pd-ba, \text{ class } 2\rangle \tag{7.3276}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb, pc-db, pd-\omega ba, \text{ class } 2\rangle \ (p=1 \bmod 3) \tag{7.3277}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb, pc-db, pd-\omega^2 ba, \text{ class } 2\rangle \ (p=1 \bmod 3) \tag{7.3278}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xdb, pc-db, pd-ba, \text{ class } 2\rangle \ (x\neq 0, x\sim x' \text{ if } x^3=x'^3 \bmod p) \tag{7.3279}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xdb, pc-db, pd-\omega ba, \text{ class } 2\rangle \ (x\neq 0,\ x\sim x' \text{ if } x^3=x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.3280}$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xdb, pc-db, pd-\omega^2 ba, \text{ class } 2\rangle \ (x \neq 0, x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \quad (7.3281)$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xba-ydb, pc-db, pd-ba, \text{ class } 2\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ \text{all } y) \quad (7.3282)$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xba-ydb, pc-db, pd-\omega ba, \text{ class } 2\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ \text{all } y,\ p = 1 \bmod 3) \quad (7.3283)$$

$$\langle a,b,c,d \,|\, ca, da, dc-ba, pa-cb, pb-xba-ydb, pc-db, pd-\omega^2 ba, \text{ class } 2\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ \text{all } y,\ p = 1 \bmod 3) \quad (7.3284)$$

$$\langle a,b,c,d \,|\, cb, da, db-ca, pa-jba-kca-mdc, pb-nba-rca-sdc, pc-tba-uca-vdc, pd-xba-yca-zdc, \text{ class } 2\rangle \quad (7.3285)$$

In presentation 7.3285 we have 12 parameters giving the values of pa, pb, pc, pd. We write

$$\begin{pmatrix} pa \\ pb \\ pc \\ pd \end{pmatrix} = A \begin{pmatrix} ba \\ ca \\ dc \end{pmatrix}$$

with A a 4×3 matrix. The isomorphism classes given by the matrices correspond to their orbits under transformations of the form

$$A \longmapsto (\alpha\delta - \beta\gamma)^{-1} \begin{pmatrix} \alpha\lambda & \beta\lambda & \beta\mu & -\alpha\mu \\ \gamma\lambda & \delta\lambda & \delta\mu & -\gamma\mu \\ \gamma\nu & \delta\nu & \delta\xi & -\gamma\xi \\ -\alpha\nu & -\beta\nu & -\beta\xi & \alpha\xi \end{pmatrix} A \begin{pmatrix} \lambda^2 & 2\lambda\mu & \mu^2 \\ \lambda\nu & \lambda\xi + \mu\nu & \mu\xi \\ \nu^2 & 2\nu\xi & \xi^2 \end{pmatrix}^{-1}.$$

We note that if we multiply $\alpha, \beta, \gamma, \delta$ through by a factor k (in the expression above), and multiply λ, μ, ν, ξ through by a factor l, then the image of A is multiplied by a factor $k^{-1}l^{-1}$. So we can ignore the factor $(\alpha\delta - \beta\gamma)^{-1}$ and still get the same orbits.

If we take $\mu = 0$ in the matrices above, then we obtain a subgroup H of the automorphism group of index $p+1$. The subspace spanned by a, b is invariant under H, and it turns out that pa, pb has 11 orbits under the action of H. So the problem of computing orbits of 12 parameters reduces to 11 problems with 6 parameters. There are 550 orbits when $p = 3$ and

$$\begin{aligned} p^5 + p^4 + 4p^3 + 6p^2 + 18p + 19 \text{ if } p &= 1 \bmod 3, \\ p^5 + p^4 + 4p^3 + 6p^2 + 16p + 17 \text{ if } p &= 2 \bmod 3 \end{aligned}$$

orbits when $p > 3$. This implies that we need presentations with at least 5 parameters to describe this situation, so 6 parameters is not at all bad! Nevertheless computing the orbits for a specific prime p require serious computational effort.

$$\langle a,b,c,d \,|\, da, db-\omega ca, dc-ba, pa-jba-kca-mcb, pb-nba-rca-scb, pc-tba-uca-vcb, pd-xba-yca-zcb, \text{ class } 2\rangle \quad (7.3286)$$

In presentation 7.3286 we again have 12 parameters giving the values of pa, pb, pc, pd. We let

$$\begin{pmatrix} pa \\ pb \\ pc \\ pd \end{pmatrix} = A \begin{pmatrix} ba \\ ca \\ cb \end{pmatrix}$$

where A is a 4×3 matrix. The isomorphism classes given by the matrices correspond to their orbits under transformations of the form

$$A \mapsto \begin{pmatrix} \alpha & 0 & 0 & \delta \\ \pm\lambda & \pm\gamma & \pm\omega\beta & \pm\mu \\ \nu & \beta & \gamma & \xi \\ \pm\omega\delta & 0 & 0 & \pm\alpha \end{pmatrix} AB^{-1},$$

where

$$B = \begin{pmatrix} \pm(\alpha\gamma - \omega\beta\delta) & \pm(\omega\alpha\beta - \omega\gamma\delta) & 0 \\ \alpha\beta - \gamma\delta & \alpha\gamma - \omega\beta\delta & 0 \\ \pm(\beta\lambda - \gamma\nu + \omega\beta\xi - \gamma\mu) & \pm(\gamma\lambda - \omega\beta\mu + \omega\gamma\xi - \omega\beta\nu) & \pm(\gamma^2 - \omega\beta^2) \end{pmatrix}.$$

We note that $\langle a, d\rangle + L^2$ is a characteristic subalgebra, and first investigate the orbits of pa, pd. We consider three separate cases: $pa = pd = 0$, pa and pd span a one dimensional subspace, and pa, pd are linearly independent. It turns out that there are $p + 4$ orbits of pa, pd. We pick a set of orbit representatives for pa, pd, and work out a set of orbit representatives for pc, pd in each case. There are $\frac{9}{2}p + \frac{13}{2} + 3p^2 + \frac{1}{2}p^4 + \frac{1}{2}p^3$ orbits of matrices A.

8.198 Descendants of 5.3

$p^4 + 5p^3 + 19p^2 + 64p + 140 + (p + 6)\gcd(p - 1, 3) + (p + 7)\gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa, pb, pc - baa, pd - bab, \text{ class } 3\rangle \tag{7.3287}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3288}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - baa, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3289}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa, pb - baa, pc - bab, pd, \text{ class } 3\rangle \tag{7.3290}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa, pb - \omega baa, pc - bab, pd, \text{ class } 3\rangle \tag{7.3291}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3292}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3293}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3294}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3295}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - baa, pb - xbab, pc, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x^{-1}) \tag{7.3296}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - baa - bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3297}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - baa - \omega bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3298}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - \omega bab, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3299}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa - xbab, pb - baa - bab, pc, pd, \text{ class } 3\rangle \ (\text{all } x \text{ with } 1 + 4x \text{ not a square } \bmod p) \tag{7.3300}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3301}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3302}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3303}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3304}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-xbaa, pb-bab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3305}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-baa-bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3306}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-baa-\omega bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3307}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-baa-xbab, pb-baa, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3308}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3309}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-bab, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3310}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega bab, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3311}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-baa-xbab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3312}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3313}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-bab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3314}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega bab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3315}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb-xbaa, pc-bab, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3316}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-baa, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.3317}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.3318}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-bab, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.3319}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega bab, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.3320}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega^2 bab, pb, pc-baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.3321}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega^3 bab, pb, pc-baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.3322}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-xbab, pb-bab, pc-baa, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.3323}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-xbab, pb-\omega bab, pc-baa, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.3324}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-xbab, pb-\omega^2 bab, pc-baa, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.3325}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb, pc-bab, pd-baa, \text{ class } 3\rangle \tag{7.3326}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.3327}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-bab, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.3328}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega bab, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.3329}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb-bab, pc, pd-baa, \text{ class } 3\rangle \tag{7.3330}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-bab, pb-bab, pc, pd-baa, \text{ class } 3\rangle \tag{7.3331}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-\omega bab, pb-bab, pc, pd-baa, \text{ class } 3\rangle \tag{7.3332}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb, pc-baa, pd-bab, \text{ class } 3\rangle \tag{7.3333}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb-baa, pc, pd-bab, \text{ class } 3\rangle \tag{7.3334}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb-\omega baa, pc, pd-bab, \text{ class } 3\rangle \tag{7.3335}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.3336}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc, pa-baa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.3337}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3338}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3339}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3340}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3341}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-\omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3342}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-\omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3343}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-bab, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3344}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-bab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3345}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-\omega bab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.3346}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-\omega bab, pb-baa, pc, pd, \text{ class } 3\rangle \ (p=3 \bmod 4) \tag{7.3347}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-xbab, pb-baa, pc, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.3348}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-xbab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.3349}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa, pb-xbab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim x^{-1}) \tag{7.3350}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-bab, pb-xbab, pc, pd, \text{ class } 3\rangle \ (x\neq 0) \tag{7.3351}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-\omega bab, pb-xbab, pc, pd, \text{ class } 3\rangle \ (x\neq 0) \tag{7.3352}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-bab, pb-xbaa-ybab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ y\neq 0,1,-1,\ y\sim y^{-1}) \tag{7.3353}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-\omega bab, pb-xbaa-ybab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ y\neq 0,1,-1,\ y\sim y^{-1}) \tag{7.3354}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-bab, pb-x^2baa+bab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.3355}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-bab, pb-x^2baa-bab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.3356}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-bab, pb-(x^2*\omega)baa+bab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.3357}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-bab, pb-(x^2*\omega)baa-bab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x) \tag{7.3358}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-\omega bab, pb-(x^2*\omega)baa+bab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x,\ p=1 \bmod 4) \tag{7.3359}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-\omega bab, pb-(x^2*\omega)baa-bab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x,\ p=1 \bmod 4) \tag{7.3360}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-\omega bab, pb-x^2baa+bab, pc, pd, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x,\ p=3 \bmod 4) \tag{7.3361}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa-\omega bab, pb-x^2baa-bab, pc, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim -x,\ p = 3 \bmod 4) \tag{7.3362}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.3363}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-bab, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.3364}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-\omega bab, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.3365}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-xbab, pb-bab, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.3366}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-xbab, pb-ybab, pc-baa-bab, pd, \text{ class } 3\rangle \ (\text{all } x,\ y,\ (x,\ y) \sim (y,x)) \tag{7.3367}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa, pb, pc-bab, pd-baa, \text{ class } 3\rangle \tag{7.3368}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.3369}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-bab, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.3370}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-\omega bab, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.3371}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa, pb-bab, pc, pd-baa, \text{ class } 3\rangle \tag{7.3372}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-bab, pb-bab, pc, pd-baa, \text{ class } 3\rangle \tag{7.3373}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-\omega bab, pb-bab, pc, pd-baa, \text{ class } 3\rangle \tag{7.3374}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa, pb, pc-baa, pd-baa-bab, \text{ class } 3\rangle \tag{7.3375}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa, pb, pc, pd-baa-bab, \text{ class } 3\rangle \tag{7.3376}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa, pb, pc, pd-baa-bab, \text{ class } 3\rangle \tag{7.3377}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-\omega baa, pb, pc, pd-baa-bab, \text{ class } 3\rangle \tag{7.3378}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa, pb-(\omega * x^2)baa, pc, pd-baa-bab, \text{ class } 3\rangle \ (x \neq 0,\ x \sim -x) \tag{7.3379}$$

$$\langle a,b,c,d \,|\, ca-baa, cb, da, db, dc, pa-baa, pb-x^2baa, pc, pd-baa-bab, \text{ class } 3\rangle \ (x \neq 0,\ \text{equivalence classes } \{x, -x, \frac{1}{x}, -\frac{1}{x}\}) \tag{7.3380}$$

$$\langle a,b,c,d\,|\,ca-baa, cb, da, db, dc, pa-\omega baa, pb-(\omega * x^2)baa, pc, pd-baa-bab, \text{ class } 3\rangle \ (x \neq 0, \text{ equivalence classes } \{x, -x, \frac{1}{x}, -\frac{1}{x}\}) \tag{7.3381}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3382}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-xbaa-bab, pb, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.3383}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-xbaa-\omega bab, pb, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.3384}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3385}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-\omega baa, pb, pc, pd, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.3386}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-xbaa-ybab, pb-zbaa-tbab, pc, pd, \text{ class } 3\rangle \tag{7.3387}$$

In presentation 7.3387, if we write the parameters x, y, z, t in a matrix $A = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$ (which is assumed to be non-singular) then two matrices give isomorphic algebras if and only if they are in the same orbit under the action

$$A \to \frac{1}{\det P} PAP^{-1},$$

where P lies in the group of non-singular matrices of the form

$$\begin{pmatrix} \alpha & \beta \\ \pm\omega\beta & \pm\alpha \end{pmatrix}.$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-xbaa, pb-ybaa, pc-bab, pd, \text{ class } 3\rangle \ (\text{all } x,\ y,\ x \sim -x) \tag{7.3388}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa, pb, pc-bab, pd-baa, \text{ class } 3\rangle \tag{7.3389}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-bab, pb-xbab, pc, pd-baa, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.3390}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa-\omega bab, pb-xbab, pc, pd-baa, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.3391}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.3392}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa, pb-bab, pc, pd-baa, \text{ class } 3\rangle \tag{7.3393}$$

$$\langle a,b,c,d\,|\,ca-bab, cb-\omega baa, da, db, dc, pa, pb-\omega bab, pc, pd-baa, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.3394}$$

$$\langle a,b,c,d\,|\,ca, cb-baa, da, db-bab, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3395}$$

$$\langle a,b,c,d\,|\,ca, cb-baa, da, db-bab, dc, pa, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3396}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3397}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3398}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - baa, pb - xbab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3399}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - baa, pb - baa - bab, pc, pd, \text{ class } 3\rangle \tag{7.3400}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - baa, pb - \omega baa - bab, pc, pd, \text{ class } 3\rangle \tag{7.3401}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3402}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - bab, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3403}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - bab, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3404}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - bab, pb - xbaa - bab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3405}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3406}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3407}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3408}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb - xbaa - bab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3409}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.3410}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb - bab, pc, pd - baa, \text{ class } 3\rangle \tag{7.3411}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - bab, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.3412}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - \omega bab, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.3413}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3414}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb - baa, pc, pd - bab, \text{ class } 3\rangle \tag{7.3415}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb - \omega baa, pc, pd - bab, \text{ class } 3\rangle \tag{7.3416}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa - baa, pb - xbaa, pc, pd - bab, \text{ class } 3\rangle \text{ (all } x) \tag{7.3417}$$

$$\langle a, b, c, d \,|\, ca, cb - baa, da, db - bab, dc, pa, pb, pc - baa, pd - xbab, \text{ class } 3\rangle \ (x \neq 0) \tag{7.3418}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb, pc-baa, pd-baa-bab, \text{ class } 3\rangle \tag{7.3419}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-xbab, pb, pc-baa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3420}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-xbab, pb-bab, pc-baa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3421}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb, pc-bab, pd-xbaa-bab, \text{ class } 3\rangle \ (x \neq 0) \tag{7.3422}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb, pc-bab, pd-baa, \text{ class } 3\rangle \tag{7.3423}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb, pc-bab, pd-\omega baa, \text{ class } 3\rangle \tag{7.3424}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.3425}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-baa, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.3426}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-\omega baa, pb, pc-bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.3427}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-\omega^2 baa, pb, pc-bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.3428}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb-baa, pc-bab, pd, \text{ class } 3\rangle \tag{7.3429}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb-\omega baa, pc-bab, pd, \text{ class } 3\rangle \tag{7.3430}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb-\omega^2 baa, pc-bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.3431}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa, pb-\omega^3 baa, pc-bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.3432}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-baa, pb-xbaa, pc-bab, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.3433}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-\omega baa, pb-xbaa, pc-bab, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.3434}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-\omega^2 baa, pb-xbaa, pc-bab, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.3435}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, da, db-bab, dc, pa-xbaa, pb-ybaa, pc-bab, pd-bab, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.3436}$$

$$\langle a,b,c,d \,|\, ca-bab, cb-baa, da, db-bab, dc, pa-rbaa-sbab, pb-tbaa-ubab, pc-vbaa-xbab, pd-ybaa-zbab, \text{ class } 3\rangle \tag{7.3437}$$

In presentation 7.3437 we write

$$\begin{pmatrix} pa \\ pb \\ pc \\ pd \end{pmatrix} = A \begin{pmatrix} baa \\ bab \end{pmatrix}$$

where A is 4×2 matrix. Two matrices A give isomorphic algebras if they lie in the same orbit under the action

$$A \to \frac{1}{\alpha^2+\beta^2} \begin{pmatrix} \alpha & -\beta & \gamma & \delta \\ \pm\beta & \pm\alpha & \pm\lambda & \pm\mu \\ 0 & 0 & \alpha^2-\beta^2 & -4\alpha\beta \\ 0 & 0 & \pm\alpha\beta & \pm(\alpha^2-\beta^2) \end{pmatrix} A \begin{pmatrix} \pm\alpha & \mp\beta \\ \beta & \alpha \end{pmatrix}^{-1}.$$

$$\langle a,b,c,d \,|\, ca-bab, cb-\omega baa, da, db-bab, dc, pa-rbaa-sbab, pb-tbaa-ubab, pc-vbaa-xbab, pd-ybaa-zbab, \text{ class } 3\rangle \tag{7.3438}$$

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$$A \to \frac{1}{\alpha^2+\omega\beta^2} \begin{pmatrix} \alpha & \beta & \gamma & \delta \\ \mp\omega\beta & \pm\alpha & \pm\lambda & \pm\mu \\ 0 & 0 & \alpha^2-\omega\beta^2 & 4\omega\alpha\beta \\ 0 & 0 & \mp\alpha\beta & \pm(\alpha^2-\omega\beta^2) \end{pmatrix} A \begin{pmatrix} \pm\alpha & \pm\beta \\ -\omega\beta & \alpha \end{pmatrix}^{-1}.$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3439}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3440}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3441}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3442}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-baa, pb-xbab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3443}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-baa, pb-baa-bab, pc, pd, \text{ class } 3\rangle \tag{7.3444}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-baa, pb-\omega baa-bab, pc, pd, \text{ class } 3\rangle \tag{7.3445}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3446}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-bab, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3447}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-bab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3448}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-bab, pb-xbaa-bab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3449}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb,pc,pd, \text{ class } 3\rangle \tag{7.3450}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-baa,pc,pd, \text{ class } 3\rangle \tag{7.3451}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-\omega baa,pc,pd, \text{ class } 3\rangle \tag{7.3452}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-xbaa-bab,pc,pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3453}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb,pc-baa,pd, \text{ class } 3\rangle \tag{7.3454}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-baa,pc-baa,pd, \text{ class } 3\rangle \tag{7.3455}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-\omega baa,pc-baa,pd, \text{ class } 3\rangle \tag{7.3456}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-bab,pc-baa,pd, \text{ class } 3\rangle \tag{7.3457}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa,pb-xbab,pc-baa,pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3458}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa,pb-baa-bab,pc-baa,pd, \text{ class } 3\rangle \tag{7.3459}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa,pb-\omega baa-bab,pc-baa,pd, \text{ class } 3\rangle \tag{7.3460}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb,pc-baa,pd, \text{ class } 3\rangle \tag{7.3461}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb-baa,pc-baa,pd, \text{ class } 3\rangle \tag{7.3462}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb-\omega baa,pc-baa,pd, \text{ class } 3\rangle \tag{7.3463}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb-xbaa-bab,pc-baa,pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3464}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb,pc-baa,pd, \text{ class } 3\rangle \tag{7.3465}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-baa,pc-baa,pd, \text{ class } 3\rangle \tag{7.3466}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-\omega baa,pc-baa,pd, \text{ class } 3\rangle \tag{7.3467}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-xbaa-bab,pc-baa,pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3468}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa-xbab,pb-baa-ybab,pc-bab,pd, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.3469}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa-xbab,pb-\omega baa-ybab,pc-bab,pd, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.3470}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa,pb-xbab,pc-bab,pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3471}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa-bab,pb-xbab,pc-bab,pd,\text{ class 3}\rangle \text{ (all } x) \tag{7.3472}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-baa-\omega bab,pb-xbab,pc-bab,pd,\text{ class 3}\rangle \text{ (all } x) \tag{7.3473}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-xbab,pb-baa-bab,pc-bab,pd,\text{ class 3}\rangle \text{ (all } x) \tag{7.3474}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-xbab,pb-\omega baa-bab,pc-bab,pd,\text{ class 3}\rangle \text{ (all } x) \tag{7.3475}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-baa,pc-bab,pd,\text{ class 3}\rangle \tag{7.3476}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb-baa,pc-bab,pd,\text{ class 3}\rangle \tag{7.3477}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-baa,pc-bab,pd,\text{ class 3}\rangle \tag{7.3478}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-\omega baa,pc-bab,pd,\text{ class 3}\rangle \tag{7.3479}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb-\omega baa,pc-bab,pd,\text{ class 3}\rangle \tag{7.3480}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-\omega baa,pc-bab,pd,\text{ class 3}\rangle \tag{7.3481}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb,pc-bab,pd,\text{ class 3}\rangle \tag{7.3482}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb,pc-bab,pd,\text{ class 3}\rangle \tag{7.3483}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb,pc-bab,pd,\text{ class 3}\rangle \tag{7.3484}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-bab,pc-bab,pd,\text{ class 3}\rangle \tag{7.3485}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-bab,pb-bab,pc-bab,pd,\text{ class 3}\rangle \tag{7.3486}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa-\omega bab,pb-bab,pc-bab,pd,\text{ class 3}\rangle \tag{7.3487}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb,pc-baa,pd-bab,\text{ class 3}\rangle \tag{7.3488}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-baa,pc-baa,pd-bab,\text{ class 3}\rangle \tag{7.3489}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-\omega baa,pc-baa,pd-bab,\text{ class 3}\rangle \tag{7.3490}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-\omega^2 baa,pc-baa,pd-bab,\text{ class 3}\rangle \ (p=1 \bmod 4) \tag{7.3491}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-\omega^3 baa,pc-baa,pd-bab,\text{ class 3}\rangle \ (p=1 \bmod 4) \tag{7.3492}$$

$$\langle a,b,c,d \,|\, ca,cb,da,db,dc-baa,pa,pb-bab,pc-baa,pd-bab,\text{ class 3}\rangle \tag{7.3493}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-baa, pb-xbab, pc-baa, pd-bab, \text{ class } 3\rangle \text{ (all } x) \tag{7.3494}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-baa, pb-xbaa-bab, pc-baa, pd-bab, \text{ class } 3\rangle \ (x \neq 0) \tag{7.3495}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-bab, pb-xbaa-ybab, pc-baa, pd-bab, \text{ class } 3\rangle \text{ (all } x,\ y,\ y \sim -y) \tag{7.3496}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-baa, pa-\omega bab, pb-xbaa-ybab, pc-baa, pd-bab, \text{ class } 3\rangle \text{ (all } x,\ y,\ y \sim -y) \tag{7.3497}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3498}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3499}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3500}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3501}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa, pb-xbab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3502}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa, pb-baa-bab, pc, pd, \text{ class } 3\rangle \tag{7.3503}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa, pb-\omega baa-bab, pc, pd, \text{ class } 3\rangle \tag{7.3504}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3505}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3506}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3507}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb-xbaa-bab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3508}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3509}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.3510}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3511}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb-xbaa-bab, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3512}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.3513}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-baa, pc-baa, pd, \text{ class } 3\rangle \tag{7.3514}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega baa, pc-baa, pd, \text{ class } 3\rangle \tag{7.3515}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-bab, pc-baa, pd, \text{ class } 3\rangle \tag{7.3516}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega bab, pc-baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3517}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega^2 bab, pc-baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3518}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa, pb-xbab, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.3519}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega baa, pb-xbab, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.3520}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^2 baa, pb-xbab, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.3521}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa, pb-xbaa-bab, pc-baa, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.3522}$$

In the next two presentations we have $x \neq 0,\ x \sim x'$ if $x^3 = x'^3 \bmod p,\ p = 1 \bmod 3$:

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega baa, pb-xbaa-\omega bab, pc-baa, pd, \text{ class } 3\rangle \tag{7.3523}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^2 baa, pb-xbaa-\omega^2 bab, pc-baa, pd, \text{ class } 3\rangle \tag{7.3524}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb-xbaa-ybab, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ y,\ y \sim -y,\ p=3 \bmod 4) \tag{7.3525}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb-xbaa-ybab, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ y,\ y \sim -y,\ p=3 \bmod 4) \tag{7.3526}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb-xbaa, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p=1 \bmod 4) \tag{7.3527}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb-xbaa, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p=1 \bmod 4) \tag{7.3528}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^2 bab, pb-xbaa, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p=1 \bmod 4) \tag{7.3529}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^3 bab, pb-xbaa, pc-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p=1 \bmod 4) \tag{7.3530}$$

In the next four presentations we have all $x,\ y \neq 0,\ y \sim y'$ if $y^4 = y'^4 \bmod p,\ p = 1 \bmod 4$:

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb-xbaa-ybab, pc-baa, pd, \text{ class } 3\rangle \tag{7.3531}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb-xbaa-ybab, pc-baa, pd, \text{ class } 3\rangle \tag{7.3532}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^2 bab, pb-xbaa-ybab, pc-baa, pd, \text{ class } 3\rangle \tag{7.3533}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^3 bab, pb-xbaa-ybab, pc-baa, pd, \text{ class } 3\rangle \tag{7.3534}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa-xbab, pb-ybaa-zbab, pc-bab, pd, \text{ class } 3\rangle \text{ (all } x,\ y, z) \tag{7.3535}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-bab, pc-bab, pd, \text{ class } 3\rangle \text{ (all } x,\ y) \tag{7.3536}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa, pc-bab, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3537}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-bab, pb-xbaa, pc-bab, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3538}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega bab, pb-xbaa, pc-bab, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3539}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb, pc, pd-baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3540}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-baa, pc, pd-baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3541}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-\omega baa, pc, pd-baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3542}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-bab, pc, pd-baa, \text{ class } 3\rangle \ (x \neq -2,\ \text{all } y) \tag{7.3543}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb, pc, pd-\omega baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3544}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-baa, pc, pd-\omega baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3545}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-\omega baa, pc, pd-\omega baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3546}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-bab, pc, pd-\omega baa, \text{ class } 3\rangle \ (x \neq -2\omega,\ \text{all } y) \tag{7.3547}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa+2bab, pb-xbaa, pc, pd-baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3548}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa+2\omega bab, pb-xbaa, pc, pd-\omega baa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3549}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-zbab, pc-bab, pd-baa, \text{ class } 3\rangle \ (x \neq -2,\ \text{all } y, z, z \sim -z) \tag{7.3550}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-zbab, pc-bab, pd-\omega baa, \text{ class } 3\rangle \ (x \neq -2\omega, \text{ all } y, z, \ z \sim -z) \tag{7.3551}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa+2bab, pb-ybaa, pc-bab, pd-baa, \text{ class } 3\rangle \ (\text{all } x, \ y, \ x \sim -x) \tag{7.3552}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa+2\omega bab, pb-ybaa, pc-bab, pd-\omega baa, \text{ class } 3\rangle \ (\text{all } x, \ y, \ x \sim -x) \tag{7.3553}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-bab, pb-ybaa-zbab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, z) \tag{7.3554}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-\omega bab, pb-ybaa-zbab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, z) \tag{7.3555}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa, pb-xbaa-ybab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3556}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega baa, pb-xbaa-ybab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3557}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^2 baa, pb-xbaa-ybab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3558}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega^3 baa, pb-xbaa-ybab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3559}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-baa, pb-xbaa-ybab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, \ p = 3 \bmod 4) \tag{7.3560}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-\omega baa, pb-xbaa-ybab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ y, \ p = 3 \bmod 4) \tag{7.3561}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-bab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3562}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-\omega bab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3563}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-\omega^2 bab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3564}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-\omega^3 bab, pc, pd-bab, \text{ class } 3\rangle \ (\text{all } x, \ x \sim -x, \ p = 1 \bmod 4) \tag{7.3565}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-bab, pc, pd-bab, \text{ class } 3\rangle \text{ (all } x,\ p=3 \bmod 4) \tag{7.3566}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-\omega bab, pc, pd-bab, \text{ class } 3\rangle \text{ (all } x,\ p=3 \bmod 4) \tag{7.3567}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.3568}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-baa, pc, pd-bab, \text{ class } 3\rangle \tag{7.3569}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega baa, pc, pd-bab, \text{ class } 3\rangle \tag{7.3570}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega^2 baa, pc, pd-bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3571}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega^3 baa, pc, pd-bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3572}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega^4 baa, pc, pd-bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3573}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-\omega^5 baa, pc, pd-bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3574}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-ybab, pb-zbaa-tbab, pc-baa, pd-bab, \text{ class } 3\rangle \text{ (all } x,\ y,z,t,\ p\neq 1 \bmod 5) \tag{7.3575}$$

In the next 5 presentations we have $x \neq 0$, $x \sim x'$ if $x^5 = x'^5 \bmod p$, all y, z, t, $p = 1 \bmod 5$:

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-ybab, pb-zbaa-tbab, pc-baa, pd-bab, \text{ class } 3\rangle \tag{7.3576}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-ybab, pb-zbaa-tbab, pc-\omega baa, pd-bab, \text{ class } 3\rangle \tag{7.3577}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-ybab, pb-zbaa-tbab, pc-\omega^2 baa, pd-bab, \text{ class } 3\rangle \tag{7.3578}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-ybab, pb-zbaa-tbab, pc-\omega^3 baa, pd-bab, \text{ class } 3\rangle \tag{7.3579}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbaa-ybab, pb-zbaa-tbab, pc-\omega^4 baa, pd-bab, \text{ class } 3\rangle \tag{7.3580}$$

In the next 5 presentations we have $x \neq 0$, $x \sim x'$ if $x^5 = x'^5 \bmod p$, all y, z, $p = 1 \bmod 5$:

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-zbab, pc-baa, pd-bab, \text{ class } 3\rangle \tag{7.3581}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-zbab, pc-\omega baa, pd-bab, \text{ class } 3\rangle \tag{7.3582}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-zbab, pc-\omega^2 baa, pd-bab, \text{ class } 3\rangle \tag{7.3583}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-zbab, pc-\omega^3 baa, pd-bab, \text{ class } 3\rangle \tag{7.3584}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa-xbab, pb-ybaa-zbab, pc-\omega^4 baa, pd-bab, \text{ class } 3\rangle \tag{7.3585}$$

In the next 5 presentations we have $x \neq 0$, $x \sim x'$ if $x^5 = x'^5 \bmod p$, all y, $p = 1 \bmod 5$:

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-ybab, pc-baa, pd-bab, \text{ class } 3\rangle \tag{7.3586}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-ybab, pc-\omega baa, pd-bab, \text{ class } 3\rangle \tag{7.3587}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-ybab, pc-\omega^2 baa, pd-bab, \text{ class } 3\rangle \tag{7.3588}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-ybab, pc-\omega^3 baa, pd-bab, \text{ class } 3\rangle \tag{7.3589}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbaa-ybab, pc-\omega^4 baa, pd-bab, \text{ class } 3\rangle \tag{7.3590}$$

And in the next 5 presentations we have $x \neq 0$, $x \sim x'$ if $x^5 = x'^5 \bmod p$, $p = 1 \bmod 5$:

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbab, pc-baa, pd-bab, \text{ class } 3\rangle \tag{7.3591}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbab, pc-\omega baa, pd-bab, \text{ class } 3\rangle \tag{7.3592}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbab, pc-\omega^2 baa, pd-bab, \text{ class } 3\rangle \tag{7.3593}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbab, pc-\omega^3 baa, pd-bab, \text{ class } 3\rangle \tag{7.3594}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb-xbab, pc-\omega^4 baa, pd-bab, \text{ class } 3\rangle \tag{7.3595}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc-baa, pd-bab, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.3596}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc-\omega baa, pd-bab, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.3597}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc-\omega^2 baa, pd-bab, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.3598}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc-\omega^3 baa, pd-bab, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.3599}$$

$$\langle a,b,c,d \,|\, ca-bab, cb, da, db, dc-baa, pa, pb, pc-\omega^4 baa, pd-bab, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.3600}$$

8.199 Descendants of 6.10

8 algebras

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3601}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db - p^2 d, dc, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3602}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc - p^2 d, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3603}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db - p^2 d, dc - p^2 d, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3604}$$

$$\langle a, b, c, d \,|\, ca - p^2 d, cb, da, db, dc, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3605}$$

$$\langle a, b, c, d \,|\, ca - p^2 d, cb, da, db - p^2 d, dc, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3606}$$

$$\langle a, b, c, d \,|\, ca - p^2 d, cb, da, db, dc - p^2 d, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3607}$$

$$\langle a, b, c, d \,|\, ca - p^2 d, cb, da, db - p^2 d, dc - p^2 d, pa, pb, pc - ba, \text{ class } 3\rangle \tag{7.3608}$$

8.200 Descendants of 6.11

39 algebras

$$\langle a, b, c, d \,|\, baa, bab, ca, cb, da, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{7.3609}$$

$$\langle a, b, c, d \,|\, baa, bab, ca - p^2 a, cb, da, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{7.3610}$$

$$\langle a, b, c, d \,|\, baa, bab, ca, cb - p^2 a, da, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{7.3611}$$

$$\langle a, b, c, d \,|\, baa, bab, ca, cb - p^2 a, da - p^2 a, db, dc, pb, pc, pd, \text{ class } 3\rangle \tag{7.3612}$$

$$\langle a, b, c, d \,|\, baa, bab, ca, cb, da, db, dc - p^2 a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3613}$$

$$\langle a, b, c, d \,|\, baa, ca, cb, da, db, dc - bab, p^2 a - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3614}$$

$$\langle a, b, c, d \,|\, baa, ca, cb, da, db, dc, p^2 a - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3615}$$

$$\langle a, b, c, d \,|\, baa, ca - bab, cb, da, db, dc, p^2 a - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3616}$$

$$\langle a, b, c, d \,|\, baa, ca, cb, da, db, dc - bab, p^2 a - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3617}$$

$$\langle a, b, c, d \,|\, baa, ca, cb, da, db, dc, p^2 a - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3618}$$

$$\langle a, b, c, d \,|\, baa, ca - bab, cb, da, db, dc, p^2 a - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3619}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dc - bab, p^2a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3620}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dc - bab, p^2a, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3621}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dc - bab, p^2a, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3622}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dc, p^2a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3623}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dc, p^2a, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3624}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dc, p^2a, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3625}$$

$$\langle a,b,c,d \,|\, baa, ca - bab, cb, da, db, dc, p^2a, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3626}$$

$$\langle a,b,c,d \,|\, baa, ca - bab, cb, da, db, dc, p^2a, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3627}$$

$$\langle a,b,c,d \,|\, baa, ca - bab, cb, da, db, dc, p^2a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3628}$$

$$\langle a,b,c,d \,|\, baa, ca - bab, cb, da, db, dc, p^2a, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3629}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc, p^2a - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3630}$$

$$\langle a,b,c,d \,|\, bab, ca, cb - baa, da, db, dc, p^2a - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3631}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, p^2a - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3632}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, p^2a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3633}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, p^2a, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3634}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, p^2a, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3635}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, p^2a, pb, pc - baa, pd, \text{ class } 3\rangle \tag{7.3636}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, p^2a, pb - baa, pc - baa, pd, \text{ class } 3\rangle \tag{7.3637}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, p^2a, pb - \omega baa, pc - baa, pd, \text{ class } 3\rangle \tag{7.3638}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc, p^2a, pb, pc - baa, pd, \text{ class } 3\rangle \tag{7.3639}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc, p^2a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3640}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc, p^2a, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3641}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, p^2a, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3642}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, p^2a, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.3643}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, p^2a, pb, pc - baa, pd, \text{ class } 3\rangle \tag{7.3644}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, p^2a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3645}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, p^2a, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3646}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, p^2a, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3647}$$

8.201 Descendants of 6.12

8 algebras

$$\langle a, b, c, d \,|\, baa, ca, cb, da, db, dc, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3648}$$

$$\langle a, b, c, d \,|\, baa, ca - p^2a, cb, da, db, dc, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3649}$$

$$\langle a, b, c, d \,|\, baa, ca, cb - p^2a, da - p^2a, db, dc, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3650}$$

$$\langle a, b, c, d \,|\, baa, ca, cb - p^2a, da, db, dc, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3651}$$

$$\langle a, b, c, d \,|\, baa, ca, cb, da, db, dc - p^2a, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3652}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, p^2a, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3653}$$

$$\langle a, b, c, d \,|\, ca - baa, cb, da, db, dc, p^2a, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3654}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc - baa, p^2a, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3655}$$

8.202 Descendants of 6.13

11 algebras

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3656}$$

$$\langle a, b, c, d \,|\, ca - p^2a, cb, da, db, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3657}$$

$$\langle a, b, c, d \,|\, ca, cb - p^2a, da, db, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3658}$$

$$\langle a, b, c, d \,|\, ca, cb - \omega p^2a, da, db, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3659}$$

$$\langle a, b, c, d \,|\, ca, cb, da - p^2a, db, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3660}$$

$$\langle a, b, c, d \,|\, ca, cb - p^2a, da - p^2a, db, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3661}$$

$$\langle a, b, c, d \,|\, ca, cb - \omega p^2a, da - p^2a, db, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3662}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db - p^2a, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3663}$$

$$\langle a, b, c, d \,|\, ca - p^2a, cb, da, db - p^2a, dc, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3664}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc - p^2a, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3665}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db - p^2a, dc - p^2a, pb, pc - ba, pd, \text{ class } 3\rangle \tag{7.3666}$$

8.203 Descendants of 6.14

39 algebras

$$\langle a, b, c, d \,|\, baa, bab, ca, cb, da, db, dc, pa, pb, pd, \text{ class } 3\rangle \tag{7.3667}$$

$$\langle a, b, c, d \,|\, baa, bab, ca - p^2c, cb, da, db, dc, pa, pb, pd, \text{ class } 3\rangle \tag{7.3668}$$

$$\langle a, b, c, d \,|\, baa, bab, ca, cb, da - p^2c, db, dc, pa, pb, pd, \text{ class } 3\rangle \tag{7.3669}$$

$$\langle a, b, c, d \,|\, baa, bab, ca, cb - p^2c, da - p^2c, db, dc, pa, pb, pd, \text{ class } 3\rangle \tag{7.3670}$$

$$\langle a, b, c, d \,|\, baa, bab, ca, cb, da, db, dc - p^2c, pa, pb, pd, \text{ class } 3\rangle \tag{7.3671}$$

$$\langle a, b, c, d \,|\, baa, bab, ca, cb, da - p^2c, db, dc - p^2c, pa, pb, pd, \text{ class } 3\rangle \tag{7.3672}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb, p^2c, pd - baa, \text{ class } 3\rangle \tag{7.3673}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb - baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3674}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb - \omega baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3675}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3676}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa - baa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3677}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, pa, pb, p^2c, pd - baa, \text{ class } 3\rangle \tag{7.3678}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, pa, pb - baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3679}$$

$$\langle a, b, c, d \,|\, bab, ca, cb - baa, da, db, dc, pa, pb - \omega baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3680}$$

$$\langle a,b,c,d \,|\, bab, ca, cb - baa, da, db, dc, pa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3681}$$

$$\langle a,b,c,d \,|\, bab, ca, cb - baa, da, db, dc, pa - baa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3682}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc, pa, pb, p^2c, pd - baa, \text{ class } 3\rangle \tag{7.3683}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc, pa, pb - baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3684}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc, pa, pb - \omega baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3685}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc, pa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3686}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc, pa - baa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3687}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, pa, pb, p^2c, pd - baa, \text{ class } 3\rangle \tag{7.3688}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, pa, pb - baa, p^2c, pd - baa, \text{ class } 3\rangle \tag{7.3689}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, pa, pb - \omega baa, p^2c, pd - baa, \text{ class } 3\rangle \tag{7.3690}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, pa, pb - baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3691}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, pa, pb - \omega baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3692}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, pa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3693}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baa, pa - baa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3694}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc - baa, pa, pb, p^2c, pd - baa, \text{ class } 3\rangle \tag{7.3695}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc - baa, pa, pb - baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3696}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc - baa, pa, pb - \omega baa, p^2c, pd, \text{ class } 3\rangle \tag{7.3697}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc - baa, pa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3698}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc - baa, pa - baa, pb, p^2c, pd, \text{ class } 3\rangle \tag{7.3699}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc, pa, pb, p^2c - baa, pd, \text{ class } 3\rangle \tag{7.3700}$$

$$\langle a,b,c,d \,|\, bab, ca, cb - baa, da, db, dc, pa, pb, p^2c - baa, pd, \text{ class } 3\rangle \tag{7.3701}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db - baa, dc, pa, pb, p^2c - baa, pd, \text{ class } 3\rangle \tag{7.3702}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc - baa, pa, pb, p^2c - baa, pd, \text{ class } 3\rangle \tag{7.3703}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db - baa, dc - baa, pa, pb, p^2c - baa, pd, \text{ class } 3\rangle \tag{7.3704}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db - \omega baa, dc - baa, pa, pb, p^2c - baa, pd, \text{ class } 3\rangle \tag{7.3705}$$

8.204 Descendants of 6.15

$2p + 6$ algebras

$$\langle a, b, c, d \,|\, ca, cb - xp^2c, da, db, dc, pa - ba, pb, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3706}$$

$$\langle a, b, c, d \,|\, ca - p^2c, cb, da, db, dc, pa - ba, pb, pd, \text{ class } 3\rangle \tag{7.3707}$$

$$\langle a, b, c, d \,|\, ca, cb - xp^2c, da - p^2c, db, dc, pa - ba, pb, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3708}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db - p^2c, dc, pa - ba, pb, pd, \text{ class } 3\rangle \tag{7.3709}$$

$$\langle a, b, c, d \,|\, ca - p^2c, cb, da, db - p^2c, dc, pa - ba, pb, pd, \text{ class } 3\rangle \tag{7.3710}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc - p^2c, pa - ba, pb, pd, \text{ class } 3\rangle \tag{7.3711}$$

$$\langle a, b, c, d \,|\, ca, cb, da - p^2c, db, dc - p^2c, pa - ba, pb, pd, \text{ class } 3\rangle \tag{7.3712}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db - p^2c, dc - p^2c, pa - ba, pb, pd, \text{ class } 3\rangle \tag{7.3713}$$

8.205 Descendants of 6.16

6 algebras

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc - ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3714}$$

$$\langle a, b, c, d \,|\, ca - p^2a, cb, da, db, dc - ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3715}$$

$$\langle a, b, c, d \,|\, ca, cb, da, db, dc - ba - p^2a, pb, pc, pd, \text{ class } 3\rangle \tag{7.3716}$$

$$\langle a, b, c, d \,|\, ca, cb - p^2a, da, db, dc - ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3717}$$

$$\langle a, b, c, d \,|\, ca, cb - p^2a, da - p^2a, db, dc - ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3718}$$

$$\langle a, b, c, d \,|\, ca, cb - p^2a, da - \omega p^2a, db, dc - ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3719}$$

8.206 Descendants of 6.17

$p+7$ algebras

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3720}$$

$$\langle a,b,c,d \,|\, ca-p^2a, cb, da, db, dc-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3721}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-ba-p^2a, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3722}$$

$$\langle a,b,c,d \,|\, ca-p^2a, cb, da, db, dc-ba-p^2a, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3723}$$

$$\langle a,b,c,d \,|\, ca, cb-p^2a, da, db, dc-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3724}$$

$$\langle a,b,c,d \,|\, ca, cb-p^2a, da-p^2a, db, dc-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3725}$$

$$\langle a,b,c,d \,|\, ca, cb-p^2a, da-\omega p^2a, db, dc-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3726}$$

$$\langle a,b,c,d \,|\, ca, cb-p^2a, da-xp^2a, db, dc-ba-p^2a, pb-ba, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3727}$$

8.207 Descendants of 6.18

$3p+5$ algebras

$$\langle a,b,c,d \,|\, ca, cb, da-xp^2a, db, dc-ba, pb, pc-ba, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3728}$$

$$\langle a,b,c,d \,|\, ca-p^2a, cb, da, db, dc-ba, pb, pc-ba, pd, \text{ class } 3\rangle \tag{7.3729}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db, dc-ba-p^2a, pb, pc-ba, pd, \text{ class } 3\rangle \tag{7.3730}$$

$$\langle a,b,c,d \,|\, ca, cb, da, db-p^2a, dc-ba, pb, pc-ba, pd, \text{ class } 3\rangle \tag{7.3731}$$

$$\langle a,b,c,d \,|\, ca-p^2a, cb, da, db-p^2a, dc-ba, pb, pc-ba, pd, \text{ class } 3\rangle \tag{7.3732}$$

$$\langle a,b,c,d \,|\, ca-\omega p^2a, cb, da, db-p^2a, dc-ba, pb, pc-ba, pd, \text{ class } 3\rangle \tag{7.3733}$$

$$\langle a,b,c,d \,|\, ca, cb-p^2a, da-xp^2a, db, dc-ba, pb, pc-ba, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3734}$$

$$\langle a,b,c,d \,|\, ca, cb-\omega p^2a, da-xp^2a, db, dc-ba, pb, pc-ba, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3735}$$

8.208 Descendants of 6.19

97 algebras

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3736}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3737}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3738}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3739}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa - bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3740}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3741}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3742}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc, pa, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3743}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3744}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3745}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa - bab, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3746}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa - \omega bab, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3747}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3748}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3749}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3750}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa - bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3751}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3752}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da, db, dc - bab, pa - \omega bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3753}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3754}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3755}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3756}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3757}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3758}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa - bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3759}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3760}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac, da - bab, db, dc, pa - \omega bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3761}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3762}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3763}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3764}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc, pa - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3765}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3766}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3767}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc, pa, pb, pc - \omega bab, pd, \text{ class } 3\rangle \tag{7.3768}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3769}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.3770}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc - \omega bab, pd, \text{ class } 3\rangle \tag{7.3771}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.3772}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3773}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3774}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa - bab, cac, da, db, dc - bab, pa - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3775}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac + bab, da, db, dc, pa, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.3776}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac + bab, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3777}$$

$$\langle a, b, c, d \,|\, cb, baa, bac, caa, cac + bab, da, db, dc, pa - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3778}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da, db, dc, pa, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3779}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da, db, dc, pa-bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3780}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da, db, dc, pa-\omega bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3781}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da, db, dc, pa, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.3782}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da, db, dc, pa-bab, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.3783}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da, db, dc, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.3784}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3785}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da, db, dc, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3786}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da, db, dc, pa, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3787}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da, db, dc, pa-bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3788}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da, db, dc, pa-\omega bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3789}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.3790}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3791}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3792}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3793}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa-bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3794}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa-\omega bab, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.3795}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.3796}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+bab, da-bab, db, dc, pa-bab, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.3797}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da-bab, db, dc, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.3798}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da-bab, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3799}$$

$$\langle a,b,c,d \,|\, cb, baa, bac, caa, cac+\omega bab, da-bab, db, dc, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.3800}$$

$$\langle a,b,c,d\,|\,cb,baa,bac,caa,cac+\omega bab,da-bab,db,dc,pa,pb-bab,pc,pd,\text{ class 3}\rangle \tag{7.3801}$$

$$\langle a,b,c,d\,|\,cb,baa,bac,caa,cac+\omega bab,da-bab,db,dc,pa-bab,pb-bab,pc,pd,\text{ class 3}\rangle \tag{7.3802}$$

$$\langle a,b,c,d\,|\,cb,baa,bac,caa,cac+\omega bab,da-bab,db,dc,pa-\omega bab,pb-bab,pc,pd,\text{ class 3}\rangle \tag{7.3803}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc,pa,pb,pc,pd-baa,\text{ class 3}\rangle \tag{7.3804}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc,pa,pb,pc,pd,\text{ class 3}\rangle \tag{7.3805}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc,pa-baa,pb,pc,pd,\text{ class 3}\rangle \tag{7.3806}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc,pa,pb-baa,pc,pd,\text{ class 3}\rangle \tag{7.3807}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc,pa,pb-\omega baa,pc,pd,\text{ class 3}\rangle \tag{7.3808}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc,pa,pb,pc-baa,pd,\text{ class 3}\rangle \tag{7.3809}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db-baa,dc,pa,pb,pc,pd-baa,\text{ class 3}\rangle \tag{7.3810}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db-baa,dc,pa,pb,pc-baa,pd-baa,\text{ class 3}\rangle \tag{7.3811}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db-baa,dc,pa,pb,pc-baa,pd,\text{ class 3}\rangle \tag{7.3812}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db-baa,dc,pa,pb-baa,pc,pd,\text{ class 3}\rangle \tag{7.3813}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db-baa,dc,pa,pb-\omega baa,pc,pd,\text{ class 3}\rangle \tag{7.3814}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db-baa,dc,pa,pb,pc,pd,\text{ class 3}\rangle \tag{7.3815}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db-baa,dc,pa-baa,pb,pc,pd,\text{ class 3}\rangle \tag{7.3816}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc-baa,pa,pb,pc,pd-baa,\text{ class 3}\rangle \tag{7.3817}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc-baa,pa,pb-baa,pc,pd-baa,\text{ class 3}\rangle \tag{7.3818}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc-baa,pa,pb-\omega baa,pc,pd-baa,\text{ class 3}\rangle \tag{7.3819}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc-baa,pa,pb,pc-baa,pd,\text{ class 3}\rangle \tag{7.3820}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc-baa,pa,pb-baa,pc-baa,pd,\text{ class 3}\rangle \tag{7.3821}$$

$$\langle a,b,c,d\,|\,cb,bab,bac,caa,cac,da,db,dc-baa,pa,pb-\omega baa,pc-baa,pd,\text{ class 3}\rangle \tag{7.3822}$$

$$\langle a, b, c, d \,|\, cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3823}$$

$$\langle a, b, c, d \,|\, cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3824}$$

$$\langle a, b, c, d \,|\, cb, bab, bac, caa, cac, da, db, dc - baa, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3825}$$

$$\langle a, b, c, d \,|\, cb, bab, bac, caa, cac, da, db, dc - baa, pa - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3826}$$

$$\langle a, b, c, d \,|\, cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.3827}$$

$$\langle a, b, c, d \,|\, cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3828}$$

$$\langle a, b, c, d \,|\, cb - baa, bab, bac, caa, cac, da, db, dc, pa - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.3829}$$

$$\langle a, b, c, d \,|\, cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.3830}$$

$$\langle a, b, c, d \,|\, cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.3831}$$

$$\langle a, b, c, d \,|\, cb - baa, bab, bac, caa, cac, da, db, dc, pa, pb, pc - baa, pd, \text{ class } 3\rangle \tag{7.3832}$$

8.209 Descendants of 6.20

$6p + 35$ algebras

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba, pb, pc, pd - cac, \text{ class } 3\rangle \tag{7.3833}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba, pb - cac, pc, pd, \text{ class } 3\rangle \tag{7.3834}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3835}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba, pb, pc - cac, pd, \text{ class } 3\rangle \tag{7.3836}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba - cac, pb, pc, pd, \text{ class } 3\rangle \tag{7.3837}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba - cac, pb, pc - cac, pd, \text{ class } 3\rangle \tag{7.3838}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba - \omega cac, pb, pc, pd, \text{ class } 3\rangle \tag{7.3839}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, caa, pa - ba - \omega cac, pb, pc - cac, pd, \text{ class } 3\rangle \tag{7.3840}$$

$$\langle a, b, c, d \,|\, cb, da - cac, db, dc, caa, pa - ba, pb, pc, pd - cac, \text{ class } 3\rangle \tag{7.3841}$$

$$\langle a, b, c, d \,|\, cb, da - cac, db, dc, caa, pa - ba, pb - cac, pc, pd, \text{ class } 3\rangle \tag{7.3842}$$

$$\langle a,b,c,d \,|\, cb, da-cac, db, dc, caa, pa-ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3843}$$

$$\langle a,b,c,d \,|\, cb, da-cac, db, dc, caa, pa-ba, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.3844}$$

$$\langle a,b,c,d \,|\, cb, da, db-cac, dc, caa, pa-ba, pb, pc, pd-xcac, \text{ class } 3\rangle \text{ (all } x) \tag{7.3845}$$

$$\langle a,b,c,d \,|\, cb, da, db-cac, dc, caa, pa-ba, pb-cac, pc, pd, \text{ class } 3\rangle \tag{7.3846}$$

$$\langle a,b,c,d \,|\, cb, da, db-cac, dc, caa, pa-ba, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.3847}$$

$$\langle a,b,c,d \,|\, cb, da, db-cac, dc, caa, pa-ba-cac, pb, pc, pd+cac, \text{ class } 3\rangle \tag{7.3848}$$

$$\langle a,b,c,d \,|\, cb, da, db-cac, dc, caa, pa-ba-\omega cac, pb, pc, pd+cac, \text{ class } 3\rangle \tag{7.3849}$$

$$\langle a,b,c,d \,|\, cb, da, db-caa, dc, cac, pa-ba, pb, pc, pd-xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3850}$$

$$\langle a,b,c,d \,|\, cb, da, db-caa, dc, cac, pa-ba, pb, pc-caa, pd-xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3851}$$

$$\langle a,b,c,d \,|\, cb, da, db-caa, dc, cac, pa-ba, pb, pc-\omega caa, pd-xcaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3852}$$

$$\langle a,b,c,d \,|\, cb, da, db-caa, dc, cac, pa-ba, pb-caa, pc, pd, \text{ class } 3\rangle \tag{7.3853}$$

$$\langle a,b,c,d \,|\, cb, da, db-caa, dc, cac, pa-ba, pb-caa, pc-caa, pd, \text{ class } 3\rangle \tag{7.3854}$$

$$\langle a,b,c,d \,|\, cb, da, db-caa, dc, cac, pa-ba, pb-caa, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.3855}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc-caa, cac, pa-ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3856}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc-caa, cac, pa-ba, pb-caa, pc, pd, \text{ class } 3\rangle \tag{7.3857}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc-caa, cac, pa-ba, pb, pc, pd-caa, \text{ class } 3\rangle \tag{7.3858}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc-caa, cac, pa-ba, pb-caa, pc, pd-caa, \text{ class } 3\rangle \tag{7.3859}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc-caa, cac, pa-ba, pb-\omega caa, pc, pd-caa, \text{ class } 3\rangle \tag{7.3860}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc-caa, cac, pa-ba, pb, pc-caa, pd, \text{ class } 3\rangle \tag{7.3861}$$

$$\langle a,b,c,d \,|\, cb, da, db, dc-caa, cac, pa-ba, pb, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.3862}$$

$$\langle a,b,c,d \,|\, cb-caa, da, db, dc, cac, pa-ba, pb, pc, pd-caa, \text{ class } 3\rangle \tag{7.3863}$$

$$\langle a,b,c,d \,|\, cb-caa, da, db, dc, cac, pa-ba, pb, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3864}$$

$$\langle a, b, c, d \,|\, cb - caa, da, db, dc, cac, pa - ba, pb - caa, pc, pd, \text{ class } 3\rangle \tag{7.3865}$$

$$\langle a, b, c, d \,|\, cb - \omega caa, da, db, dc, cac, pa - ba, pb, pc, pd - caa, \text{ class } 3\rangle \tag{7.3866}$$

$$\langle a, b, c, d \,|\, cb - \omega caa, da, db, dc, cac, pa - ba, pb, pc - xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3867}$$

$$\langle a, b, c, d \,|\, cb - \omega caa, da, db, dc, cac, pa - ba, pb - caa, pc, pd, \text{ class } 3\rangle \tag{7.3868}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, cac, pa - ba, pb, pc, pd - caa, \text{ class } 3\rangle \tag{7.3869}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, cac, pa - ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.3870}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, cac, pa - ba, pb - caa, pc, pd, \text{ class } 3\rangle \tag{7.3871}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, cac, pa - ba, pb, pc - caa, pd, \text{ class } 3\rangle \tag{7.3872}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, cac, pa - ba, pb, pc - \omega caa, pd, \text{ class } 3\rangle \tag{7.3873}$$

8.210 Descendants of 6.21

$13p + 27$ algebras

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa, pb - ba, pc, pd - cac, \text{ class } 3\rangle \tag{7.3874}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3875}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa - cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3876}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa - \omega cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3877}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa - xcac, pb - ba, pc - cac, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3878}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa, pb - ba - cac, pc, pd, \text{ class } 3\rangle \tag{7.3879}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa - cac, pb - ba - cac, pc, pd, \text{ class } 3\rangle \tag{7.3880}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa - \omega cac, pb - ba - cac, pc, pd, \text{ class } 3\rangle \tag{7.3881}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db, dc, pa - xcac, pb - ba - cac, pc - cac, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3882}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db, dc, pa, pb - ba, pc, pd - xcac, \text{ class } 3\rangle \text{ (all } x) \tag{7.3883}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db, dc, pa, pb - ba - cac, pc, pd - cac, \text{ class } 3\rangle \tag{7.3884}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db, dc, pa - cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3885}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db, dc, pa - \omega cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3886}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db, dc, pa - xcac, pb - ba, pc - cac, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3887}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db - cac, dc, pa, pb - ba, pc, pd - cac, \text{ class } 3\rangle \tag{7.3888}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db - cac, dc, pa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3889}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db - cac, dc, pa - cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3890}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db - cac, dc, pa - \omega cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3891}$$

$$\langle a, b, c, d \,|\, caa, cb, da, db - cac, dc, pa - xcac, pb - ba, pc - cac, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3892}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db - cac, dc, pa, pb - ba, pc, pd - xcac, \text{ class } 3\rangle \text{ (all } x) \tag{7.3893}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db - cac, dc, pa, pb - ba - cac, pc, pd - cac, \text{ class } 3\rangle \tag{7.3894}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db - cac, dc, pa, pb - ba - \omega cac, pc, pd - cac, \text{ class } 3\rangle \tag{7.3895}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db - cac, dc, pa - cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3896}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db - cac, dc, pa - \omega cac, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3897}$$

$$\langle a, b, c, d \,|\, caa, cb, da - cac, db - cac, dc, pa - xcac, pb - ba, pc - cac, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3898}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db, dc, pa, pb - ba, pc, pd - caa, \text{ class } 3\rangle \tag{7.3899}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db, dc, pa - caa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3900}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db, dc, pa, pb - ba, pc - xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3901}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db, dc - caa, pa, pb - ba, pc, pd - caa, \text{ class } 3\rangle \tag{7.3902}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db, dc - caa, pa - caa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3903}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db, dc - caa, pa, pb - ba, pc - xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3904}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - caa, dc, pa, pb - ba, pc - xcaa, pd - caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3905}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - caa, dc, pa - caa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.3906}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-caa, dc, pa, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3907}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-caa, dc-caa, pa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \tag{7.3908}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-caa, dc-caa, pa-caa, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3909}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-caa, dc-caa, pa, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3910}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db, dc, pa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \tag{7.3911}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db, dc, pa-caa, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.3912}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db, dc, pa, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3913}$$

8.211 Descendants of 6.23

$3p+41+8\gcd(p-1,3)+2\gcd(p-1,4)$ algebras

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa, pb-ca, pc, pd-bab, \text{ class } 3\rangle \tag{7.3914}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa, pb-ca, pc-bab, pd, \text{ class } 3\rangle \tag{7.3915}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3916}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa-bab, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3917}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa-\omega bab, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3918}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa, pb-ca-bab, pc, pd, \text{ class } 3\rangle \tag{7.3919}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa-bab, pb-ca-bab, pc, pd, \text{ class } 3\rangle \tag{7.3920}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc, pa-\omega bab, pb-ca-bab, pc, pd, \text{ class } 3\rangle \tag{7.3921}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa, pb-ca, pc, pd-bab, \text{ class } 3\rangle \tag{7.3922}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3923}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-bab, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3924}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-\omega bab, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3925}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa, pb-ca, pc-bab, pd, \text{ class } 3\rangle \tag{7.3926}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-bab, pb-ca, pc-bab, pd, \text{ class } 3\rangle \tag{7.3927}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-\omega bab, pb-ca, pc-bab, pd, \text{ class } 3\rangle \tag{7.3928}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-\omega^2 bab, pb-ca, pc-bab, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.3929}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-\omega^3 bab, pb-ca, pc-bab, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.3930}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb-ca, pc, pd-bab, \text{ class } 3\rangle \tag{7.3931}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb-ca, pc-bab, pd, \text{ class } 3\rangle \tag{7.3932}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3933}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-bab, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3934}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-\omega bab, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3935}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb-ca, pc, pd-baa, \text{ class } 3\rangle \tag{7.3936}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3937}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa-baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3938}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb-ca, pc-baa, pd, \text{ class } 3\rangle \tag{7.3939}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb-ca, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3940}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb-ca, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3941}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc, pd-baa, \text{ class } 3\rangle \tag{7.3942}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc-baa, pd-baa, \text{ class } 3\rangle \tag{7.3943}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc-\omega baa, pd-baa, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3944}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc-\omega^2 baa, pd-baa, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3945}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3946}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa-baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3947}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc-baa, pd, \text{ class } 3\rangle \tag{7.3948}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3949}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-ca, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3950}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-ca, pc, pd-baa, \text{ class } 3\rangle \tag{7.3951}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3952}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa-baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3953}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-ca, pc-baa, pd, \text{ class } 3\rangle \tag{7.3954}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-ca, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3955}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-ca, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3956}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb-ca, pc, pd-baa, \text{ class } 3\rangle \tag{7.3957}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3958}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa-baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3959}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa-\omega baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3960}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa-\omega^2 baa, pb-ca, pc, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.3961}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa-\omega^3 baa, pb-ca, pc, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.3962}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb-ca, pc-baa, pd, \text{ class } 3\rangle \tag{7.3963}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb-ca, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3964}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb-ca, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3965}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa, pb-ca, pc, pd-baa, \text{ class } 3\rangle \tag{7.3966}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3967}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa-baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3968}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa-\omega baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3969}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa-xbaa, pb-ca-baa, pc, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.3970}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa-xbaa, pb-ca-\omega baa, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.3971}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa, pb-ca, pc-baa, pd, \text{ class } 3\rangle \tag{7.3972}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa, pb-ca, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3973}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc, pa, pb-ca, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3974}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da-baa, db, dc, pa, pb-ca, pc, pd-baa, \text{ class } 3\rangle \tag{7.3975}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da-baa, db, dc, pa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3976}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da-baa, db, dc, pa-baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3977}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da-baa, db, dc, pa-\omega baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3978}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da-baa, db, dc, pa, pb-ca, pc-baa, pd, \text{ class } 3\rangle \tag{7.3979}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da-baa, db, dc, pa, pb-ca, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3980}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da-baa, db, dc, pa, pb-ca, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3981}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa, pb-ca, pc, pd-xbaa, \text{ class } 3\rangle \text{ (all } x) \tag{7.3982}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa-baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3983}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa-\omega baa, pb-ca, pc, pd, \text{ class } 3\rangle \tag{7.3984}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa, pb-ca, pc-baa, pd, \text{ class } 3\rangle \tag{7.3985}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa, pb-ca, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3986}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa, pb-ca, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.3987}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa, pb-ca-baa, pc, pd-baa, \text{ class } 3\rangle \tag{7.3988}$$

$$\langle a,b,c,d \,|\, bab-baa, cb, da, db, dc-baa, pa, pb-ca-\omega baa, pc, pd-baa, \text{ class } 3\rangle \tag{7.3989}$$

8.212 Descendants of 6.24

5 algebras

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa - ba, pb - ca, pc, pd - bab, \text{ class } 3\rangle \tag{7.3990}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa - ba, pb - ca, pc, pd, \text{ class } 3\rangle \tag{7.3991}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa - ba, pb - ca, pc - bab, pd, \text{ class } 3\rangle \tag{7.3992}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc, pa - ba, pb - ca, pc - \omega bab, pd, \text{ class } 3\rangle \tag{7.3993}$$

$$\langle a, b, c, d \,|\, cb, da, db, dc - bab, pa - ba, pb - ca, pc, pd, \text{ class } 3\rangle \tag{7.3994}$$

8.213 Descendants of 6.29

$10p + 69 + \gcd(p-1, 3) + \gcd(p-1, 4)$ algebras

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa, pb, pc - bab, pd - ca, \text{ class } 3\rangle \tag{7.3995}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa, pb, pc - bab, pd - ca - bab, \text{ class } 3\rangle \tag{7.3996}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa, pb, pc, pd - ca, \text{ class } 3\rangle \tag{7.3997}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - bab, pb, pc, pd - ca, \text{ class } 3\rangle \tag{7.3998}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - \omega bab, pb, pc, pd - ca, \text{ class } 3\rangle \tag{7.3999}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa, pb - bab, pc, pd - ca, \text{ class } 3\rangle \tag{7.4000}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - bab, pb - bab, pc, pd - ca, \text{ class } 3\rangle \tag{7.4001}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - \omega bab, pb - bab, pc, pd - ca, \text{ class } 3\rangle \tag{7.4002}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa, pb, pc, pd - ca - bab, \text{ class } 3\rangle \tag{7.4003}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - bab, pb, pc, pd - ca - bab, \text{ class } 3\rangle \tag{7.4004}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - \omega bab, pb, pc, pd - ca - bab, \text{ class } 3\rangle \tag{7.4005}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa, pb - bab, pc, pd - ca - bab, \text{ class } 3\rangle \tag{7.4006}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - bab, pb - bab, pc, pd - ca - bab, \text{ class } 3\rangle \tag{7.4007}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db, dc, pa - \omega bab, pb - bab, pc, pd - ca - bab, \text{ class } 3\rangle \tag{7.4008}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb, pc-xbab, pd-ca-bab, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4009}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb, pc-bab, pd-ca, \text{ class } 3\rangle \tag{7.4010}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb, pc-\omega bab, pd-ca, \text{ class } 3\rangle \tag{7.4011}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb, pc, pd-ca-bab, \text{ class } 3\rangle \tag{7.4012}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-bab, pb, pc, pd-ca-bab, \text{ class } 3\rangle \tag{7.4013}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-\omega bab, pb, pc, pd-ca-bab, \text{ class } 3\rangle \tag{7.4014}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-xbab, pb-bab, pc, pd-ca-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4015}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4016}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-bab, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4017}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-\omega bab, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4018}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4019}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-bab, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4020}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db, dc, pa-\omega bab, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4021}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-xbab, pb, pc-bab, pd-ca-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4022}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa, pb, pc-bab, pd-ca, \text{ class } 3\rangle \tag{7.4023}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-bab, pb, pc-bab, pd-ca, \text{ class } 3\rangle \tag{7.4024}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-\omega bab, pb, pc-bab, pd-ca, \text{ class } 3\rangle \tag{7.4025}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-xbab, pb, pc, pd-ca-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4026}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-xbab, pb-bab, pc, pd-ca-bab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4027}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4028}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-bab, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4029}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-\omega bab, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4030}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4031}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-bab, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4032}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-bab, pa-\omega bab, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4033}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-xbab, pb, pc-bab, pd-ca-bab, \text{ class } 3\rangle \text{ (all } x) \tag{7.4034}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa, pb, pc-bab, pd-ca, \text{ class } 3\rangle \tag{7.4035}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-bab, pb, pc-bab, pd-ca, \text{ class } 3\rangle \tag{7.4036}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-\omega bab, pb, pc-bab, pd-ca, \text{ class } 3\rangle \tag{7.4037}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-xbab, pb, pc, pd-ca-bab, \text{ class } 3\rangle \text{ (all } x) \tag{7.4038}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-xbab, pb-bab, pc, pd-ca-bab, \text{ class } 3\rangle \text{ (all } x) \tag{7.4039}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4040}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-bab, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4041}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-\omega bab, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4042}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4043}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-bab, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4044}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db, dc-\omega bab, pa-\omega bab, pb-bab, pc, pd-ca, \text{ class } 3\rangle \tag{7.4045}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4046}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb-baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4047}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb-\omega baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4048}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4049}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc, pa-baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4050}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4051}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa-baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4052}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa-\omega baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4053}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4054}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-\omega baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4055}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4056}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-baa, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4057}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db, dc-baa, pa, pb-\omega baa, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4058}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4059}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa-baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4060}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa-\omega baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4061}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa-\omega^2 baa, pb, pc, pd-ca, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4062}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa-\omega^3 baa, pb, pc, pd-ca, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4063}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa, pb, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4064}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa, pb, pc-\omega baa, pd-ca, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4065}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa, pb, pc-\omega^2 baa, pd-ca, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4066}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa, pb-baa, pc-xbaa, pd-ca, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.4067}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc-baa, pa, pb-\omega baa, pc-xbaa, pd-ca, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.4068}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4069}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb-baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4070}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb-\omega baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4071}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4072}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db, dc, pa-baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4073}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4074}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4075}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb-\omega baa, pc, pd-ca, \text{ class } 3\rangle \tag{7.4076}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4077}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-baa, dc, pa-baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4078}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-baa, dc, pa, pb, pc-baa, pd-ca, \text{ class } 3\rangle \tag{7.4079}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-baa, dc, pa, pb-xbaa, pc, pd-ca, \text{ class } 3\rangle \text{ (all } x) \tag{7.4080}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-baa, dc, pa-baa, pb, pc, pd-ca, \text{ class } 3\rangle \tag{7.4081}$$

8.214 Descendants of 6.33

$39 + \gcd(p-1, 3)$ algebras

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4082}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.4083}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4084}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa-\omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4085}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4086}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.4087}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa-bab, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4088}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd, pa-\omega bab, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4089}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4090}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.4091}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4092}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa-\omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4093}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4094}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.4095}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa-bab, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4096}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa-\omega bab, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4097}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.4098}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa, pb-bab, pc, pd-bab, \text{ class } 3\rangle \tag{7.4099}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa-bab, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.4100}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da-bab, db, dcc, dcd, pa-\omega bab, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.4101}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4102}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.4103}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4104}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa, pb, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4105}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc, pd-bab, \text{ class } 3\rangle \tag{7.4106}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.4107}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4108}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa-bab, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4109}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa-bab, pb, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4110}$$

$$\langle a,b,c,d \,|\, baa, ca, cb, da, db, dcc, dcd-bab, pa-\omega bab, pb, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4111}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4112}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb, pc, pd-bab, \text{ class } 3\rangle \tag{7.4113}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4114}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4115}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc, pd-bab, \text{ class } 3\rangle \tag{7.4116}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc-bab, pd, \text{ class } 3\rangle \tag{7.4117}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4118}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa-bab, pb, pc-bab, pd, \text{ class } 3\rangle \tag{7.4119}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa-bab, pb, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4120}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa-\omega bab, pb, pc-\omega bab, pd, \text{ class } 3\rangle \tag{7.4121}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc, pd-\omega bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4122}$$

$$\langle a,b,c,d \,|\, baa, ca-bab, cb, da, db, dcc, dcd-bab, pa, pb-bab, pc, pd-\omega^2 bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4123}$$

8.215 Descendants of 6.34

$5p+38+2\gcd(p-1,4)$ algebras

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.4124}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-cac, pb, pc, pd, \text{ class } 3\rangle \tag{7.4125}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-\omega cac, pb, pc, pd, \text{ class } 3\rangle \tag{7.4126}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.4127}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-cac, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.4128}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-\omega cac, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.4129}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba, pb-cac, pc, pd, \text{ class } 3\rangle \tag{7.4130}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-cac, pb-cac, pc, pd, \text{ class } 3\rangle \tag{7.4131}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-\omega cac, pb-cac, pc, pd, \text{ class } 3\rangle \tag{7.4132}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba, pb-cac, pc-cac, pd, \text{ class } 3\rangle \tag{7.4133}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-cac, pb-cac, pc-cac, pd, \text{ class } 3\rangle \tag{7.4134}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-\omega cac, pb-cac, pc-cac, pd, \text{ class } 3\rangle \tag{7.4135}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.4136}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-cac, pb, pc, pd, \text{ class } 3\rangle \tag{7.4137}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-\omega cac, pb, pc, pd, \text{ class } 3\rangle \tag{7.4138}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba, pb, pc, pd-cac, \text{ class } 3\rangle \tag{7.4139}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba+cac, pb, pc, pd-cac, \text{ class } 3\rangle \tag{7.4140}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba+(1-\omega)cac, pb, pc, pd-cac, \text{ class } 3\rangle \tag{7.4141}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.4142}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-cac, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.4143}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-\omega cac, pb, pc-cac, pd, \text{ class } 3\rangle \tag{7.4144}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb, pc-cac, pd-cac, \text{ class } 3\rangle \text{ (all } x) \tag{7.4145}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-cac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ p=3 \bmod 4) \tag{7.4146}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-cac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ p=3 \bmod 4) \tag{7.4147}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega cac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ p=3 \bmod 4) \tag{7.4148}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega cac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ p=3 \bmod 4) \tag{7.4149}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-cac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4150}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-cac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4151}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega cac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4152}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega cac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4153}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega^2 cac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4154}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega^2 cac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4155}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega^3 cac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4156}$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-\omega^3 cac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ x\sim -x,\ p=1 \bmod 4) \tag{7.4157}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.4158}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb, pc-caa, pd, \text{ class } 3\rangle \tag{7.4159}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.4160}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-caa, pc, pd, \text{ class } 3\rangle \tag{7.4161}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-caa, pc-caa, pd, \text{ class } 3\rangle \tag{7.4162}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-caa, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.4163}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba-caa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4164}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba-caa, pb-caa, pc, pd, \text{ class } 3\rangle \tag{7.4165}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb, pc, pd, \text{ class } 3\rangle \tag{7.4166}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba-caa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4167}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-caa, pc, pd, \text{ class } 3\rangle \tag{7.4168}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba-caa, pb-caa, pc, pd, \text{ class } 3\rangle \tag{7.4169}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb, pc-caa, pd, \text{ class } 3\rangle \tag{7.4170}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.4171}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-caa, pc-caa, pd, \text{ class } 3\rangle \tag{7.4172}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-caa, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.4173}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb, pc, pd-caa, \text{ class } 3\rangle \tag{7.4174}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa - ba + caa, pb, pc, pd - caa, \text{ class } 3\rangle \tag{7.4175}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa - ba + (1-\omega)caa, pb, pc, pd - caa, \text{ class } 3\rangle \tag{7.4176}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc - caa, pd - caa, \text{ class } 3\rangle \tag{7.4177}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa - ba, pb, pc - \omega caa, pd - caa, \text{ class } 3\rangle \tag{7.4178}$$

8.216 Descendants of 6.35

$10p + 26$ algebras

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.4179}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa - caa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.4180}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba - caa, pc, pd, \text{ class } 3\rangle \tag{7.4181}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa - caa, pb - ba - caa, pc, pd, \text{ class } 3\rangle \tag{7.4182}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba, pc - caa, pd, \text{ class } 3\rangle \tag{7.4183}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba, pc - \omega caa, pd, \text{ class } 3\rangle \tag{7.4184}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba - caa, pc - caa, pd, \text{ class } 3\rangle \tag{7.4185}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba - caa, pc - \omega caa, pd, \text{ class } 3\rangle \tag{7.4186}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba, pc, pd - caa, \text{ class } 3\rangle \tag{7.4187}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa + caa, pb - ba, pc, pd - caa, \text{ class } 3\rangle \tag{7.4188}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba, pc - caa, pd - caa, \text{ class } 3\rangle \tag{7.4189}$$

$$\langle a, b, c, d \,|\, cac, cb, da, db - ba, dc, pa, pb - ba, pc - \omega caa, pd - caa, \text{ class } 3\rangle \tag{7.4190}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.4191}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa - caa, pb - ba, pc, pd, \text{ class } 3\rangle \tag{7.4192}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa, pb - ba - caa, pc, pd, \text{ class } 3\rangle \tag{7.4193}$$

$$\langle a, b, c, d \,|\, cac, cb - caa, da, db - ba, dc, pa - caa, pb - ba - caa, pc, pd, \text{ class } 3\rangle \tag{7.4194}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa, pb-ba, pc-caa, pd, \text{ class } 3\rangle \tag{7.4195}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa, pb-ba, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.4196}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa, pb-ba-caa, pc-caa, pd, \text{ class } 3\rangle \tag{7.4197}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa, pb-ba-caa, pc-\omega caa, pd, \text{ class } 3\rangle \tag{7.4198}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \tag{7.4199}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa+caa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \tag{7.4200}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa, pb-ba, pc-caa, pd-caa, \text{ class } 3\rangle \tag{7.4201}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa, pb-ba, pc-\omega caa, pd-caa, \text{ class } 3\rangle \tag{7.4202}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4203}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa, pb-ba, pc-xcaa, pd-caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.4204}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa, pb-ba-caa, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4205}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa, pb-ba-caa, pc-xcaa, pd-caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.4206}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-caa, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.4207}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-caa, pb-ba-caa, pc, pd, \text{ class } 3\rangle \tag{7.4208}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-xcaa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \ (x \neq 0, -2,\ x \sim -x-2) \tag{7.4209}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-xcaa, pb-ba-caa, pc, pd-caa, \text{ class } 3\rangle \ (x \neq 0, -2, x \sim -x-2) \tag{7.4210}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4211}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa, pb-ba, pc-xcaa, pd-caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.4212}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa, pb-ba-caa, pc-xcaa, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4213}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa, pb-ba-caa, pc-xcaa, pd-caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.4214}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-caa, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.4215}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-caa, pb-ba-caa, pc, pd, \text{ class } 3\rangle \tag{7.4216}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-xcaa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \ (x \neq 0,-2,\ x \sim -x-2) \tag{7.4217}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-xcaa, pb-ba-caa, pc, pd-caa, \text{ class } 3\rangle \ (x \neq 0,-2,\ x \sim -x-2) \tag{7.4218}$$

8.217 Descendants of 6.36

$p^3+9p^2+20p+18+\gcd(p-1,3)+\gcd(p-1,4)$ algebras

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.4219}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-cac, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.4220}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-\omega cac, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.4221}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-xcac, pb-ba-cac, pc, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4222}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-xcac, pb-ba-\omega cac, pc, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4223}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba, pb-ba, pc, pd-cac, \text{ class } 3\rangle \tag{7.4224}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba+cac, pb-ba, pc, pd-cac, \text{ class } 3\rangle \tag{7.4225}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba+(1-\omega)cac, pb-ba, pc, pd-cac, \text{ class } 3\rangle \tag{7.4226}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba, pb-ba, pc-cac, pd, \text{ class } 3\rangle \tag{7.4227}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-cac, pb-ba, pc-cac, pd, \text{ class } 3\rangle \tag{7.4228}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-\omega cac, pb-ba, pc-cac, pd, \text{ class } 3\rangle \tag{7.4229}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-xcac, pb-ba-cac, pc-cac, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4230}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-xcac, pb-ba-\omega cac, pc-cac, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4231}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba, dc, pa-ba-xcac, pb-ba, pc-cac, pd-cac, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4232}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-xcac, pb-ba, pc-ycac, pd-cac, \text{ class } 3\rangle \ (\text{all } x,\ y,\ y \sim -y,\ p = 3 \bmod 4) \tag{7.4233}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-xcac, pb-ba, pc-ycac, pd-\omega cac, \text{ class } 3\rangle \ (\text{all } x,\ y,\ y\sim -y,\ p=3 \bmod 4) \tag{7.4234}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-xcac, pb-ba, pc-ycac, pd-cac, \text{ class } 3\rangle \ (x\neq 0,\ x\sim -x,\ \text{all } y,\ y\sim -y,\ p=1 \bmod 4) \tag{7.4235}$$

In the next presentation we have $x\neq 1-\omega$, $x\sim -x+2(1-\omega)$, all y, $y\sim -y$, $p=1 \bmod 4$:

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-xcac, pb-ba, pc-ycac, pd-\omega cac, \text{ class } 3\rangle \tag{7.4236}$$

In the next presentation we have $x\neq 1-\omega^2$, $x\sim -x+2(1-\omega^2)$, all y, $y\sim -y$, $p=1 \bmod 4$:

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-xcac, pb-ba, pc-ycac, pd-\omega^2 cac, \text{ class } 3\rangle \tag{7.4237}$$

In the next presentation we have $x\neq 1-\omega^3$, $x\sim -x+2(1-\omega^3)$, all y, $y\sim -y$, $p=1 \bmod 4$:

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-xcac, pb-ba, pc-ycac, pd-\omega^3 cac, \text{ class } 3\rangle \tag{7.4238}$$

In the next four presentations we have all y, $y\sim y'$ if $y^4=y'^4 \bmod p$, $p=1 \bmod 4$:

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba, pb-ba, pc-ycac, pd-cac, \text{ class } 3\rangle \tag{7.4239}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-(1-\omega)cac, pb-ba, pc-ycac, pd-\omega cac, \text{ class } 3\rangle \tag{7.4240}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-(1-\omega^2)cac, pb-ba, pc-ycac, pd-\omega^2 cac, \text{ class } 3\rangle \tag{7.4241}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-(1-\omega^3)cac, pb-ba, pc-ycac, pd-\omega^3 cac, \text{ class } 3\rangle \tag{7.4242}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-cac, pb-ba-cac, pc, pd, \text{ class } 3\rangle \tag{7.4243}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-cac, pb-ba-cac, pc-cac, pd, \text{ class } 3\rangle \tag{7.4244}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-cac, pb-ba-cac, pc-\omega cac, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4245}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-cac, pb-ba-cac, pc-\omega^2 cac, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4246}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-cac, pb-ba-2cac, pc-xcac, pd, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x) \tag{7.4247}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-cac, pb-ba-(1+\omega)cac, pc-xcac, pd, \text{ class } 3\rangle \ (\text{all } x,\ x\sim -x) \tag{7.4248}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-2cac, pb-ba-xcac, pc-ycac, pd, \text{ class } 3\rangle \ (\text{all } x,\ \text{all } y,\ y\sim -y) \tag{7.4249}$$

$$\langle a,b,c,d \,|\, caa, cb, da, db-ba-cac, dc, pa-ba-(1+\omega)cac, pb-ba-xcac, pc-ycac, pd, \text{ class } 3\rangle \text{ (all } x, \text{ all } y,\ y \sim -y) \quad (7.4250)$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-ba-ycac, pc-zcac, pd-cac, \text{ class } 3\rangle \text{ (all } x,\ y, z,\ z \sim -z) \quad (7.4251)$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-ba-ycac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ y) \quad (7.4252)$$

$$\langle a,b,c,d \,|\, caa, cb, da-cac, db-ba, dc, pa-ba-xcac, pb-ba-ycac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ y) \quad (7.4253)$$

$$\langle a,b,c,d \,|\, caa, cb, da-\omega cac, db-ba, dc, pa-ba-xcac, pb-ba-ycac, pc-zcac, pd-cac, \text{ class } 3\rangle \text{ (all } x,\ y, z,\ z \sim -z) \quad (7.4254)$$

$$\langle a,b,c,d \,|\, caa, cb, da-\omega cac, db-ba, dc, pa-ba-xcac, pb-ba-ycac, pc, pd, \text{ class } 3\rangle \text{ (all } x,\ y) \quad (7.4255)$$

$$\langle a,b,c,d \,|\, caa, cb, da-\omega cac, db-ba, dc, pa-ba-xcac, pb-ba-ycac, pc-cac, pd, \text{ class } 3\rangle \text{ (all } x,\ y) \quad (7.4256)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba, pc, pd, \text{ class } 3\rangle \quad (7.4257)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba-caa, pc, pd, \text{ class } 3\rangle \quad (7.4258)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba-caa, pb-ba-xcaa, pc, pd, \text{ class } 3\rangle \text{ (all } x) \quad (7.4259)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba, pc, pd-caa, \text{ class } 3\rangle \quad (7.4260)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba+caa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \quad (7.4261)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba, pc-caa, pd, \text{ class } 3\rangle \quad (7.4262)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba-caa, pc-caa, pd, \text{ class } 3\rangle \quad (7.4263)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba, pc-\omega caa, pd, \text{ class } 3\rangle \quad (7.4264)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba-caa, pc-\omega caa, pd, \text{ class } 3\rangle \quad (7.4265)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba, pc-caa, pd-caa, \text{ class } 3\rangle \quad (7.4266)$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc, pa-ba, pb-ba, pc-\omega caa, pd-caa, \text{ class } 3\rangle \quad (7.4267)$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-ba, pc, pd, \text{ class } 3\rangle \quad (7.4268)$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-ba-caa, pc, pd, \text{ class } 3\rangle \tag{7.4269}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba-caa, pb-ba-xcaa, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4270}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4271}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-ba-caa, pc-xcaa, pd, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4272}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba-xcaa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \text{ (all } x) \tag{7.4273}$$

$$\langle a,b,c,d \,|\, cac, cb-caa, da, db-ba, dc, pa-ba, pb-ba, pc-xcaa, pd-caa, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4274}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.4275}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba, pb-ba-caa, pc, pd, \text{ class } 3\rangle \tag{7.4276}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba-caa, pb-ba-xcaa, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4277}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4278}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba, pb-ba-caa, pc-xcaa, pd, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4279}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba-ycaa, pb-ba-xcaa, pc, pd-caa, \text{ class } 3\rangle \ (x \neq 0,\ x \sim -x,\ \text{ all } y) \tag{7.4280}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba-ycaa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \text{ (all } y,\ y \sim -y-2) \tag{7.4281}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-caa, pa-ba, pb-ba-ycaa, pc-xcaa, pd-caa, \text{ class } 3\rangle \ (x \neq 0,\ \text{ all } y,\ y \sim -y) \tag{7.4282}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba, pb-ba, pc, pd, \text{ class } 3\rangle \tag{7.4283}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba, pb-ba-caa, pc, pd, \text{ class } 3\rangle \tag{7.4284}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba-caa, pb-ba-xcaa, pc, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4285}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba, pb-ba, pc-xcaa, pd, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4286}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba, pb-ba-caa, pc-xcaa, pd, \text{ class } 3\rangle \ (x \neq 0) \tag{7.4287}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba-ycaa, pb-ba-xcaa, pc, pd-caa, \text{ class } 3\rangle \ (x \neq 0,\ x \sim -x,\ \text{ all } y) \tag{7.4288}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba-ycaa, pb-ba, pc, pd-caa, \text{ class } 3\rangle \text{ (all } y,\ y \sim -y-2) \tag{7.4289}$$

$$\langle a,b,c,d \,|\, cac, cb, da, db-ba, dc-\omega caa, pa-ba, pb-ba-ycaa, pc-xcaa, pd-caa, \text{ class } 3\rangle \ (x \neq 0,\ \text{ all } y,\ y \sim -y) \tag{7.4290}$$

8.218 Descendants of 6.48

$27 + 3\gcd(p-1,3) + 2\gcd(p-1,4)$ algebras

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc, pa, pb, pc - baa, pd, \text{ class } 3\rangle \tag{7.4291}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc, pa, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.4292}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4293}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc, pa - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4294}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc, pa, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.4295}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.4296}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb, pc - baa, pd, \text{ class } 3\rangle \tag{7.4297}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.4298}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4299}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb - baa, da, db - ca, dc, pa - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4300}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.4301}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb - baa, da, db - ca, dc, pa, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.4302}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc - baa, pd, \text{ class } 3\rangle \tag{7.4303}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc - \omega baa, pd, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4304}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc - \omega^2 baa, pd, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4305}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa - baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4306}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4307}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb, pc, pd - baa, \text{ class } 3\rangle \tag{7.4308}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - baa, pc, pd, \text{ class } 3\rangle \tag{7.4309}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - baa, pc, pd - baa, \text{ class } 3\rangle \tag{7.4310}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db - ca, dc - baa, pa, pb - \omega baa, pc, pd, \text{ class } 3\rangle \tag{7.4311}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-baa, pa, pb-\omega baa, pc, pd-baa, \text{ class } 3\rangle \tag{7.4312}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-baa, pa, pb-\omega^2 baa, pc, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4313}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-baa, pa, pb-\omega^2 baa, pc, pd-baa, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4314}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-baa, pa, pb-\omega^3 baa, pc, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4315}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-baa, pa, pb-\omega^3 baa, pc, pd-baa, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4316}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb, da, db-ca, dc, pa, pb, pc-bad, pd, \text{ class } 3\rangle \tag{7.4317}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb, da, db-ca, dc, pa, pb, pc-\omega bad, pd, \text{ class } 3\rangle \tag{7.4318}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb, da, db-ca, dc, pa-bad, pb, pc, pd, \text{ class } 3\rangle \tag{7.4319}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb, da, db-ca, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4320}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb, da, db-ca, dc, pa, pb, pc, pd-bad, \text{ class } 3\rangle \tag{7.4321}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb, da, db-ca, dc, pa, pb-bad, pc, pd, \text{ class } 3\rangle \tag{7.4322}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb, da, db-ca, dc, pa, pb-bad, pc, pd-bad, \text{ class } 3\rangle \tag{7.4323}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb, pc-bad, pd, \text{ class } 3\rangle \tag{7.4324}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb, pc-\omega bad, pd, \text{ class } 3\rangle \tag{7.4325}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa-bad, pb, pc, pd, \text{ class } 3\rangle \tag{7.4326}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4327}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb-bad, pc, pd, \text{ class } 3\rangle \tag{7.4328}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb, pc, pd-bad, \text{ class } 3\rangle \tag{7.4329}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb-bad, pc, pd-bad, \text{ class } 3\rangle \tag{7.4330}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb, pc, pd-\omega bad, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4331}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb-bad, pc, pd-\omega bad, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4332}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb, pc, pd-\omega^2 bad, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4333}$$

$$\langle a,b,c,d \,|\, baa, bab, bac, cb-bad, da, db-ca, dc, pa, pb-bad, pc, pd-\omega^2 bad, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4334}$$

8.219 Descendants of 6.51

$8p + 15 + (2p + 5)\gcd(p - 1, 3) + (p + 2)\gcd(p - 1, 4) + \gcd(p - 1, 5)$ algebras

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.4335}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd - bab, \text{ class } 3\rangle \tag{7.4336}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd - bab, \text{ class } 3\rangle \tag{7.4337}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd - \omega bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4338}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd - \omega bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4339}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd - \omega^2 bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4340}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc - bab, pd - \omega^2 bab, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4341}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb, pc, pd, \text{ class } 3\rangle \tag{7.4342}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca - bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4343}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca - \omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4344}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.4345}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca - bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.4346}$$

$$\langle a, b, c, d \,|\, baa, cb, da, db - ca, dc, pa - ca - \omega bab, pb - bab, pc, pd, \text{ class } 3\rangle \tag{7.4347}$$

$$\langle a, b, c, d \,|\, baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - xbab, pd - bab, \text{ class } 3\rangle \ (\text{all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.4348}$$

$$\langle a, b, c, d \,|\, baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - xbab, pd - \omega bab, \text{ class } 3\rangle \ (\text{all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.4349}$$

$$\langle a, b, c, d \,|\, baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - xbab, pd - \omega^2 bab, \text{ class } 3\rangle \ (\text{all } x,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.4350}$$

$$\langle a, b, c, d \,|\, baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - bab, pd, \text{ class } 3\rangle \tag{7.4351}$$

$$\langle a, b, c, d \,|\, baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - \omega bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.4352}$$

$$\langle a, b, c, d \,|\, baa, cb, da - bab, db - ca, dc, pa - ca, pb, pc - \omega^2 bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.4353}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca, pb, pc-\omega^3 bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.4354}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca, pb, pc-\omega^4 bab, pd, \text{ class } 3\rangle \ (p = 1 \bmod 5) \tag{7.4355}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-xbab, pb-bab, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.4356}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-xbab, pb-\omega bab, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.4357}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-xbab, pb-\omega^2 bab, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.4358}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-xbab, pb-\omega^3 bab, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x,\ p = 1 \bmod 4) \tag{7.4359}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-xbab, pb-bab, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ p = 3 \bmod 4) \tag{7.4360}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-xbab, pb-\omega bab, pc, pd, \text{ class } 3\rangle \ (\text{all } x,\ p = 3 \bmod 4) \tag{7.4361}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca, pb, pc, pd, \text{ class } 3\rangle \tag{7.4362}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4363}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-ca-\omega bab, pb, pc, pd, \text{ class } 3\rangle \tag{7.4364}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-ca, pb, pc, pd-xbab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4365}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-ca, pb, pc-bab, pd-xbab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4366}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-ca, pb-bab, pc, pd, \text{ class } 3\rangle \tag{7.4367}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-ca-ybab, pb, pc, pd+bab, \text{ class } 3\rangle \ (y \neq 0,\ y \sim y' \text{ if } y^3 = y'^3 \bmod p) \tag{7.4368}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-ca-ybab, pb, pc-bab, pd+bab, \text{ class } 3\rangle \ (y \neq 0,\ y \sim y' \text{ if } y^3 = y'^3 \bmod p) \tag{7.4369}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega bab, pa-ca, pb, pc, pd-xbab, \text{ class } 3\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.4370}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega bab, pa-ca, pb, pc-bab, pd-xbab, \text{ class } 3\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.4371}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega bab, pa-ca, pb-bab, pc, pd, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4372}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega bab, pa-ca-ybab, pb, pc, pd+\omega bab, \text{ class } 3\rangle \ (y \neq 0,\ y \sim y' \text{ if } y^3 = y'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.4373}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega bab, pa-ca-ybab, pb, pc-bab, pd+\omega bab, \text{ class } 3\rangle \ (y \neq 0,\ y \sim y' \text{ if } y^3 = y'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.4374}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega^2 bab, pa-ca, pb, pc, pd-xbab, \text{ class } 3\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.4375}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega^2 bab, pa-ca, pb, pc-bab, pd-xbab, \text{ class } 3\rangle \ (\text{all } x,\ p = 1 \bmod 3) \tag{7.4376}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega^2 bab, pa-ca, pb-bab, pc, pd, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4377}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega^2 bab, pa-ca-ybab, pb, pc, pd+\omega^2 bab, \text{ class } 3\rangle \ (y \neq 0,\ y \sim y' \text{ if } y^3 = y'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.4378}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-\omega^2 bab, pa-ca-ybab, pb, pc-bab, pd+\omega^2 bab, \text{ class } 3\rangle \ (y \neq 0,\ y \sim y' \text{ if } y^3 = y'^3 \bmod p,\ p = 1 \bmod 3) \tag{7.4379}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-ca, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.4380}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-ca, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.4381}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-ca, pb, pc, pd, \text{ class } 3\rangle \tag{7.4382}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-ca-baa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4383}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-ca, pb-baa, pc, pd, \text{ class } 3\rangle \tag{7.4384}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-ca, pb-\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.4385}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa-ca, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.4386}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa-ca, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.4387}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa-ca, pb, pc, pd-\omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 4) \tag{7.4388}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa-ca, pb-xbaa, pc, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4389}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa-ca-baa, pb+baa, pc, pd, \text{ class } 3\rangle \tag{7.4390}$$

$$\langle a,b,c,d \,|\, bab, cb-\omega baa, da, db-ca, dc, pa-ca, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.4391}$$

$$\langle a,b,c,d \,|\, bab, cb-\omega baa, da, db-ca, dc, pa-ca, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.4392}$$

$$\langle a,b,c,d \,|\, bab, cb-\omega baa, da, db-ca, dc, pa-ca, pb, pc, pd-\omega baa, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4393}$$

$$\langle a,b,c,d \,|\, bab, cb-\omega baa, da, db-ca, dc, pa-ca, pb-xbaa, pc, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4394}$$

$$\langle a,b,c,d \,|\, bab, cb-\omega baa, da, db-ca, dc, pa-ca-baa, pb+\omega baa, pc, pd, \text{ class } 3\rangle \tag{7.4395}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.4396}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb, pc-\omega baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4397}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb, pc-\omega^2 baa, pd, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4398}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb, pc, pd-xbaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4399}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb-baa, pc, pd-xbaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4400}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb-\omega baa, pc, pd-xbaa, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4401}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb-\omega^2 baa, pc, pd-xbaa, \text{ class } 3\rangle \ (\text{all } x,\ p=1 \bmod 4) \tag{7.4402}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-ca, pb-\omega^3 baa, pc, pd-xbaa, \text{ class } 3\rangle \ (\text{all } x,\ p=1 \bmod 4) \tag{7.4403}$$

8.220 Descendants of 6.52

$4p^2+15p+15+(p+1)\gcd(p-1,3)+\gcd(p-1,4)$ algebras

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa, pb, pc-ca, pd-bab, \text{ class } 3\rangle \tag{7.4404}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa, pb-bab, pc-ca, pd, \text{ class } 3\rangle \tag{7.4405}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa, pb-bab, pc-ca-bab, pd, \text{ class } 3\rangle \tag{7.4406}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4407}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa-bab, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4408}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa-\omega bab, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4409}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa, pb, pc-ca-bab, pd, \text{ class } 3\rangle \tag{7.4410}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa-bab, pb, pc-ca-bab, pd, \text{ class } 3\rangle \tag{7.4411}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc, pa-\omega bab, pb, pc-ca-bab, pd, \text{ class } 3\rangle \tag{7.4412}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa, pb, pc-ca, pd-bab, \text{ class } 3\rangle \tag{7.4413}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa, pb, pc-ca, pd-\omega bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4414}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa, pb, pc-ca, pd-\omega^2 bab, \text{ class } 3\rangle \ (p=1 \bmod 3) \tag{7.4415}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-bab, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4416}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-\omega bab, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4417}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-\omega^2 bab, pb, pc-ca, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4418}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa-\omega^3 bab, pb, pc-ca, pd, \text{ class } 3\rangle \ (p=1 \bmod 4) \tag{7.4419}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4420}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa, pb-bab, pc-ca, pd, \text{ class } 3\rangle \tag{7.4421}$$

$$\langle a,b,c,d \,|\, baa, cb, da, db-ca, dc-bab, pa, pb-\omega bab, pc-ca, pd, \text{ class } 3\rangle \tag{7.4422}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa, pb, pc-ca, pd-xbab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4423}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa, pb, pc-ca-bab, pd-bab, \text{ class } 3\rangle \tag{7.4424}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-xbab, pb-bab, pc-ca, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4425}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-bab, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4426}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc, pa-\omega bab, pb, pc-ca, pd, \text{ class } 3\rangle \tag{7.4427}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-bab, pa, pb, pc-ca, pd-xbab, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4428}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-bab, pa, pb, pc-ca-xbab, pd-bab, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.4429}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-bab, pa-xbab, pb, pc-ca, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.4430}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-bab, pa, pb-xbab, pc-ca, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p) \tag{7.4431}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega bab, pa, pb, pc-ca, pd-xbab, \text{ class } 3\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.4432}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega bab, pa, pb, pc-ca-xbab, pd-bab, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.4433}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega bab, pa-xbab, pb, pc-ca, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.4434}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega bab, pa, pb-xbab, pc-ca, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.4435}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega^2 bab, pa, pb, pc-ca, pd-xbab, \text{ class } 3\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.4436}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega^2 bab, pa, pb, pc-ca-xbab, pd-bab, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.4437}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega^2 bab, pa-xbab, pb, pc-ca, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.4438}$$

$$\langle a,b,c,d \,|\, baa, cb, da-bab, db-ca, dc-\omega^2 bab, pa, pb-xbab, pc-ca, pd, \text{ class } 3\rangle \ (x \neq 0,\ x \sim x' \text{ if } x^3 = x'^3 \bmod p,\ p=1 \bmod 3) \tag{7.4439}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa, pb, pc-ca, pd-baa, \text{ class } 3\rangle \tag{7.4440}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa, pb, pc-ca-baa, pd-baa, \text{ class } 3\rangle \tag{7.4441}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa, pb-xbaa, pc-ca, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4442}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa, pb-xbaa, pc-ca-baa, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4443}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-baa, pb-xbaa, pc-ca, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4444}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc, pa-baa, pb-xbaa, pc-ca-baa, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4445}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa, pb, pc-ca-xbaa, pd-baa, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.4446}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa, pb, pc-ca-xbaa, pd-\omega baa, \text{ class } 3\rangle \ (\text{all } x,\ x \sim -x) \tag{7.4447}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa-xbaa, pb-ybaa, pc-ca-baa, pd, \text{ class } 3\rangle \ (\text{all } x,\ y) \tag{7.4448}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa, pb-xbaa, pc-ca, pd, \text{ class } 3\rangle \ (\text{all } x) \tag{7.4449}$$

$$\langle a,b,c,d \,|\, bab, cb-baa, da, db-ca, dc, pa-baa, pb-xbaa, pc-ca, pd, \text{ class } 3\rangle \text{ (all } x) \tag{7.4450}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa, pb-ybaa, pc-ca-xbaa, pd-baa, \text{ class } 3\rangle \text{ (all } x,\ y,\ x \sim -x) \tag{7.4451}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa, pb-ybaa, pc-ca-xbaa, pd, \text{ class } 3\rangle \text{ (all } x,\ y,\ x \sim -x) \tag{7.4452}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-baa, pa-baa, pb-ybaa, pc-ca-xbaa, pd, \text{ class } 3\rangle \text{ (all } x,\ y,\ x \sim -x) \tag{7.4453}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-\omega baa, pa, pb-ybaa, pc-ca-xbaa, pd-baa, \text{ class } 3\rangle \text{ (all } x,\ y,\ x \sim -x) \tag{7.4454}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-\omega baa, pa, pb-ybaa, pc-ca-xbaa, pd, \text{ class } 3\rangle \text{ (all } x,\ y,\ x \sim -x) \tag{7.4455}$$

$$\langle a,b,c,d \,|\, bab, cb, da, db-ca, dc-\omega baa, pa-baa, pb-ybaa, pc-ca-xbaa, pd, \text{ class } 3\rangle \text{ (all } x,\ y, x \sim -x) \tag{7.4456}$$

8.221 Descendants of 6.60

$7 + \gcd(p-1, 3)$ algebras

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-\omega ba, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4457}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-\omega ba, pa, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.4458}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-\omega ba, pa, pb-baa, pc-xbaa, pd, \text{ class } 3\rangle \text{ (} x \text{ FIXED with } x^2-\omega \text{ not a square } \bmod p) \tag{7.4459}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb, da, db-ca, dc-\omega ba, pa, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.4460}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb-baa, da, db-ca, dc-\omega ba, pa, pb, pc, pd, \text{ class } 3\rangle \tag{7.4461}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb-baa, da, db-ca, dc-\omega ba, pa, pb, pc-baa, pd, \text{ class } 3\rangle \tag{7.4462}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb-baa, da, db-ca, dc-\omega ba, pa, pb-baa, pc-xbaa, pd, \text{ class } 3\rangle \text{ (} x \text{ FIXED with } x^2-\omega \text{ not a square } \bmod p) \tag{7.4463}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb-baa, da, db-ca, dc-\omega ba, pa, pb, pc, pd-baa, \text{ class } 3\rangle \tag{7.4464}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb-baa, da, db-ca, dc-\omega ba, pa, pb, pc, pd-\omega baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4465}$$

$$\langle a,b,c,d \,|\, bab, bac, bad, cb-baa, da, db-ca, dc-\omega ba, pa, pb, pc, pd-\omega^2 baa, \text{ class } 3\rangle \ (p = 1 \bmod 3) \tag{7.4466}$$

8.222 Descendants of 6.63

4 algebras

$$\langle a, b, c, d \,|\, ba, ca, cb, da, db, dc, pb, pc, pd, \text{ class } 4\rangle \tag{7.4467}$$

$$\langle a, b, c, d \,|\, ba - p^3a, ca, cb, da, db, dc, pb, pc, pd, \text{ class } 4\rangle \tag{7.4468}$$

$$\langle a, b, c, d \,|\, ba, ca, cb - p^3a, da, db, dc, pb, pc, pd, \text{ class } 4\rangle \tag{7.4469}$$

$$\langle a, b, c, d \,|\, ba, ca, cb - p^3a, da - p^3a, db, dc, pb, pc, pd, \text{ class } 4\rangle \tag{7.4470}$$

8.223 Descendants of 6.67

$18 + 8\gcd(p-1, 3) + 3\gcd(p-1, 4)$ algebras

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb, pc - baaa, pd, \text{ class } 4\rangle \tag{7.4471}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb - baaa, pc, pd, \text{ class } 4\rangle \tag{7.4472}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb - \omega baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4473}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4474}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4475}$$

$$\langle a, b, c, d \,|\, bab, ca, cb, da, db, dc, pa - baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4476}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa, pb, pc - baaa, pd, \text{ class } 4\rangle \tag{7.4477}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa, pb - baaa, pc, pd, \text{ class } 4\rangle \tag{7.4478}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa, pb - \omega baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4479}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4480}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4481}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa - baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4482}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa - \omega baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4483}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa - \omega^2 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.4484}$$

$$\langle a, b, c, d \,|\, bab - baaa, ca, cb, da, db, dc, pa - \omega^3 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.4485}$$

$$\langle a,b,c,d \,|\, bab, ca, cb-baaa, da, db, dc, pa, pb, pc, pd-baaa, \text{ class } 4\rangle \tag{7.4486}$$

$$\langle a,b,c,d \,|\, bab, ca, cb-baaa, da, db, dc, pa, pb, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4487}$$

$$\langle a,b,c,d \,|\, bab, ca, cb-baaa, da, db, dc, pa, pb-baaa, pc, pd, \text{ class } 4\rangle \tag{7.4488}$$

$$\langle a,b,c,d \,|\, bab, ca, cb-baaa, da, db, dc, pa, pb-\omega baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4489}$$

$$\langle a,b,c,d \,|\, bab, ca, cb-baaa, da, db, dc, pa, pb-\omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4490}$$

$$\langle a,b,c,d \,|\, bab, ca, cb-baaa, da, db, dc, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4491}$$

$$\langle a,b,c,d \,|\, bab, ca, cb-baaa, da, db, dc, pa-baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4492}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa, pb, pc, pd-baaa, \text{ class } 4\rangle \tag{7.4493}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa, pb, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4494}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa, pb, pc-\omega baaa, pd, \text{ class } 4\rangle \tag{7.4495}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa, pb-baaa, pc, pd, \text{ class } 4\rangle \tag{7.4496}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa, pb-\omega baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4497}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa, pb-\omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4498}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4499}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa-baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4500}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa-\omega baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4501}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa-\omega^2 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.4502}$$

$$\langle a,b,c,d \,|\, bab-baaa, ca, cb-baaa, da, db, dc, pa-\omega^3 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.4503}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc-baaa, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4504}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc-baaa, pa, pb, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4505}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc-baaa, pa, pb-baaa, pc, pd, \text{ class } 4\rangle \tag{7.4506}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc-baaa, pa, pb-baaa, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4507}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4508}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc - baaa, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4509}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4510}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc - baaa, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4511}$$

$$\langle a,b,c,d \,|\, bab, ca, cb, da, db, dc - baaa, pa - baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4512}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4513}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb, pc - baaa, pd, \text{ class } 4\rangle \tag{7.4514}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - baaa, pc, pd, \text{ class } 4\rangle \tag{7.4515}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - baaa, pc - baaa, pd, \text{ class } 4\rangle \tag{7.4516}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4517}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega baaa, pc - baaa, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4518}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4519}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa, pb - \omega^2 baaa, pc - baaa, pd, \text{ class } 4\rangle \ (p = 1 \bmod 3) \tag{7.4520}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa - baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4521}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa - \omega baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4522}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa - \omega^2 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.4523}$$

$$\langle a,b,c,d \,|\, bab - baaa, ca, cb, da, db, dc - baaa, pa - \omega^3 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p = 1 \bmod 4) \tag{7.4524}$$

8.224 Descendants of 6.72

$17 + (p+7)\gcd(p-1,3) + \gcd(p-1,4)$ algebras

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb, pc, pd-baaa, \text{ class } 4\rangle \tag{7.4525}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4526}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4527}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb-baaa, pc, pd, \text{ class } 4\rangle \tag{7.4528}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb-baaa, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4529}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb-\omega baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4530}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb-\omega baaa, pc-baaa, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4531}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb-\omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4532}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa, pb-\omega^2 baaa, pc-baaa, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4533}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa-baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4534}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc, pa-baaa, pb, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4535}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb, pc, pd-baaa, \text{ class } 4\rangle \tag{7.4536}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4537}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4538}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb-baaa, pc, pd, \text{ class } 4\rangle \tag{7.4539}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb-baaa, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4540}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb-\omega baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4541}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb-\omega baaa, pc-baaa, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4542}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb-\omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4543}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa, pb-\omega^2 baaa, pc-baaa, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4544}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc, pa-baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4545}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb, pc, pd-baaa, \text{ class } 4\rangle \tag{7.4546}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb, pc, pd-\omega baaa, \text{ class } 4\rangle \tag{7.4547}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4548}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4549}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb-baaa, pc, pd, \text{ class } 4\rangle \tag{7.4550}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb-baaa, pc-baaa, pd, \text{ class } 4\rangle \tag{7.4551}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb-\omega baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4552}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb-\omega baaa, pc-baaa, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4553}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb-\omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4554}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa, pb-\omega^2 baaa, pc-baaa, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4555}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db, dc-baaa, pa-baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4556}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb, pc, pd-baaa, \text{ class } 4\rangle \tag{7.4557}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb, pc, pd-\omega baaa, \text{ class } 4\rangle \tag{7.4558}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb-xbaaa, pc-baaa, pd, \text{ class } 4\rangle \ (\text{all } x) \tag{7.4559}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb-xbaaa, pc-\omega baaa, pd, \text{ class } 4\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.4560}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb-xbaaa, pc-\omega^2 baaa, pd, \text{ class } 4\rangle \ (\text{all } x,\ p=1 \bmod 3) \tag{7.4561}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb-baaa, pc, pd, \text{ class } 4\rangle \tag{7.4562}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb-\omega baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4563}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb-\omega^2 baaa, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 3) \tag{7.4564}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4565}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa-baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4566}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa-\omega baaa, pb, pc, pd, \text{ class } 4\rangle \tag{7.4567}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa-\omega^2 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.4568}$$

$$\langle a,b,c,d \,|\, ca, cb-baa, bab, da, db-baaa, dc-baaa, pa-\omega^3 baaa, pb, pc, pd, \text{ class } 4\rangle \ (p=1 \bmod 4) \tag{7.4569}$$

8.225 Descendants of 6.9

7 algebras

$$\langle a,b,c,d \,|\, ba, ca, cb, da, db, dc, p^2b, pc, pd, \text{ class } 3\rangle \tag{7.4570}$$

$$\langle a,b,c,d \,|\, ba-p^2a, ca, cb, da, db, dc, p^2b, pc, pd, \text{ class } 3\rangle \tag{7.4571}$$

$$\langle a,b,c,d \,|\, ba, ca-p^2a, cb, da, db, dc, p^2b, pc, pd, \text{ class } 3\rangle \tag{7.4572}$$

$$\langle a,b,c,d \,|\, ba, ca, cb-p^2a, da, db, dc, p^2b, pc, pd, \text{ class } 3\rangle \tag{7.4573}$$

$$\langle a,b,c,d \,|\, ba, ca, cb-p^2a, da-p^2a, db, dc, p^2b, pc, pd, \text{ class } 3\rangle \tag{7.4574}$$

$$\langle a,b,c,d \,|\, ba, ca, cb, da, db, dc-p^2a, p^2b, pc, pd, \text{ class } 3\rangle \tag{7.4575}$$

$$\langle a,b,c,d \,|\, ba-p^2a, ca, cb, da, db, dc-p^2a, p^2b, pc, pd, \text{ class } 3\rangle \tag{7.4576}$$

8.226 Descendants of 5.1

178 algebras if $p=3$, and $p^2+15p+125$ otherwise

$$\langle a,b,c,d,e \,|\, ba, ca, da, ea, cb, db, eb, dc, ec, ed, pc, pd, pe, \text{ class } 2\rangle \tag{7.4577}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4578}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pd, pe, \text{ class } 2\rangle \tag{7.4579}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pd-ba, pe, \text{ class } 2\rangle \tag{7.4580}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pa-ba, pb, pd, pe, \text{ class } 2\rangle \tag{7.4581}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4582}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc, ec, ed, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4583}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc-ba, ec, ed, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4584}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc-ba, ec, ed, pa, pb, pc, pd, \text{ class } 2\rangle \tag{7.4585}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc-ba, ec, ed, pa-ba, pb, pc, pd, \text{ class } 2\rangle \tag{7.4586}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc-ba, ec, ed, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4587}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc-ba, ec, ed, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4588}$$

$$\langle a,b,c,d,e \,|\, ca, da, ea, cb, db, eb, dc-ba, ec, ed, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4589}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4590}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ba, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4591}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4592}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc-\omega ba, ec, ed, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4593}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4594}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4595}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa-ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4596}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4597}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4598}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4599}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4600}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4601}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4602}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba-ca, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4603}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba-ca, pb-ba-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4604}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4605}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe-ba-ca, \text{ class } 2\rangle \tag{7.4606}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa-ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4607}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4608}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa-ca, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4609}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb, pc-ca, pd, pe, \text{ class } 2\rangle \tag{7.4610}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4611}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe-ca, \text{ class } 2\rangle \tag{7.4612}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc-\omega ba, ea, eb, ec, ed, pa-ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4613}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc-\omega ba, ea, eb, ec, ed, pa, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4614}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4615}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4616}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-ca, pd, pe, \text{ class } 2\rangle \tag{7.4617}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc, pd-ba, pe, \text{ class } 2\rangle \tag{7.4618}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd-ba, pe, \text{ class } 2\rangle \tag{7.4619}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4620}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ca, pd-ca, pe, \text{ class } 2\rangle \tag{7.4621}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4622}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ca, pd, pe, \text{ class } 2\rangle \tag{7.4623}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4624}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ca, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4625}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4626}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd, pe-ba, \text{ class } 2\rangle \tag{7.4627}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-\omega ba, pd, pe-ba, \text{ class } 2\rangle \tag{7.4628}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4629}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4630}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ba, pb, pc, pd, pe, \text{ class } 2\rangle \tag{7.4631}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc, pd-ba, pe, \text{ class } 2\rangle \tag{7.4632}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed, pa, pb, pc, pd-ba, pe-ca, \text{ class } 2\rangle \tag{7.4633}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa-ca, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4634}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa-ba, pb-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4635}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb-ba, pc-xca, pd, pe, \text{ class } 2\rangle \ (x \neq 0, x \sim x^{-1}) \tag{7.4636}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb-ba-ca, pc-ca, pd, pe, \text{ class } 2\rangle \tag{7.4637}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb-\omega ca, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4638}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb-xca, pc-ba-ca, pd, pe, \text{ class } 2\rangle \ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.4639}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa-ba, pb, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4640}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4641}$$

$$\langle a,b,c,d,e \,|\, cb, da, db, dc, ea, eb, ec, ed, pa, pb, pc-ba, pd-ca, pe, \text{ class } 2\rangle \tag{7.4642}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba, pb-ba, pc-ca, pd-ca, pe, \text{ class } 2\rangle \tag{7.4643}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4644}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba+ca, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4645}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa+ca, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4646}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4647}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba-ca, pb-ba-xca, pc, pd, pe, \text{ class } 2\rangle \ (x \neq 0, 1) \tag{7.4648}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba-ca, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4649}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba-ca, pb-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4650}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ba, pb-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4651}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ca, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4652}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ca, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4653}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa+ca, pb, pc, pd-ca, pe-ba, \text{ class } 2\rangle \tag{7.4654}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa, pb, pc, pd-ca, pe-ba, \text{ class } 2\rangle \tag{7.4655}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa-ca, pb, pc, pd, pe-ba-ca, \text{ class } 2\rangle \tag{7.4656}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ba, dc, ea, eb, ec, ed, pa, pb, pc, pd-ca, pe-ba-ca, \text{ class } 2\rangle \tag{7.4657}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa-ba, pb-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4658}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa-ca, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4659}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb-ca, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4660}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb-\omega ca, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4661}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa-ba, pb, pc-ca, pd, pe, \text{ class } 2\rangle \tag{7.4662}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb-xba, pc-ca, pd, pe, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4663}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb, pc-ba, pd-ca, pe, \text{ class } 2\rangle \tag{7.4664}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa-ca, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4665}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb, pc-ca, pd, pe-ba, \text{ class } 2\rangle \tag{7.4666}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa-ba, pb, pc, pd, pe-ca, \text{ class } 2\rangle \tag{7.4667}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc, ea, eb, ec, ed, pa, pb, pc-ba, pd, pe-ca, \text{ class } 2\rangle \tag{7.4668}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc-\omega ba, ea, eb, ec, ed, pa, pb, pc-ba, pd-ca, pe, \text{ class } 2\rangle \tag{7.4669}$$

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc-\omega ba, ea, eb, ec, ed, pa, pb-ba, pc-xba-yca, pd, pe, \text{ class } 2\rangle \tag{7.4670}$$

In presentation 7.4670 we have two parameters x, y with $y \neq 0$. Parameter pairs (x, y) and (z, t) give isomorphic algebras if and only if

$$\begin{pmatrix} 1 & 0 \\ z & t \end{pmatrix} = \begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix} \begin{pmatrix} 1 & 0 \\ x & y \end{pmatrix} \begin{pmatrix} \mu+\nu x & \nu y \\ \omega\nu y & \mu+\nu x \end{pmatrix}^{-1}$$

for some non-singular matrix $\begin{pmatrix} \mu & \nu \\ \omega\nu & \mu \end{pmatrix}$.

$$\langle a,b,c,d,e \,|\, cb, da, db-ca, dc-\omega ba, ea, eb, ec, ed, pa-ca, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4671}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc, pd-ba, pe-ca, \text{ class } 2\rangle \tag{7.4672}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd-ba, pe-ca, \text{ class } 2\rangle \tag{7.4673}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ca, pd-ba, pe-ca, \text{ class } 2\rangle \tag{7.4674}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ca, pb, pc, pd-ba, pe, \text{ class } 2\rangle \tag{7.4675}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4676}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ca, pb, pc-ba, pd-ba, pe, \text{ class } 2\rangle \tag{7.4677}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd-ca, pe, \text{ class } 2\rangle \tag{7.4678}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb, pc-ca, pd-ca, pe, \text{ class } 2\rangle \tag{7.4679}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ca, pd-ba, pe, \text{ class } 2\rangle \tag{7.4680}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb, pc-ca, pd-ba-ca, pe, \text{ class } 2\rangle \tag{7.4681}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ca, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4682}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb, pc-ca, pd, pe, \text{ class } 2\rangle \tag{7.4683}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ba, pc-xca, pd, pe, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4684}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ba, pc-xca, pd-ca, pe, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4685}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4686}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ca, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4687}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ca, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4688}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ba, pc-ba-ca, pd, pe, \text{ class } 2\rangle \tag{7.4689}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ba, pc-ba-ca, pd-ca, pe, \text{ class } 2\rangle \tag{7.4690}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb-ca, pc, pd, pe, \text{ class } 2\rangle \tag{7.4691}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc, pd-ba, pe, \text{ class } 2\rangle \tag{7.4692}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb-ca, pc, pd-ba, pe, \text{ class } 2\rangle \tag{7.4693}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb-ca, pc-ca, pd, pe, \text{ class } 2\rangle \tag{7.4694}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-ca, pd-ba, pe, \text{ class } 2\rangle \tag{7.4695}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa-ba, pb-ca, pc-ca, pd-ba, pe, \text{ class } 2\rangle \tag{7.4696}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4697}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-ba, pd-ba, pe, \text{ class } 2\rangle \tag{7.4698}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-\omega ba, pd, pe, \text{ class } 2\rangle \tag{7.4699}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-\omega ba, pd-ba, pe, \text{ class } 2\rangle \tag{7.4700}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-xba-ca, pd, pe, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4701}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db, eb, dc, ec, ed-ba, pa, pb-ca, pc-xba-ca, pd-ba, pe, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4702}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ba, pb, pc, pd-xca, pe, \text{ class } 2\rangle \ (x \neq 0, x \sim x^{-1}) \tag{7.4703}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ba-ca, pb, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4704}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-\omega ca, pb, pc, pd-ba, pe, \text{ class } 2\rangle \tag{7.4705}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-xca, pb, pc, pd-ba-ca, pe, \text{ class } 2\rangle \ (\text{all } x \text{ with } 1+4x \text{ not a square } \bmod p) \tag{7.4706}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ca, pb, pc, pd, pe-ba, \text{ class } 2\rangle \tag{7.4707}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc, pd-ca, pe-ba, \text{ class } 2\rangle \tag{7.4708}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ca, pb, pc, pd-ca, pe-ba, \text{ class } 2\rangle \tag{7.4709}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ca, pb, pc-ba, pd, pe-ba, \text{ class } 2\rangle \tag{7.4710}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-xca, pb, pc-ba, pd-ca, pe-ba, \text{ class } 2\rangle \ (x \sim -x) \tag{7.4711}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ca, pb, pc-\omega ba, pd, pe-ba, \text{ class } 2\rangle \tag{7.4712}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-xca, pb, pc-\omega ba, pd-ca, pe-ba, \text{ class } 2\rangle \ (x \sim -x) \tag{7.4713}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4714}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ca, pb-ba, pc, pd, pe, \text{ class } 2\rangle \tag{7.4715}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ca, pb-ba, pc, pd-ca, pe, \text{ class } 2\rangle \tag{7.4716}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd-ca, pe, \text{ class } 2\rangle \tag{7.4717}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa-ca, pb, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4718}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-xba-ca, pd, pe-ba-yca, \text{ class } 2\rangle \ (\text{all } x,\ y \text{ with } xy \neq 1, (x,\ y) \sim (y,x)) \tag{7.4719}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-xba, pd, pe-ba-ca, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4720}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd, pe-ca, \text{ class } 2\rangle \tag{7.4721}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb, pc-ba, pd, pe-\omega ca, \text{ class } 2\rangle \ (p = 1 \bmod 3) \tag{7.4722}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc-tba-xca, pd, pe-yba-zca, \text{ class } 2\rangle \tag{7.4723}$$

We consider the orbits of matrices

$$\begin{pmatrix} u & v \\ t & x \\ y & z \end{pmatrix}$$

where $(tz-xy)^2-(ux-vt)(uz-vy)$ is not a square, under the action of non-singular matrices $\begin{pmatrix} \alpha & \varepsilon \\ \gamma & \eta \end{pmatrix}$ given by

$$\begin{pmatrix} u & v \\ t & x \\ y & z \end{pmatrix} \to (\alpha\eta-\gamma\varepsilon)^{-2} \begin{pmatrix} (\alpha\eta+\gamma\varepsilon) & 2\gamma\eta & -2\alpha\varepsilon \\ \varepsilon\eta & \eta^2 & -\varepsilon^2 \\ -\alpha\gamma & -\gamma^2 & \alpha^2 \end{pmatrix} \begin{pmatrix} u & v \\ t & x \\ y & z \end{pmatrix} \begin{pmatrix} \eta & -\gamma \\ -\varepsilon & \alpha \end{pmatrix}.$$

Each orbit contains a matrix with $u=0$ and $v=1$, and we pick one matrix of this form to obtain a representative set of parameters t,x,y,z for presentation 7.4723. The number of orbits is 4 when $p=3$, $(p^2-1)/2$ when $p=1 \bmod 3$, and $(p^2+1)/2$ when $p=2 \bmod 3$.

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc, pd, pe-xba, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4724}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc, pd-ca, pe-xba, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4725}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc, pd, pe+ba-ca, \text{ class } 2\rangle \tag{7.4726}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc, pd-ca, pe+ba-ca, \text{ class } 2\rangle \tag{7.4727}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc-ba, pd, pe, \text{ class } 2\rangle \tag{7.4728}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc-ba, pd-ba, pe, \text{ class } 2\rangle \tag{7.4729}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc-\omega ba, pd, pe, \text{ class } 2\rangle \tag{7.4730}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc-\omega ba, pd-ba, pe, \text{ class } 2\rangle \tag{7.4731}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc-xba-ca, pd, pe, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4732}$$

$$\langle a,b,c,d,e \,|\, da, ea, cb, db-ca, eb, dc, ec, ed-ba, pa, pb-ca, pc-xba-ca, pd-ba, pe, \text{ class } 2\rangle \ (x \neq 0) \tag{7.4733}$$

8.227 Descendants of 6.2

5 algebras

$$\langle a,b,c,d,e \,|\, ba, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4734}$$

$$\langle a,b,c,d,e \,|\, ba-p^2a, ca, da, ea, cb, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4735}$$

$$\langle a,b,c,d,e \,|\, ba, ca, da, ea, cb-p^2a, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4736}$$

$$\langle a,b,c,d,e \,|\, ba, ca, da-p^2a, ea, cb-p^2a, db, eb, dc, ec, ed, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4737}$$

$$\langle a,b,c,d,e \,|\, ba, ca, da, ea, cb-p^2a, db, eb, dc, ec, ed-p^2a, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4738}$$

8.228 Descendants of 6.3

25 algebras

$$\langle a,b,c,d,e \,|\, ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4739}$$

$$\langle a,b,c,d,e \,|\, ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa-baa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4740}$$

$$\langle a,b,c,d,e \,|\, ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb-baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4741}$$

$$\langle a,b,c,d,e \,|\, ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb-\omega baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4742}$$

$$\langle a,b,c,d,e \,|\, ca, cb, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc-baa, pd, pe, \text{ class } 3\rangle \tag{7.4743}$$

$$\langle a,b,c,d,e \,|\, ca, cb-baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4744}$$

$$\langle a,b,c,d,e \,|\, ca, cb-baa, bab, da, db, dc, ea, eb, ec, ed, pa-baa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4745}$$

$$\langle a,b,c,d,e \,|\, ca, cb-baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb-baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4746}$$

$$\langle a,b,c,d,e \,|\, ca, cb-baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb-\omega baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4747}$$

$$\langle a,b,c,d,e \,|\, ca, cb-baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc-baa, pd, pe, \text{ class } 3\rangle \tag{7.4748}$$

$$\langle a,b,c,d,e \,|\, ca, cb-baa, bab, da, db, dc, ea, eb, ec, ed, pa, pb, pc, pd-baa, pe, \text{ class } 3\rangle \tag{7.4749}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4750}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa-baa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4751}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa, pb-baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4752}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa, pb-\omega baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4753}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa, pb, pc-baa, pd, pe, \text{ class } 3\rangle \tag{7.4754}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa, pb-baa, pc-baa, pd, pe, \text{ class } 3\rangle \tag{7.4755}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa, pb-\omega baa, pc-baa, pd, pe, \text{ class } 3\rangle \tag{7.4756}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb, ec, ed, pa, pb, pc, pd, pe-baa, \text{ class } 3\rangle \tag{7.4757}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb-baa, ec, ed, pa, pb, pc, pd, pe-baa, \text{ class } 3\rangle \tag{7.4758}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb-baa, ec, ed, pa, pb, pc-baa, pd, pe, \text{ class } 3\rangle \tag{7.4759}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb-baa, ec, ed, pa, pb-baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4760}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb-baa, ec, ed, pa, pb-\omega baa, pc, pd, pe, \text{ class } 3\rangle \tag{7.4761}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb-baa, ec, ed, pa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4762}$$

$$\langle a,b,c,d,e \,|\, bab, ca, cb, da, db, dc-baa, ea, eb-baa, ec, ed, pa-baa, pb, pc, pd, pe, \text{ class } 3\rangle \tag{7.4763}$$

8.229 Descendants of 6.1

9 algebras

$$\langle a,b,c,d,e,f \mid ba,ca,da,ea,fa,cb,db,eb,fb,dc,ec,fc,ed,fd,fe,pb,pc,pd,pe,pf,\text{ class } 2\rangle \tag{7.4764}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc,ec,fc,ed,fd,fe,pa,pb,pc,pd,pe,pf,\text{ class } 2\rangle \tag{7.4765}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc,ec,fc,ed,fd,fe,pa-ba,pb,pc,pd,pe,pf,\text{ class } 2\rangle \tag{7.4766}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc,ec,fc,ed,fd,fe,pa,pb,pc-ba,pd,pe,pf,\text{ class } 2\rangle \tag{7.4767}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc-ba,ec,fc,ed,fd,fe,pa,pb,pc,pd,pe,pf,\text{ class } 2\rangle \tag{7.4768}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc-ba,ec,fc,ed,fd,fe,pa-ba,pb,pc,pd,pe,pf,\text{ class } 2\rangle \tag{7.4769}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc-ba,ec,fc,ed,fd,fe,pa,pb,pc,pd,pe-ba,pf,\text{ class } 2\rangle \tag{7.4770}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc-ba,ec,fc,ed,fd,fe-ba,pa,pb,pc,pd,pe,pf,\text{ class } 2\rangle \tag{7.4771}$$

$$\langle a,b,c,d,e,f \mid ca,da,ea,fa,cb,db,eb,fb,dc-ba,ec,fc,ed,fd,fe-ba,pa-ba,pb,pc,pd,pe,pf,\text{ class } 2\rangle \tag{7.4772}$$

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